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BILINEAR *R*-PARITY VIOLATION IN NEUTRINOLESS DOUBLE BETA DECAY

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1 Introduction

In the standard model (SM) of the electro-weak interactions the baryon B and lepton L numbers conservation is protected to all orders of perturbation theory by an accidental $U_{1B}\times U_{1L}$ symmetry existing at the level of renormalizable operators. In the minimal supersymmetric (SUSY) extension of the standard model (MSSM) [1] this symmetry is absent and the L and B violating processes are not forbidden. A conventional way of eliminating the phenomenologically dangerous L,B-violation in this case exploits a discrete symmetry known as R-parity [2], [3] which is imposed on the model. This is a multiplicative Z_2 symmetry defined as $R_p = (-1)^{3B+L+2S}$, where S, B and L are the spin, the baryon and the lepton quantum numbers. R-parity conservation has a distinctive phenomenology. It prevents lepton and baryon number violating processes, the superpartners are produced in associated production and the lightest SUSY particle is stable. The latter leads to the celebrated missing E_T signature of the SUSY event in high energy detector and renders a cold dark matter particle candidate. Although desirable for many reasons the R-parity conservation has no well motivated theoretical grounds.

On the other hand relaxing the R-parity conservation we may get a new insight into the long standing problems of particle physics, in particular, to the neutrino mass problem. Remarkable, that in this framework neutrino can acquire the tree level supersymmetric mass via the mixing with the gauginos and higgsinos at the weak-scale [3], [4]-[7]. This mechanism does not involve the physics at the large energy scales $M_{int} \sim \mathcal{O}(10^{12}\text{GeV})$ in contrast to the see-saw mechanism but relates the neutrino mass to the weak-scale physics accessible for the experimental searches.

The R-parity can be broken ($\not{R}_p$) either explicitly [3] or spontaneously [8]. The first option allows one to establish the most general phenomenological consequences of R-parity violation while a predictive power in this case is rather weak due to the large number of free parameters. Spontaneous realization of $\not{R}_p$ SUSY is much more predictive scheme leading to many interesting phenomenological consequences [9]. However, it represents a particular model of the R-parity violation. At present it is an open question which underlying high-energy scale physics stands behind the R-parity, protecting or violating it at the weak scale.

Many aspects of the $\not{R}_p$ SUSY models in high and low energy processes had been investigated in the literature [3]-[14], [16]-[20].

Recently, a growing interest to the supersymmetric models without R-parity was stimulated by the exciting news from the HERA experiments, reported the anomaly in deep inelastic e^+p-scattering [15] which can be elegantly explained within these theoretical framework in terms of the lepton number violating interactions.

Since the lepton number is not conserved without R-parity some low-energy exotic processes become possible within the $\not{R}_p$ MSSM. Among them the neutrinoless nuclear double beta decay ($0\nu\beta\beta$) is known to be very sensitive to the certain $\not{R}_p$ interactions [18]. Provided an unprecedented accuracy of the modern $0\nu\beta\beta$-decay experiments [21] this allows one to establish stringent constraints on the $\not{R}_p$ SUSY [16]-[20].

In the present paper we consider the implications of the bilinear lepton-Higgs $\not{R}_p$ terms on $0\nu\beta\beta$-decay. In the general case of the explicitly broken R-parity these terms are present in the superpotential and in the soft SUSY breaking potential. Previously the main attention was paid to the phenomenology of the trilinear $\not{R}_p$ Yukawa couplings. It was widely believed that the bilinear $\not{R}_p$ terms can be rotated away by a proper field redefinition.

However, it is not the case in the presence of the soft SUSY breaking interactions [6], [9]. It was realized that the bilinear $\not{R}_p$ violation, generically leading to the non-zero vacuum expectation values (VEV) of the sneutrino fields and to the lepton-gaugino-higgsino and slepton-Higgs mixing, provides a number of interesting phenomenological issues [4]-[7], [11]-[13].

In particular, this mixing generates the new effective lepton number violation operators which contribute to the nuclear $0\nu\beta\beta$-decay. In what follows we derive these operators and analyze their net effect in the presence of the nuclear media.

The paper is organized as follows. Basic ingredients of the $\not{R}_p$MSSM with the general setting of the explicit R-parity violation are shortly described in Section 2. In Section 3 we discuss the bilinear $\not{R}_p$ mechanism of the nuclear $0\nu\beta\beta$-decay. Here we analyze all the tree-level $\not{R}_p$ MSSM contributions to the $0\nu\beta\beta$-decay amplitude. We start with the quark level and derive the corresponding low energy effective Lagrangian. In Section 4 we take into account the effect of nuclear structure and derive the corresponding nuclear matrix elements. Then we calculate their values within the renormalized Quasiparticle Random Phase Approximation (pn-RQRPA) [22]. The pn-RQRPA is an extension of the pn-QRPA by taking into account the effects of the Pauli principle for the fermion pairs. In this approach the sensitivity of the nuclear matrix elements to the details of the nuclear Hamiltonian is reduced considerably. Using experimental lower bound on the ^{76}Ge half-life we extract in Section 5 stringent constraints on the 1st generation lepton-Higgs mixing mass parameter and on the electron sneutrino VEV. We close our discussion with the short comments on some implications of these constraints for the other experiments.

2 Minimal SUSY model with R-parity violation

In order to set up our notations let us briefly recapitulate the main ingredients of the minimal SUSY standard model (MSSM) with explicit R-parity violation ($\not{R}_p$ MSSM).

The R_p violation is introduced into the theory through the superpotential and soft SUSY breaking sector.

For the minimal MSSM field contents the most general gauge invariant form of the renormalizable superpotential reads

$$W = W_{R_p} + W_{\not{R}_p}. \tag{1}$$

The R_p conserving part has the standard MSSM form

$$W_{R_p} = h_L H_1 L E^c + h_D H_1 Q D^c + h_U H_2 Q U^c + \mu H_1 H_2. \tag{2}$$

Here L, Q stand for lepton and quark doublet left-handed superfields while E^c, U^c, D^c for lepton and *up*, *down* quark singlet superfields; H_1 and H_2 are the Higgs doublet superfields with a weak hypercharge $Y = -1,\ +1$, respectively. Summation over the generations is implied.

The R_p violating part of the superpotential (1) can be written as [2], [3]

$$W_{\not{R}_p} = \lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k + \mu_i L_j H_2 + \lambda''_{ijk} U^c_i D^c_j D^c_k, \tag{3}$$

The coupling constants λ (λ'') are antisymmetric in the first (last) two indices. The first two terms violate lepton number while the last one violates baryon number conservation.

Another source of the R-parity violation is the soft supersymmetry breaking part of the scalar potential. It contains the $\not{R}_p$-terms

$$\begin{aligned} V^{soft}_{\not{R}_p} = \tilde\lambda_{ijk} \tilde L_i \tilde L_j \tilde E^c_k + \tilde\lambda'_{ijk} \tilde L_i \tilde Q_j \tilde D^c_k + \tilde\lambda''_{ijk} \tilde U^c_i \tilde D^c_j \tilde D^c_k + \tilde\mu_i \tilde L_i H_2 + \\ + m^2_{LH} \tilde L_i H_1^\dagger + \text{H.c.} \end{aligned} \tag{4}$$

The simultaneous presence of lepton and baryon number violating terms in Eqs. (3), (4) (unless the couplings are very small) would cause unsuppressed proton decay. Therefore, either the lepton or the baryon number violating couplings can be present. There may exist in the theory an underlying discrete symmetry such as the B-parity [3], [23] which forbids dangerous combinations of these couplings. Henceforth we simply set $\lambda'' = \tilde\lambda'' = 0$.

The remaining R-parity conserving part of the soft SUSY breaking sector includes the scalar field interactions

$$\begin{aligned} V^{soft}_{R_p} = & \sum_{i=scalars} m_i^2 |\phi_i|^2 + h_L A_L H_1 \tilde L \tilde E^c + h_D A_D H_1 \tilde Q \tilde D^c - \\ & - h_U A_U H_2 \tilde Q \tilde U^c - \mu B H_1 H_2 + \text{ H.c.} \end{aligned} \tag{5}$$

and the "soft" gaugino mass terms

$$\mathcal{L}_{GM} = -\frac{1}{2}\left[M_1 \tilde B \tilde B + M_2 \tilde W^k \tilde W^k + M_3 \tilde g^a \tilde g^a\right] - \text{ H.c.} \tag{6}$$

As usual, $M_{3,2,1}$ denote the masses of the $SU(3)\times SU(2)\times U(1)$ gauginos $\tilde g, \tilde W, \tilde B$ while m_i stand for the masses of the scalar fields. The gluino $\tilde g$ soft mass M_3 coincides in this framework with its physical mass denoted hereafter as $m_{\tilde g} = M_3$. A_L, A_D, A_U and B in Eq. (5) are trilinear and bilinear "soft" supersymmetry breaking parameters. All these quantities are free SUSY model parameters which due to the renormalization effect depend on the energy scale.

In this paper we assume for simplicity the universal gaugino soft masses at the grand unification scale M_{GUT}. At the weak scale this leads to the following relations

$$M_1 = (5/3)\tan\theta_W^2 M_2, \quad M_2 \approx 0.3 M_3, \tag{7}$$

An impact of the R-parity violation on the low energy phenomenology is twofold. First, it leads the lepton number (LNV) and lepton flavor (LFV) violating interactions directly from the trilinear terms in $W_{\not{R}_p}$. Second, bilinear terms in $W_{\not{R}_p}$ and in $V^{soft}_{\not{R}_p}$ generate the non-zero vacuum expectation value for the sneutrino fields $\langle\tilde\nu_i\rangle \neq 0$ and cause neutrino-neutralino as well as electron-chargino mixing. The mixing brings in the new LNV and LFV interactions in the physical mass eigenstate basis. Below we will specify those interactions which are relevant for the $0\nu\beta\beta$-decay.

The trilinear terms of the R-parity breaking part of the superpotential $W_{\not{R}_p}$ lead to the following $\Delta L = 1$ lepton-quark operators

$$\begin{aligned} \mathcal{L}_\lambda = & \lambda_{ijk}[\tilde\nu_{iL}\bar e_k P_L e_j + \tilde e_{jL}\bar e_k P_L \nu_i + \tilde e^*_{kR}\bar e_j P_R \nu^c_i - (i \leftrightarrow j)] + \\ + & \lambda'_{ijk}[\tilde\nu_{iL}\bar d_k P_L d_i + \tilde d_{jL}\bar d_k P_L \nu_i + \tilde d^*_{kR}\bar d_j P_R \nu^c_i - \tilde e_{iL}\bar d_k P_L u_j \\ - & \tilde u_{jL}\bar d_k P_L e_i - \tilde d^*_{kR}\bar u_j P_R e^c_i] + \text{ H.c.} \end{aligned} \tag{8}$$

Here, as usual $P_{L,R} = (1 \mp \gamma_5)/2$.

The presence of the bilinear terms in the Eqs. (3),(4) leads to the terms in the scalar potential linear in the sneutrino fields $\tilde{\nu}_i$. As a result, at the minimum of the potential $\langle\tilde{\nu}_i\rangle \neq 0$. Thus, the MSSM vertices $\tilde{Z}\nu\tilde{\nu}$ and $\tilde{W}e\tilde{\nu}$ create the gaugino-lepton mixing mass terms $\tilde{Z}\nu\langle\tilde{\nu}\rangle, \tilde{W}e\langle\tilde{\nu}\rangle$ (with $\tilde{W}, \tilde{Z}$ being wino and zino fields). Combining this terms with the lepton-higgsino $\mu_i L_i \tilde{H}_1$ mixing from the superpotential Eq. (3) we end up with 7×7 neutral fermion and 5×5 charged fermion mass matrices (see Appendix A). The mass eigenstate fields can be written in the form

$$\Psi_{(0)i} = \Xi_{ij}\ \Psi'_{(0)j}, \qquad \Psi_{(\pm)i} = \Delta^{\pm}_{ij}\ \Psi'_{(\pm)j}, \tag{9}$$

with the weak eigenstate fields in two component notation

$$\Psi^T_{(0)} = (\nu_i,\ -i\lambda',\ -i\lambda_3,\ \tilde{H}^0_1,\ \tilde{H}^0_2), \tag{10}$$
$$\Psi^T_{(-)} = (e^-_L,\ \mu^-_L,\ \tau^-_L,\ -i\lambda_-,\ \tilde{H}^-_1), \tag{11}$$
$$\Psi^T_{(+)} = (e^+_L,\ \mu^+_L,\ \tau^+_L,\ -i\lambda_+,\ \tilde{H}^+_2). \tag{12}$$

Here ν_i are the neutrino fields, λ' and λ_3, λ_- are the U_{1Y} and SU_{2L} gauginos, respectively while higgsinos are denoted as $\tilde{H}^0_{1,2}, \tilde{H}^\pm_{1,2}$. The mixing matrices Ξ and $\Delta^\pm$ diagonalize the neutralino-neutrino and the chargino-charged lepton mass matrices respectively. The lightest mass eigenstates are identified with the physical neutrinos and the charged leptons. Remarkable, that as a result of the minimal field content and the gauge invariance the neutral fermion mass matrix $\mathcal{M}_0$ (A.4) before diagonalization has such a texture that its first three rows and the last one are linearly dependent and, as a result, two neutrino mass eigenstates are degenerate massless states. The third neutrino state acquires the tree level mass which approximate form is (see Appendix B)

$$m_\nu = \frac{2}{3}\frac{g_1^2 M_2}{Det M_\chi}|\vec{\Lambda}|^2, \tag{13}$$

It is natural to identify the massive neutrino state with the tau neutrino ν_τ while the two massless states with the ν_e and ν_μ. The $\nu_e - \nu_\mu$ mass degeneracy is lifted by the 1-loop corrections as well as by the non-renormalizable terms in the superpotential giving to $\nu_{e,\mu}$ the small non-equal masses [7]. As to the tau neutrino mass in Eq. (13) it is subject to the experimental constraint $m_{\nu_\tau} \leq 23$MeV [24]. Assuming no cancellation in Eq. (13) this leads to the upper bounds

$$\mu_i \lesssim 15\text{GeV}, \qquad \langle\tilde{\nu}_i\rangle \lesssim 7\text{GeV}. \tag{14}$$

at the typical sample values of the MSSM parameters $\mu \sim M_2 \sim M_W$. Of course, these bounds are only indicative and may essentially vary from point to point in the MSSM parameter space.

The m_{ν_τ} constraints can be evaded assuming an approximate alignment between two vectors $a_i = (\mu_i, \mu)$ and $b_i = (\langle\tilde{\nu}_i\rangle, \langle H^0_1\rangle)$ which leads to the cancellation in Eq. (13) since $|\vec{\Lambda}|^2 = |\vec{a}|^2|\vec{b}|^2 - \left(\vec{a}\cdot\vec{b}\right)^2$. This might be guaranteed by a special global symmetry [5] or by some dynamical reasons [4].

Rotating the MSSM Lagrangian to the mass eigenstate basis one obtains the RPM generated lepton number violating interactions which bring many interesting implications for the low and high energy phenomenology. Below we are studying they contribution to the $0\nu\beta\beta$-decay.

3 LH-induced $0\nu\beta\beta$-decay. Quark level transitions

We have analyzed all the possible tree level contributions to the $0\nu\beta\beta$-decay amplitude which include the RPM interactions and the superpotential λ, λ' couplings from Eq. (8). The leading diagrams are presented in the Fig. 1. The diagrams in Fig.1(a.b) incorporate only the RPM generated vertices, and in Fig. 1(c.d) these vertices are accompanied by one λ' type vertex (on the top of the diagrams). The diagram in Fig. 1(a) has in the intermediate state either neutrinos or neutralinos and two W-bosons while the diagrams in Fig. 1(c,d) neutrinos, squarks/selectron and one W-boson. The diagram in Fig.1(b) is mediated by the gluino and double squark exchange. The diagram Fig. 1(a) with the neutrino exchange is the conventional Majorana neutrino contribution to the $0\nu\beta\beta$-decay. Recall that in the $R_p\!\!\!\!/\ $ MSSM with the bilinear R-parity violation the neutrino masses and mixing angles are derived at the tree level in terms of $\mu_i, \langle\tilde{\nu}_i\rangle$ and the MSSM parameters (see Appendix A). Therefore, this contribution inherently pertains to this model. We did not include in this list those diagrams which do not contain RPM vertices. These diagrams constructed of the λ, λ' couplings were previously analyzed in Refs. [16]-[20]. All the other diagrams in this order of perturbation theory have extra suppression factors and, therefore, can be neglected. The suppression factors originate from the smallness of neutrino mass, when it appears in a positive power, from the 1st generation left-right sfermion mixing proportional to $m_{u,d,e}/M_{SUSY}$ and/or from the fermion-sfermion-higgsino couplings proportional to $m_{u,d,e}/M_W$ with $m_{u,d,e}$ being the u,d quark and the electron masses respectively while M_{SUSY} denotes the typical SUSY breaking mass scale.

Now let us specify those RPM generated operators which are encountered in the diagrams in Fig. 1. They are

$$\mathcal{L}_{LH} = -\frac{g_2}{\sqrt{2}}\kappa_n W^-_\mu \bar{e}\gamma^\mu P_L \chi_n + \tag{15}$$
$$+\sqrt{2}g_2\left(\beta^d_k \bar{\nu}_k P_R d\, \tilde{d}^*_R + \beta^u_k \bar{\nu}_k P_R u^c\, \tilde{u}_L + \beta^e_{ki}\bar{\nu}_k P_R e^c\, \tilde{e}_{Li} + \zeta\bar{u}P_R e^c\, \tilde{d}_L\right) + H.c.$$

The subscripts k, i denote generations.

The first term is generated from the standard model $W-e-\nu$ and the MSSM $W-\chi^\pm-\chi$ interaction terms while the rest originates from the MSSM neutralino (chargino)-fermion-sfermion interactions $\chi - q - \tilde{q}$, $\chi^\pm - q - \tilde{q}$ (for the MSSM Lagrangian see [1]).

Note that the trilinear fermion-sfermion couplings in $\mathcal{L}_{LH}$ are not present among the superpotential trilinear λ, λ' terms in Eq. (8).

The coefficients in Eqs. (15) depend on the mixing matrix elements introduced in Eq. (9):

$$\kappa_n = \sum_{j=1}^{3}\Delta^-_{1j}\Xi^*_{n+3,j} + \sqrt{2}\Delta^-_{14}\Xi^*_{n+3,5} + \Delta^-_{15}\Xi^*_{n+3,6}, \tag{16}$$
$$\beta^e_{ki} = -\frac{1}{\sqrt{2}}\Delta^-_{14}\Xi_{ki} + \frac{1}{2}\left(\tan\theta_W\Xi_{k4} + \Xi_{k5}\right)\delta_{i1},$$
$$\beta^u_k = -\frac{1}{6}\left(\tan\theta_W\Xi_{k4} + 3\Xi_{k5}\right),$$
$$\beta^d_k = -\frac{1}{3}\tan\theta_W\Xi_{k4}, \quad \zeta = -\frac{1}{\sqrt{2}}\Delta^-_{14}.$$

In what follows, for the derivation of the constraints on parameters $\langle\tilde{\nu}_i\rangle, \mu_i$ characterizing the bilinear $R_p\!\!\!\!/\ $ we employ the approximate analytical diagonalization method of the Ref.

[12]. It allows one to represent the mixing matrices in a convenient analytic form and express the dependence of the coefficients in Eqs. (16) on the afore-mentioned $R\!\!\!/_p$ parameters explicitly. In the leading order in small parameters μ_i/M_Z, $\langle\tilde{\nu}_i\rangle/M_Z$ we obtain

$$\begin{aligned}
\kappa_n &= \xi^*_{1k}N^*_{nk} - \sqrt{2}\xi^L_{11}N^*_{n2} - \xi^L_{12}N^*_{n3}, \qquad (17)\\
\beta^e_{ki} &= \frac{1}{\sqrt{2}}\xi^L_{11}V^{(\nu)*}_{ik} - \frac{1}{2}V^{(\nu)*}_{jk}\left(\tan\theta_W\xi^*_{j1} + \xi^*_{j2}\right)\delta_{i1},\\
\beta^u_k &= \frac{1}{6}V^{(\nu)*}_{jk}\left(\tan\theta_W\xi^*_{j1} + 3\xi^*_{j2}\right),\\
\beta^d_k &= \frac{1}{3}\tan\theta_W V^{(\nu)*}_{jk}\xi^*_{j1}, \ \zeta = \frac{1}{\sqrt{2}}\xi^L_{11}.
\end{aligned}$$

The notations used in these formulas are explained in Appendix B.

The MSSM gluino-quark-squark vertex in the diagram Fig. 1(b) is described by the Lagrangian term

$$\mathcal{L}_{\tilde{g}} = -\sqrt{2}g_3\frac{\lambda^{(a)}_{\alpha\beta}}{2}\left(\bar{q}^\alpha P_R\tilde{g}^{(a)}\tilde{q}^\beta_L - \bar{q}^\alpha P_L\tilde{g}^{(a)}\tilde{q}^\beta_R\right) + h.c., \qquad (18)$$

Here $\lambda^{(a)}$ are 3×3 Gell-Mann matrices ($a = 1, ..., 8$). Superscripts α, β denote the color indices.

The diagrams in Fig.1 describe the $R\!\!\!/_p$ SUSY induced quark transitions which proceed in the nuclear media and trigger the nuclear $0\nu\beta\beta$-decay. Our goal is to derive the corresponding half-life for a certain isotope assuming for simplicity that there is no other contributions to this nuclear process. In order to apply the standard approach [18], [20], [25] based on the non-relativistic impulse approximation one has to derive first the effective low energy Lagrangian describing the basic $0\nu\beta\beta$-quark transition $dd \longrightarrow uuee$ in terms of the color singlet quark charged currents which can be embedded then into the corresponding hadronic (nucleon or pi-meson) currents inside a nucleus. One has also to separate the short and long ranged parts of the quark level transition operators since they are treated within this approach in different ways. It is understood that the short ranged parts involve only heavy particles in the intermediate states ($\chi, \chi^\pm, W, \tilde{q}, \tilde{e}$) while the long distance ones include the neutrino exchange.

Integrating out the heavy fields from the diagrams in Fig. 1 and carrying out Fierz reshuffling we obtain the desired effective Lagrangian which allows one to reproduce the low energy contribution of these diagrams in the first or in the second order of perturbation theory. It takes the form

$$\begin{aligned}
\mathcal{L}_{eff}(x) &= \frac{G_F^2}{2m_P}\left[\eta_{\tilde{g}}(J\,J - \frac{1}{4}J^{\mu\nu}\,J_{\mu\nu}) + \eta_\chi J^\mu J_\mu\right](\bar{e}P_R e^c) - \qquad (19)\\
&- \sqrt{2}G_F\lambda'_{i11}\cdot\eta_\lambda^{(ki)}\cdot(\bar{\nu}_k P_R e^c)\,J + G_F\sqrt{2}(\bar{e}\gamma^\mu P_L\nu_k)V^{(\nu)}_{1k}J_\mu.
\end{aligned}$$

Here we introduced the color singlet quark currents

$$J = \bar{u}^\alpha P_R d_\alpha, \quad J^{\mu\nu} = \bar{u}^\alpha\sigma^{\mu\nu}P_R d_\alpha, \quad J^\mu = \bar{u}^\alpha\gamma^\mu P_L d_\alpha, \qquad (20)$$

The effective parameters η accumulating the dependence on the initial $R\!\!\!/_p$ SUSY parameters are defined as

$$\eta_{\tilde{g}} = \frac{4\pi\alpha_s}{9}\frac{g_2^2\zeta^2}{G_F^2 m^4_{\tilde{d}_L}}\left(\frac{m_p}{m_{\tilde{g}}}\right); \quad \eta_\chi = \sum_{i=1}^{4}\frac{m_p}{m_{\chi_i}}\kappa_i^2 \equiv \frac{m_p}{\langle m_\chi\rangle}, \qquad (21)$$

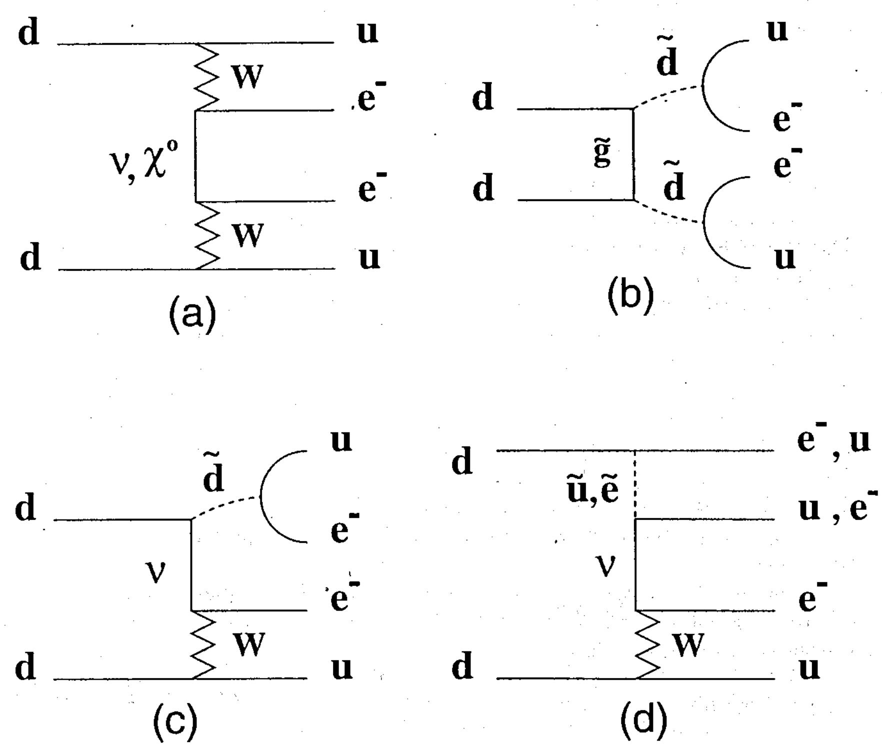

Fig.1.: Feynman graphs contributing to neutrinoless double beta ($0\nu\beta\beta$) decay for the case of the bilinear R-parity violation. (a) the Majorana neutrino or neutralino exchange with two accompanying W-bosons; (b) the gluino-squark-squark exchange; (c,d) the neutrino-squark/slepton exchange with one accompanying W-boson.

$$\eta_\lambda^{(ki)} = \frac{g_2}{2G_F}\left(2\frac{\beta^e_{ki}}{m_{\tilde{e}_{Li}}} - \frac{\beta^d_k}{m_{\tilde{d}_R}}\delta_{i1} - \frac{\beta^u_k}{m_{\tilde{u}_L}}\delta_{i1}\right).$$

Here $m_{\tilde{g}}, m_{\tilde{q}}$ and m_{χ_i} are the gluino, squark and neutralino masses.

In the $R\!\!\!/_p$MSSM we have for the neutrino mixing matrix element (see Appendix B) the following expression

$$V^{(\nu)}_{1k} = \delta_{1k}\cos\theta - \delta_{3k}\sin\theta \tag{22}$$

with

$$\sin\theta = -\Lambda_1/|\vec{\Lambda}|. \tag{23}$$

In Eq. (19) the first and the second terms reproduce the contribution of the gluino Fig. 1 (b) and the neutralino Fig. 1(a) exchange graphs in the 1st order of perturbation theory while the third and the last terms reproduce the contribution of the neutrino exchange graphs in Fig. 1(a) and Fig. 1(c,d) in the 2nd order of perturbation theory. Note, that the last term in Eq.(19) is the ordinary lepton-number L conserving standard model interaction term. Since the $0\nu\beta\beta$-decay requires the L-violation $\Delta L = 2$ the neutrinos in Fig. 1 (a) propagate in the Majorana lepton-number violating mode. In this case the source of the L-violation is given by the neutrino Majorana mass term. That is why the contribution corresponding to this diagram is proportional to the neutrino mass or, more precisely, to the average neutrino mass $\langle m_\nu\rangle$ defined below. On the contrary, the neutrino exchange diagrams in Fig. 1 (c,d) are not proportional to $\langle m_\nu\rangle$ and survive in the limit $m_\nu = 0$ since the lepton-number violation $\Delta L = 2$ is produced by the interaction term $\bar{\nu}P_R e^c$ in Eq. (19) itself. Therefore, the neutrinos in diagrams Fig. 1 (c,d) propagate in the L-conserving Dirac mode.

So far we concentrated on the $0\nu\beta\beta$-transitions at the quark level described by the effective Lagrangian (19). The aim of this paper is the calculation of the amplitude for the nuclear $0\nu\beta\beta$-decay taking into account nuclear structure.

The next section deals with the derivation of the amplitude for the nuclear $0\nu\beta\beta$-decay triggered by the quark transitions in Fig.1.

4 Nuclear $0\nu\beta\beta$-decay

Let us write down the following formal expression for the amplitude of $0\nu\beta\beta$-decay

$$\begin{aligned} &< (A,Z+2), 2e^-|S-1|(A,Z) >= \\ &= \ < (A,Z+2), 2e^-|Texp[i\int d^4x\mathcal{L}_{eff}(x)]|(A,Z)> \end{aligned} \tag{24}$$

where the effective Lagrangian $\mathcal{L}_{eff}$ is given by Eq. (19). The nuclear structure is involved via the initial (A,Z) and the final (A, Z+2) nuclear states having the same atomic weight A, but different electric charges Z and Z+2. The standard framework for the calculation of this nuclear matrix element is the non-relativistic impulse approximation (NRIA) [25].

It is straightforward to derive the following formula for the amplitude of the $0^+ \to 0^+$ transition with two outgoing S-wave electrons

$$\begin{aligned} &\mathcal{R}_{0\nu\beta\beta}(0^+ \to 0^+) = C_{0\nu} f_A^2 \bar{e}(1+\gamma_5)e^c \times \\ &\left[\eta_{\tilde{g}}\mathcal{M}_{\tilde{g}} + \lambda'_{111}\eta_\lambda^{(k1)}V^{(\nu)}_{1k}\mathcal{M}_\lambda + \frac{m_p}{< m_\chi >}\mathcal{M}_N + \frac{< m_\nu >}{m_e}\mathcal{M}_\nu\right], \end{aligned} \tag{25}$$

The normalization factor is

$$C_{0\nu} = (G_F^2 2m_e)/(8\sqrt{2}\pi R). \tag{26}$$

Here, m_e and $f_A \approx 1.261$ are the electron mass and the nucleon axial coupling. $R = r_0 A^{1/3}$ is the nuclear radius ($r_0 = 1.1fm$).

The last term is the conventional Majorana neutrino mass contribution proportional to the average neutrino mass. In the $R\!\!\!/_p$MSSM we have

$$\langle m_\nu\rangle = \sum_i m_{\nu_i}\left(V^{(\nu)}_{1i}\right)^2 = m_{\nu_\tau}\left(V^{(\nu)}_{13}\right)^2 = \frac{2}{3}\frac{g_1^2 M_2}{Det M_\chi}\Lambda_1^2. \tag{27}$$

Here we neglected the small loop induced neutrino masses $m_{\nu_e} \approx m_{\nu_\mu} \approx 0$ and used Eq. (22).

Let us specify the nuclear matrix elements involved in the formula for the $0\nu\beta\beta$-decay amplitude (25). They are

$$\begin{aligned} \mathcal{M}_{\tilde{g}} &= \left(\frac{m_A}{m_p}\right)^2 \frac{m_p}{m_e}\left(M_{GT,\tilde{g}} + M_{T,\tilde{g}}\right), \\ \mathcal{M}_\nu &= \left(\frac{f_V}{f_A}\right)^2 \mathcal{M}_{F,\nu} - \mathcal{M}_{GT,\nu}, \\ \mathcal{M}_\lambda &= \frac{\alpha_P}{m_e R}\left[\frac{1}{3}\mathcal{M}_{GT,\lambda} + \mathcal{M}_{T,\lambda}\right], \\ \mathcal{M}_N &= \left(\frac{m_A}{m_p}\right)^2 \frac{m_p}{m_e}\left\{\left(\frac{f_V}{f_A}\right)^2 \mathcal{M}_{F,N} - \mathcal{M}_{GT,N}\right\}. \end{aligned} \tag{28}$$

Here m_p and m_e stand for the proton and electron masses, $f_V \approx 1.0$ is the vector nucleon constants, m_A= 0.85GeV to be defined below. The coefficient $\alpha_P = 1.75$ is related to the nucleon matrix element of the pseudoscalar current. Its numerical value calculated in the quark bag model we take from Ref. [18].

The partial nuclear matrix elements in the closure approximation we write down in the form

$$\begin{aligned} \mathcal{M}_{F,i} &= \langle 0_f^+|\sum_{a\neq b}\tau_a^+\tau_b^+\ \mathcal{F}_i(r_{ab})\left(\frac{R}{r_{ab}}\right)^{\delta_i}|0_i^+\rangle, \\ \mathcal{M}_{GT,i} &= \langle 0_f^+|\sum_{a\neq b}\tau_a^+\tau_b^+\ \mathcal{G}_i(r_{ab})\left(\frac{R}{r_{ab}}\right)^{\delta_i}\sigma_{ab}\,|0_i^+\rangle, \\ \mathcal{M}_{T,i} &= \langle 0_f^+|\sum_{a\neq b}\tau_a^+\tau_b^+\ \mathcal{T}_i(r_{ab})\left(\frac{R}{r_{ab}}\right)^{\delta_i} S_{ab}\,|0_i^+\rangle, \end{aligned} \tag{29}$$

where $i = \tilde{g}, \lambda, N, \nu$. The exponent takes the values $\delta_i = \{1, 0, 1, 0\}$. We use the shorthand notations

$$\begin{aligned} \mathcal{F}_i &= \{0, 0, F_N(x_A), h_+(r_{ab})\}, \quad \mathcal{T}_i = \{F_2(x_\pi), h_{T'}(r_{ab}), 0, 0\}, \\ \mathcal{G}_i &= \{F_1(x_\pi), h_R(r_{ab}), F_N(x_A), h_+(r_{ab})\}, \end{aligned} \tag{30}$$

for the following form factor functions and neutrino potentials

$$
\begin{aligned}
F_1(x) &= \left[\alpha^{1\pi} + \alpha^{2\pi}(x-2)\right] e^{-x}, \quad F_N(x) = \frac{x}{48}(3+3x+x^2)e^{-x}, \\
F_2(x) &= \left[\alpha^{1\pi}\frac{3+3x+x^2}{x^2} + \alpha^{2\pi}(x+1)\right] e^{-x}, \\
h_+(r_{ab}) &= \frac{2}{\pi} R \int_0^\infty dq \cdot q\Phi^2(\mathrm{q}^2)\frac{j_0(qr_{ab})}{q+\bar{A}}, \\
h_R(r_{ab}) &= \frac{2}{\pi}\frac{R^2}{m_p} \int_0^\infty dq \cdot q^3\Phi^2(\mathrm{q}^2)\frac{j_0(qr_{ab})}{q+\bar{A}}, \\
h_{T'}(r_{ab}) &= -\frac{2}{\pi}\frac{R^2}{3m_p} \int_0^\infty dq \cdot q^3\Phi^2(\mathrm{q}^2)\frac{j_2(qr_{ab})}{q+\bar{A}}.
\end{aligned}
\tag{31}
$$

with $q = |\mathbf{q}|$ being an absolute value of the 3-momentum transferred between the decaying nucleons. $\alpha^{1\pi} = -4.4 \cdot 10^{-2}$ and $\alpha^{2\pi} = 0.2$ are the pion structure coefficients introduced and calculated in Ref. [20]. $\bar{A} \approx 10$MeV is the average excitation energy of the intermediate nuclear state. The spherical Bessel functions are defined in the standard way

$$
j_0(x) = \frac{\sin x}{x}, \quad j_1(x) = \frac{\sin x}{x^2} - \frac{\cos x}{x}, \quad j_2(x) = \frac{3}{x}j_1(x) - j_0(x). \tag{32}
$$

The nucleon form factor $\Phi(q^2)$ in Eqs. (31) takes into account the finite nucleon size. In our numerical analysis we employ the conventional dipole parametrization

$$
\Phi(\mathrm{q}^2) = \left(1 + \frac{\mathrm{q}^2}{m_A^2}\right)^{-2} \tag{33}
$$

with $m_A = 0.85$GeV. We also defined in Eqs. (30)-(32):

$$
\begin{aligned}
S_{ab} &= 3(\sigma_a \cdot \hat{\mathbf{r}}_{ab})(\sigma_b \cdot \hat{\mathbf{r}}_{ab}) - \sigma_a \cdot \sigma_b, \quad \sigma_{ab} = \sigma_a \cdot \sigma_b \\
\mathbf{r}_{ab} &= (\mathbf{r}_a - \mathbf{r}_b), \quad r_{ab} = |\mathbf{r}_{ab}|, \quad \hat{\mathbf{r}}_{ab} = \mathbf{r}_{ab}/r_{ab}, \\
x_A &= m_A r_{ab}, \quad x_\pi = m_\pi r_{ab},
\end{aligned}
\tag{34}
$$

where $\mathbf{r}_a$ is the coordinate of the "ath" nucleon.

The following comments on the nuclear matrix element $\mathcal{M}_{\tilde{g}}$ in Eq. (28) associated with the gluino graph Fig.1(b) are in order. As discussed in Ref. [20] it consists of the two parts $\mathcal{M}_{\tilde{g}} = \mathcal{M}_{\tilde{g}}^{2N} + \mathcal{M}_{\tilde{g}}^{\pi N}$ corresponding to the gluino graph contribution via the two-nucleon and the pion-exchange modes respectively. These two modes arise from the two possibilities of hadronization of the 1st term of the effective Lagrangian $\mathcal{L}_{eff}$ in Eq. (19). One can place the four quark fields present in this term in the two initial neutrons and two final protons separately (2N-mode). Then $nn \to pp + 2e^-$-transition is directly induced by the underlying quark subprocess $dd \to uu + 2e^-$. In this case the nucleon transition is mediated by the exchange of a heavy particle which is the gluino $\tilde{g}$ with the mass $m_{\tilde{g}} \geq 100$GeV. Therefore, the two decaying neutrons are required to come up very closely to each other what is suppressed by the nucleon repulsion. Another possibility is to incorporate quarks involved in the underlying $R\!\!\!/_p$ SUSY transition $dd \to uu + 2e^-$ not into nucleons but into two virtual pions or into one pion as well as into one initial neutron and one final proton [20]. Now $nn \to pp + 2e^-$ transition is mediated by the charged pion-exchange between the decaying nucleons (πN-mode). Since the interaction region extends to the distances $\sim 1/m_\pi$ this mode is not suppressed by the nucleon repulsion. An additional enhancement of the πN-mode comes from the hadronization of the $R\!\!\!/_p$SUSY effective vertex operator $\bar{u}\gamma_5 d \cdot \bar{u}\gamma_5 d \cdot \bar{e}P_R e^c$ replaced by its hadronic image $\pi^2 \cdot \bar{e}P_R e^c$. The enhancement occurs due to the coincidence of the pseudoscalar quark bilinears $\bar{u}\gamma_5 d$ with π-meson field. As is shown in Ref. [20] the πN-mode absolutely dominates over the 2N-mode. Therefore we neglected the subdominant 2N-mode part $\mathcal{M}_{\tilde{g}}^{2N}$ in Eq. (28).

We calculate the nuclear matrix elements within the renormalized Quasiparticle Random Phase Approximation (pn-RQRPA) [22]. This nuclear structure method has been developed from the proton-neutron QRPA approach, which has been frequently used in the $0\nu\beta\beta$-decay calculations. The pn-RQRPA is an extension of the pn-QRPA by incorporating the Pauli exclusion principle for the fermion pairs.

The limitation of the conventional pn-QRPA is traced to the quasiboson approximation (QBA), which violates the Pauli exclusion principle. In the QBA one neglects the terms coming from the commutator of the two bifermion operators by replacing the exact expression for this commutator with its expectation value in the uncorrelated BCS ground state. In this way the QBA implies the two-quasiparticle operator to be a boson operator. The QBA leads to too strong ground state correlations with increasing strength of the residual interaction in the particle-particle channel what affects the calculated nuclear matrix elements severely.

To overcome this problem the Pauli exclusion principle has to be incorporated into the formalism [22] in order to limit the number of quasiparticle pairs in the correlated ground state. The commutator is not anymore boson like, but obtains corrections to its bosonic behavior due to the fermionic constituents. The pn-RQRPA goes beyond the QBA. The Pauli effect of fermion pairs is included in the pn-RQRPA via the renormalized QBA (RQBA) [22], i.e. by calculating the commutator of two bifermion operators in the correlated RPA ground state. Now it is widely recognized that the QBA is a poor approximation and that the pn-RQRPA offers the advantages over pn-QRPA. Let us stress that there is no collapse of the pn-RQRPA solution for a physical value of the nuclear force and that the nuclear matrix elements have been found significantly less sensitive to the increasing strength of particle-particle interaction in comparison with QRPA results. Thus, the pn-RQRPA provides significantly more reliable treatment of the nuclear many-body problem for the description of the $0\nu\beta\beta$ decay.

For numerical treatment of the $0\nu\beta\beta$-decay matrix elements listed in Eqs. (30) within the pn-RQRPA we transform them by using the second quantization formalism to the form containing the two-body matrix elements in the relative coordinate. One obtains [26]:

$$
\begin{aligned}
< O_{12} > = \sum_{\substack{pnp'n' \\ J^\pi m_i m_f \mathcal{J}}} & (-)^{j_n + j_{p'} + J + \mathcal{J}}(2\mathcal{J}+1)\left\{\begin{matrix} j_p & j_n & J \\ j_{n'} & j_{p'} & \mathcal{J}\end{matrix}\right\} \times \\
& < p, p'; \mathcal{J} | f(r_{12}) \tau_1^+ \tau_2^+ O_{12} f(r_{12}) | n, n'; \mathcal{J} > \times \\
& < 0_f^+ \parallel [c_{p'}^+ \tilde{c}_{n'}]_J \parallel J^\pi m_f > < J^\pi m_f | J^\pi m_i > < J^\pi m_i \parallel [c_p^+ \tilde{c}_n]_J \parallel 0_i^+ > .
\end{aligned}
\tag{35}
$$

O_{12} represents the coordinate and spin dependent part of the two body transition operator of the $0\nu\beta\beta$-decay nuclear matrix elements in Eqs. (30). The short-range correlations between the two interacting nucleons are taken into account by a correlation function

$$
f(r) = 1 - e^{-\alpha r^2}(1 - br^2) \quad \text{with} \quad \alpha = 1.1 \text{ fm}^2 \quad \text{and} \quad b = 0.68 \text{ fm}^2. \tag{36}
$$

The one-body transition densities and other details of the nuclear structure model are given in [22, 26]. The calculated nuclear matrix elements for the $0\nu\beta\beta$-decay of A=76 isotope

Table 1: Nuclear matrix elements for the neutrinoless double beta decay $^{76}Ge(0^+) \to ^{76} Se(0^+)$ within the pn-RQRPA.

$\mathcal{M}_{GT,N}$	$\mathcal{M}_{F,N}$	$\mathcal{M}_{GT,\nu}$	$\mathcal{M}_{F,\nu}$	$\mathcal{M}_{GT,\lambda}$	$\mathcal{M}_{T,\lambda}$
0.071	−0.025	2.6	−1.2	1.20	0.21
$\mathcal{M}_{GT,\tilde{g}}$	$\mathcal{M}_{T,\tilde{g}}$	$\mathcal{M}_{\tilde{g}}$	$\mathcal{M}_{\lambda}$	$\mathcal{M}_{\nu}$	$\mathcal{M}_{N}$
−0.34	−0.089	−649	88	−3.4	−132

within the pn-RQRPA are presented in Table 1. The considered single-particle model space has been the 12-level model space (the full $2-4\hbar\omega$ major oscillator shells) introduced in Ref.[26]. The nuclear matrix elements listed in the Table 1 have been obtained for the $g_{pp}=1.0$ where g_{pp} is introduced to renormalize the particle-particle interaction strength of the nuclear Hamiltonian.

According to our numerical analysis, variations of the nuclear matrix elements $\mathcal{M}_{\tilde{g}}, \mathcal{M}_{\lambda}, \mathcal{M}_{N}$ and $\mathcal{M}_{\nu}$ do not exceed 15% and 30% respectively within the physical region of the nuclear structure parameters.

Having all the quantities in the $0\nu\beta\beta$-decay amplitude Eq. (25) specified we are ready to extract the limits on the $R\!\!\!/_p$ parameters from the non-observation of the $0\nu\beta\beta$-decay.

5 $0\nu\beta\beta$-decay constraints on bilinear R-parity violation

Starting from the Eq. (25) we derive the half-life formula

$$[T_{1/2}^{0\nu\beta\beta}(0^+ \to 0^+)]^{-1} = G_{01}|\mathcal{M}_\nu|^2|\mathcal{A}|^2. \tag{37}$$

Here G_{01} is the phase space factor tabulated for various isotopes in Ref. [27].

We introduced the dimensionless parameter

$$\mathcal{A} = \frac{\langle m_\nu\rangle}{m_e} + \frac{m_p}{< m_\chi >}\,\omega_N + \eta_{\tilde{g}}\,\omega_{\tilde{g}} + \lambda'_{111}\,\eta_\nu\,\omega_\lambda, \tag{38}$$

where $\omega_i = \mathcal{M}_i/\mathcal{M}_\nu$ with $i=\tilde{g},\lambda,N$. The first, second and third terms in this equation correspond to the contributions of the neutrino, neutralino and gluino graphs in Fig. 1(a,b). Graphs in Fig. 1(c,d) contribute to the last term in Eq. (38). It is worthwhile noticing that at typical randomly sampled values of the MSSM parameters $M_2, \mu, \tan\beta$ the neutrino exchange contribution from Fig. 1(a) dominates over the other contributions.

The most stringent experimental lower limit on the $0\nu\beta\beta$-decay half-life has been obtained for ^{76}Ge [21]

$$T_{1/2}^{0\nu\beta\beta-\exp}(0^+ \to 0^+) \;\geq\; 1.1\times 10^{25} \text{ years} \qquad 90\% \text{ c.l.} \tag{39}$$

With the nuclear matrix elements calculated in the previous section this lower limit can be cast into the following upper bound

$$|\mathcal{A}| \leq 1.0\cdot 10^{-6}. \tag{40}$$

This constraint represents a complex exclusion condition placed by the non-observation of the $0\nu\beta\beta$-decay on the $R\!\!\!/_p$ MSSM parameter space. The individual bounds on the bilinear $R\!\!\!/_p$ parameters $\mu_1, \langle\nu_1\rangle$ of our present concern depend on concrete SUSY model settings which fix the values of M_2, μ and $\tan\beta$ in the left hand side of Eq. (40).

Typical constrains for the 1st generation $R\!\!\!/_p$ parameters $\mu_1, \langle\nu_1\rangle, \mu_1\lambda'_{111}, \langle\nu_1\rangle\lambda'_{111}$ can be obtained at the typical weak scale values of the MSSM parameters $M_2=\mu=100GeV$ and $\tan\beta=1$. We also assume, as is commonly done in the similar cases, the absence of a significant cancellation between the terms in the left hand side of Eq. (40) defined in Eq. (38). Thus, we come up with the following constraints

$$|\mu_1| \leq 470\text{KeV}, \tag{41}$$
$$|\langle\tilde{\nu}_1\rangle| \leq 840\text{KeV}, \tag{42}$$
$$|\mu_1\lambda'_{111}| \leq 100\text{eV}, \tag{43}$$
$$|\langle\tilde{\nu}_1\rangle\lambda'_{111}| \leq 55\text{eV} \tag{44}$$

Recall that in our notations $\langle\tilde{\nu}_1\rangle \equiv \langle\tilde{\nu}_e\rangle$. To our knowledge these stringent constraints for the 1st generation $R\!\!\!/_p$ parameters were not previously considered in the literature except a parenthetic note in Ref. [3]. One can find in the published papers only those constraints which involve the combinations of the 1st and 2nd generation bilinear $R\!\!\!/_p$ parameters [12] or contain only the 3rd generation ones [13].

To see how stringent are the obtained constraints we can compare them with the following one

$$\lambda'_{111} \;\leq\; 1.3\cdot 10^{-4} \tag{45}$$

which is known as a most stringent constraint on the R-parity violation [20]. This constraint was previously obtained from the $0\nu\beta\beta$-decay by taking into account only the superpotential trilinear couplings in Eq. (8). Consider for comparison the dimensionless quantity

$$\lambda^{LH} \approx g_2^2\,\frac{\mu_1 \ or\ \langle\tilde{\nu}_1\rangle}{M_{SUSY}}, \tag{46}$$

with $M_{SUSY}\sim 100$GeV being the typical SUSY breaking scale. As follows from Eqs. (15)-(17) this dimensionless quantity sets the strength of the RPM induced trilinear fermion-sfermion-fermion interaction similarly to the coupling λ_{111} in Eq. (8). This makes reasonable the comparison of the constraints placed on these couplings by the experiment. From Eqs. (41)-(42) we get an estimation

$$\lambda^{LH} \leq 10^{-6}. \tag{47}$$

This constraint looks more stringent (if such a comparison is legitimate) than that for λ_{111} in Eq. (45).

After all we conclude that the R-parity violation within the 1st generation is restricted by the $0\nu\beta\beta$-decay to a very low level. Now this statement holds for the generic case of the R_p SUSY including both the superpotential trilinear couplings and the bilinear terms in the superpotential as well as in the soft supersymmetry breaking sector.

This conclusion has some immediate phenomenological consequences for the other experiments, in particular for the accelerator ones. For instance, among the two body decay modes of the neutralinos

$$\chi \longrightarrow e^{\pm}W^{\mp}, \mu^{\pm}W^{\mp}, \tau^{\pm}W^{\mp}, \quad \chi \longrightarrow \nu_{e,\mu,\tau}Z, \tag{48}$$

and similar processes open in the presence of the bilinear R_p terms one can now safely neglect the modes with electron or ν_e.

We can also generalize the arguments used in the R_pSUSY interpretation of the HERA anomaly [15]. It is believed that this anomaly can be explained by the s-channel squark exchange $q_1 e \to \tilde{q}_i^* \to q_j e, \chi q_i, \chi^{\pm} q_i'$ between the initial quark-lepton state and the final state particles. The quark-lepton vertex $q\tilde{q}e$ allowed in the R_p SUSY models receives the contributions both from the trilinear λ' couplings and from the trilinear operators induced by the bilinear terms via the lepton-gaugino-higgsino mixing. It is a common practice to neglect the 1st generation squarks in the above mentioned R_p SUSY explanation of the HERA anomaly. The argument is derived from the stringent constraint on the 1st generation λ'_{111} coupling [20] shown in Eq. (45). However, it does not take into account the effect of the bilinear R_p operators. Now, having at hand the new stringent limit on the 1st generation bilinear R-parity violation in Eq. (41)-(44) we can extend the validity of this argument to a general case of R-parity violation considered in the present paper.

6 Conclusion

In summary, we derived the contribution of the bilinear R-parity violating terms to the neutrinoless double beta decay. Alone with the analysis of the trilinear terms previously made in Refs. [16]-[20] this completes the derivation of all possible tree-level contributions to the $0\nu\beta\beta$-decay within the R_p MSSM.

From the non-observation of $0\nu\beta\beta$-decay we obtained new stringent upper limits on the 1st generation R-parity violating parameters such as the lepton-Higgs mixing mass parameter μ_1 and the vacuum expectation value of the electron sneutrino $\langle\tilde{\nu}_e\rangle$. Then we discussed some implications of these constraints on the other experiments and, in particular, on those which are running or planned at accelerators. We conclude that the R-parity violating effects within the 1st generation, if exist, are very small and in most cases can be neglected in phenomenological analysis of observable effects.

A special attention was paid to the effects of the nuclear structure in the $0\nu\beta\beta$-decay. In the framework of the pn-QRPA approach we obtained the nuclear matrix elements which are stable with respect to the variation of the nuclear model parameters within the physical domain. Thus, we believe that our conclusions concerning the particle physics side of the $0\nu\beta\beta$-decay do not suffer from the nuclear structure uncertainties.

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1 Appendix A

Below we present the mass matrices of the neutral and charged fermion sectors for the general case of the bilinear R-parity violation within the MSSM field contents.

1.1 Neutral fermion mass matrix

In the two component Weyl basis

$$\Psi'^{T}_{(0)} = (\nu_i,\ -i\lambda',\ -i\lambda_3,\ \tilde{H}_1^0,\ \tilde{H}_2^0), \tag{A.1}$$

(A.2)

the mass term of the neutral fermions is

$$\mathcal{L}^{(0)}_{mass} = -\frac{1}{2}\Psi'^{T}_{(0)}\mathcal{M}_0\Psi'_{(0)} + \text{H.c.}, \tag{A.3}$$

The 7×7 mass matrix has the distinct see-saw structure

$$\mathcal{M}_0 = \begin{pmatrix} 0 & m \\ m^T & M_\chi \end{pmatrix}. \tag{A.4}$$

with 3×4 matrix

$$m = \begin{pmatrix} -M_Z s_W c_\beta u_1 & M_Z c_W c_\beta u_1 & 0 & -\mu_1 \\ -M_Z s_W c_\beta u_2 & M_Z c_W c_\beta u_2 & 0 & -\mu_2 \\ -M_Z s_W c_\beta u_3 & M_Z c_W c_\beta u_3 & 0 & -\mu_3 \end{pmatrix}. \tag{A.5}$$

originating from the R_p bilinear terms in the superpotential and the soft SUSY breaking sector.

In Eq. (A.4) M_χ is the usual 4×4 the MSSM neutralino mass matrix in the basis $\{-i\lambda', -i\lambda_3, \tilde{H}_1, \tilde{H}_2\}$

$$M_\chi = \begin{pmatrix} M_1 & 0 & -M_Z s_W c_\beta & M_Z s_W s_\beta \\ 0 & M_2 & M_Z c_W c_\beta & -M_Z c_W s_\beta \\ -M_Z s_W c_\beta & M_Z c_W c_\beta & 0 & -\mu \\ M_Z s_W s_\beta & -M_Z c_W s_\beta & -\mu & 0 \end{pmatrix}. \tag{A.6}$$

Here $u_i = \langle\tilde{\nu}_i\rangle/\langle H_1\rangle$ and $\tan\beta = \langle H_2\rangle/\langle H_1\rangle$ and $s_W = \cos\theta_W$, $c_W = \cos\theta_W$, $s_\beta = \sin\beta$, $c_\beta = \cos\beta$..

In the mass eigenstate basis defined as

$$\Psi_{(0)i} = \Xi_{ij}\Psi'_{(0)j}, \tag{A.7}$$

the 7×7 neutral fermion mass matrix $\mathcal{M}_0$ in Eq. (A.4) becomes diagonal

$$\Xi^*\mathcal{M}_0\Xi^\dagger = Diag\{m_{\nu_i}, m_{\chi_k}\}, \tag{A.8}$$

where m_{ν_i} and m_{χ_i} are the physical neutrino and neutralino masses. For the considered case of the tree level mass matrix the only one neutrino has a non-zero mass $m_{\nu_1} = m_{\nu_2} = 0$, $m_{\nu_3} \neq 0$.

1.2 Charged fermion mass matrix

The mass term of the charged fermion sector has the following form

$$\mathcal{L}^{(\pm)}_{mass} = -\Psi'^{T}_{(-)} \mathcal{M}_{\pm} \Psi'_{(+)} + \text{H.c.} \tag{A.9}$$

in the two component Weyl spinor basis

$$\begin{aligned} \Psi'^{T}_{(-)} &= (e^-_L,\ \mu^-_L,\ \tau^-_L,\ -i\lambda_-,\ \tilde{H}^-_1), && \text{(A.10)} \\ \Psi'^{T}_{(+)} &= (e^+_L,\ \mu^+_L,\ \tau^+_L,\ -i\lambda_+,\ \tilde{H}^+_2). && \text{(A.11)} \end{aligned}$$

The 5×5 charged fermion mass matrix is

$$\mathcal{M}_{\pm} = \begin{pmatrix} M^{(l)} & E \\ E' & M_{\chi^\pm} \end{pmatrix}, \tag{A.12}$$

where $M_{\chi^\pm}$ is the MSSM chargino mass matrix

$$M_{\chi^\pm} = \begin{pmatrix} M & \sqrt{2} M_Z c_W s_\beta \\ \sqrt{2} M_Z c_W c_\beta & \mu \end{pmatrix}. \tag{A.13}$$

The sub-matrices E and E' lead to the chargino-lepton mixing. They are defined as

$$E = \begin{pmatrix} \sqrt{2} M_Z c_W c_\beta u_1 & \mu_1 \\ \sqrt{2} M_Z c_W c_\beta u_2 & \mu_2 \\ \sqrt{2} M_Z c_W c_\beta u_3 & \mu_3 \end{pmatrix}, \tag{A.14}$$

and

$$E' = -\begin{pmatrix} 0 & 0 & 0 \\ M^{(l)}_{1i} u_i & M^{(l)}_{2i} u_i & M^{(l)}_{3i} u_i \end{pmatrix}, \tag{A.15}$$

where $M^{(l)}$ is the charged lepton mass matrix. In a good approximation it can be treated as a diagonal matrix $M^{(l)} = Diag\{m^{(l)}_i\}$ with $m^{(l)}_i$ being the physical lepton masses. Also, one can safely neglect matrix E' compared to the other entries of the full mass matrix (A.12) taking into account smallness of the lepton masses.

Rotation to the mass eigenstate basis

$$\Psi_{(\pm)i} = \Delta^{\pm}_{ij} \Psi'_{(\pm)j}, \tag{A.16}$$

casts the mass matrix in Eq. (A.12) to a diagonal form

$$\left(\Delta^-\right)^* \mathcal{M}_{\pm} \left(\Delta^+\right)^\dagger = Diag\{m^{(l)}_i, m_{\chi^\pm_k}\}, \tag{A.17}$$

where $m^{(l)}_i$ and $m_{\chi^\pm_k}$ are the physical charged lepton and chargino masses.

2 Appendix B

Here we give a short account on the results of the approximate diagonalization method used in our analysis.

2.1 Neutral fermion mixing matrix

To leading order in the small expansion parameters ξ defined below, an approximate form of the neutral fermion 7×7 mixing matrix introduced in Eqs. (9), (A.7) is [12]

$$\Xi^* = \begin{pmatrix} V^{(\nu)T}(1 - \frac{1}{2}\xi\xi^\dagger) & -V^{(\nu)T}\xi \\ N^*\xi^\dagger & N^*(1 - \frac{1}{2}\xi^\dagger\xi) \end{pmatrix}, \tag{B.1}$$

Here

$$\begin{aligned} \xi_{i1} &= \frac{g_1 M_2 \mu}{2\ Det M_\chi}\Lambda_i, \quad \xi_{i2} = -\frac{g_2 M_1 \mu}{2\ Det M_\chi}\Lambda_i, && \text{(B.2)} \\ \xi_{i3} &= \frac{\mu_i}{\mu} + \frac{g_2(M_1 + \tan^2\theta_W M_2)\sin\beta\cos\theta_W M_Z}{2\ Det M_\chi}\Lambda_i, && \text{(B.3)} \\ \xi_{i4} &= -\frac{g_2(M_1 + \tan^2\theta_W M_2)\cos\beta\cos\theta_W M_Z}{2\ Det M_\chi}\Lambda_i, && \text{(B.4)} \end{aligned}$$

with $i = 1, 2, 3$. The determinant of the MSSM neutralino mass matrix (A.6) is

$$Det M_\chi = \sin 2\beta M^2_W \mu (M_1 + \tan^2\theta_W M_2) - M_1 M_2 \mu^2. \tag{B.5}$$

The 4×4 matrix N rotates the MSSM neutralino mass matrix M_χ to the diagonal form

$$N^* M_\chi N^\dagger = Diag\{m_{\tilde{\chi}_i}\} \tag{B.6}$$

where $m_{\tilde{\chi}_i}$ are the physical neutralino masses. Thus, to leading order in ξ the mixing within the neutralino sector is described as in the MSSM by

$$\chi_k = N_{kn}\chi'_n \tag{B.7}$$

with $\chi'_n = (-i\lambda', -i\lambda_3, \tilde{H}^0_1, \tilde{H}^0_2)$ being the weak basis.

The 3×3 matrix $V^{(\nu)}$ rotates the RPM induced effective neutrino mass matrix to the diagonal form

$$V^{(\nu)T}\, m_{eff}\, V^{(\nu)} = Diag\{0,\ 0,\ m_\nu\}, \tag{B.8}$$

The tree level expression for this mass matrix can be found in Ref. [12]. The only non-zero neutrino mass is given by

$$m_\nu = g_2^2 \frac{M_1 + \tan^2\theta_W M_2}{4\ Det M_\chi}|\vec{\Lambda}|^2. \tag{B.9}$$

where

$$\Lambda_i = \mu\langle\nu_i\rangle - \langle H_1\rangle\mu_i\ , \tag{B.10}$$

Let us show an explicit form of the neutrino mixing matrix

$$V^{(\nu)} = \begin{pmatrix} \cos\theta_{13} & 0 & -\sin\theta_{13} \\ \sin\theta_{23}\sin\theta_{13} & \cos\theta_{23} & \sin\theta_{23}\cos\theta_{13} \\ \sin\theta_{13} & \sin\theta_{23} & \cos\theta_{13}\cos\theta_{23} \end{pmatrix}, \tag{B.11}$$

where the mixing angles are expressed through the vector $\vec{\Lambda}$ as follows:

$$\tan\theta_{13} = -\frac{\Lambda_1}{\sqrt{\Lambda_2^2+\Lambda_3^2}}, \qquad \tan\theta_{23} = \frac{\Lambda_2}{\Lambda_3}. \tag{B.12}$$

The mixing within the neutrino sector to leading order in ξ is described by

$$\nu_k = V^{(\nu)*}_{nk}\nu'_n \tag{B.13}$$

with $\nu'_n = (\nu_e, \nu_\mu, \nu_\tau)$ being the weak basis.

2.2 Charged fermion mixing matrix

To leading order in the small expansion parameters ξ^L and ξ^R defined below, an approximate form of the charged fermion 5×5 mixing matrix introduced in Eqs. (9), (A.16) reads

$$(\Delta^-)^* = \begin{pmatrix} V_L(1-\frac{1}{2}\xi^{L*}\xi^{L^T}) & -V_L\xi^{L*} \\ U^*\xi^{L^T} & U^*(1-\frac{1}{2}\xi^{L^T}\xi^{L*}) \end{pmatrix}, \tag{B.14}$$

and

$$(\Delta^+)^\dagger = \begin{pmatrix} (1-\frac{1}{2}\xi^{R*}\xi^{R^T})V_R^\dagger & \xi^{R*}V^\dagger \\ -\xi^{R^T}V_R^\dagger & (1-\frac{1}{2}\xi^{R^T}\xi^{R*})V^\dagger \end{pmatrix} \tag{B.15}$$

Here

$$\xi^{L*}_{i1} = \frac{g_2}{\sqrt{2}\,Det M_{\chi^\pm}}\Lambda_i, \quad \xi^{L*}_{i2} = \frac{\mu_i}{\mu} - \frac{g_2 \sin\beta\cos\theta_W M_Z}{\mu\, Det M_{\chi^\pm}}\Lambda_i, \tag{B.16}$$

with $i = 1,2,3$ and

$$\xi^{R*} = M^{(l)\dagger}\xi^{L*}\left(M^{-1}_{\chi^\pm}\right)^T. \tag{B.17}$$

This matrix is much smaller than ξ^L by the factor m_l/M_{SUSY}, where m_l and M_{SUSY} are the lepton masses and the typical SUSY breaking scale M_{SUSY} ~100GeV. Thus the mixing between (e^+_L, μ^+_L, τ^+_L) and ($-i\lambda_+$, $\tilde{H}^+_2$) described by the off diagonal blocks of the Δ^+ in Eq. (B.15) is small and, therefore, neglected in our analysis.

In Eqs. (B.14)-(B.15) the determinant of the MSSM chargino mass matrix is

$$Det M_{\chi^\pm} = M_2\mu - \sin2\beta M_W^2. \tag{B.18}$$

The other matrices are defined as follows:

$$U^* M_{\chi^\pm} V^\dagger = Diag\{m_{\chi^\pm_i}\}, \tag{B.19}$$
$$V_L M^{(l)} V_R^\dagger = Diag\{m_{l_i}\},$$

with $M_{\chi^\pm}$ and $M^{(l)}$ are the MSSM chargino and charged leptons mass matrices defined in Appendix A while $m_{\chi^\pm_i}$ and m_{l_i} are the physical chargino and the charged lepton masses.

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