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PIONS IN NUCLEI AND MANIFESTATIONS OF SUPERSYMMETRY IN NEUTRINOLESS DOUBLE BETA DECAY

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1 Introduction

The observation of neutrinoless nuclear double beta decay

$$(A, Z) \longrightarrow (A, Z+2) + 2e^- \qquad (1)$$

($0\nu\beta\beta$) would undoubtedly indicate the presence of the new physics beyond the standard model (SM) of electroweak interactions. However, as yet there is no experimental evidence for this lepton-number violating ($\Delta L = 2$)exotic process. On the other hand non-observation of $0\nu\beta\beta$-decay at certain experimental sensitivity allows one to set limits on some parameters of the new physics. An unprecedented accuracy and precision of the modern $0\nu\beta\beta$-decay experiments allows one in certain cases to push these limits far out of reach of the accelerator and the other non-accelerator experiments.

A well known example is given by the upper limit on the light effective Majorana neutrino mass $\langle m_M^\nu \rangle$. From the $0\nu\beta\beta$-decay experiments [1] it was found $\langle m_M^\nu \rangle \leq \mathcal{O}(1.1$ eV) [2]. Recall that the Majorana neutrino mass term violates the lepton number $\Delta L = 2$. This is exactly what is necessary for $0\nu\beta\beta$-decay to proceed via the virtual neutrino exchange between the two neutrons. In this case the $0\nu\beta\beta$-decay amplitude is proportional to $\langle m_M^\nu \rangle$.

The Majorana neutrino exchange is not the only possible mechanism of $0\nu\beta\beta$-decay. The lepton-number violating quark-lepton interactions of the R-parity non-conserving supersymmetric extensions of the SM ($\not{R}_p$SUSY) can also induce this process [3]-[6]. R-parity is a discrete multiplicative Z_2 symmetry defined as $R_p = (-1)^{3B+L+2S}$, where S, B and L are the spin, the baryon and the lepton quantum number. At the level of renormalizable operators R-parity can be explicitly violated by the trilinear and the bilinear terms in the superpotential and in the soft SUSY breaking sector.

The impact of the R-parity violation on the low energy phenomenology is twofold. First, it leads the lepton number and lepton flavor violating interactions directly from the trilinear terms. Second, bilinear terms generate the non-zero vacuum expectation value for the sneutrino fields $\langle \tilde{\nu}_i \rangle \neq 0$ and cause neutrino-neutralino as well as electron-chargino mixing. The mixing brings in the new lepton number and lepton flavor violating interactions in the physical mass eigenstate basis.

The implications of the trilinear and the bilinear terms for the $0\nu\beta\beta$-decay were previously considered in Refs. [3]-[7] and in Ref. [8] respectively. The $0\nu\beta\beta$-decay proved to be very sensitive probe of the new interactions predicted in the $\not{R}_p$ SUSY [5]-[8].

In this paper we return to the phenomenology of the trilinear terms and perform a comprehensive analysis of their contribution to the $0\nu\beta\beta$-decay, paying special attention to the hadronization of the corresponding quark interactions and to the nuclear structure calculations. We will show that for the case of the trilinear terms the stage of hadronization plays especially important role in derivation of the short-ranged $\not{R}_p$ SUSY mechanism of $0\nu\beta\beta$-decay.

In our paper Ref. [6] we had considered the two-pion realization of the underlying $\Delta L = 2$ quark-level $0\nu\beta\beta$-transition $dd \to uu + 2e^-$. It was found that the corresponding contribution to $0\nu\beta\beta$-decay absolutely dominates over the conventional two nucleon mode realization. In this paper we generalize the previous treatment of the hadronization of the $\Delta L = 2$ quark operators.

Searching for tiny effects of the physics beyond the SM in $0\nu\beta\beta$-decay requires a reliable treatment of the nuclear structure as well. In this paper we present the results of our calculations within the proton-neutron renormalized Quasiparticle Random Phase Approximation (pn-RQRPA) [9], [10]. We are listing the nuclear matrix elements for the $R\!\!\!/_p$ SUSY mechanism of the $0\nu\beta\beta$-decay and their specific values for the experimentally interesting isotopes.

We introduce new characteristic of $0\nu\beta\beta$-decaying isotope: its sensitivity to the $R\!\!\!/_p$ SUSY signal. This characteristic depends only on the corresponding nuclear matrix element and the kinematical phase-space factor. We calculate the SUSY sensitivities of all experimentally interesting isotopes and on this basis estimate prospects for SUSY searches in $0\nu\beta\beta$-experiments. Applying our approach, we determine presently most successful $0\nu\beta\beta$-experiment which establishes the most stringent constraints on the R-parity violating Yukawa couplings.

The paper is organized as follows. In the next section we shortly outline the minimal supersymmetric standard model with the explicit R-parity violation ($R\!\!\!/_p$ MSSM) and show the effective $\Delta L = 2$ Lagrangian which describes in this model the quark-level $0\nu\beta\beta$-transition. In section 3 we derive the corresponding effective Lagrangian at the hadronic level in terms of the meson and the nucleon fields. Section 4 is devoted to derivation of the nuclear $0\nu\beta\beta$-transition operators in non-relativistic impulse approximation and to calculation of their matrix elements in the pn-RQRPA approach. Section 5 deals with the constraints on $R\!\!\!/_p$ Yukawa couplings from various $0\nu\beta\beta$-experiments.

2 $R\!\!\!/_p$ SUSY induced $\Delta L = 2$ quark-lepton interactions

Let us shortly outline the minimal supersymmetric standard model with the explicit R-parity violation ($R\!\!\!/_p$ MSSM).

For the minimal MSSM field contents the most general gauge invariant form of the renormalizable superpotential is

$$W = W_{R_p} + W_{R\!\!\!/_p}, \tag{2}$$

where the R_p conserving part has the standard MSSM form [11]

$$W_{R_p} = h_L H_1 L E^c + h_D H_1 Q D^c + h_U H_2 Q U^c + \mu H_1 H_2. \tag{3}$$

Here L, Q stand for lepton and quark doublet left-handed superfields while E^c, U^c, D^c for lepton and *up*, *down* quark singlet superfields; H_1 and H_2 are the Higgs doublet superfields with weak hypercharges $Y = -1, +1$, respectively. Summation over the generations is implied.

The R_p violating part of the superpotential (2) can be written as [12], [13]

$$W_{R\!\!\!/_p} = \lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k + \mu_j L_j H_2 + \lambda''_{ijk} U^c_i D^c_j D^c_k, \tag{4}$$

The coupling constants λ (λ'') are antisymmetric in the first (last) two indices.

The soft supersymmetry breaking part of the scalar potential also contains the $R\!\!\!/_p$-terms of the form:

$$V^{soft}_{R\!\!\!/_p} = \Lambda_{ijk} \tilde{L}_i \tilde{L}_j \tilde{E}^c_k + \Lambda'_{ijk} \tilde{L}_i \tilde{Q}_j \tilde{D}^c_k + \Lambda''_{ijk} \tilde{U}^c_i \tilde{D}^c_j \tilde{D}^c_k + \\ + \tilde{\mu}^2_{2j} \tilde{L}_j H_2 + \tilde{\mu}^2_{1j} \tilde{L}_j H_1^{\dagger} + \text{H.c.} \tag{5}$$

In Eqs. (4), (5) the trilinear terms proportional to $\lambda, \lambda', \Lambda, \Lambda'$ and the bilinear terms violate the lepton number while the trilinear terms proportional to λ'', Λ'' violate baryon number conservation.

It is well known that the simultaneous presence of lepton and baryon number violating terms in Eqs. (4), (5) (unless the couplings are very small) leads to unsuppressed proton decay. Therefore, either only the lepton or the baryon number violating couplings can be present. Certain discrete symmetries such as the B-parity [13], [14] may originate from the underlying high energy scale theory and forbid dangerous combinations of these couplings. Henceforth we simply set $\lambda'' = \Lambda'' = 0$.

The remaining R-parity conserving part of the soft SUSY breaking sector includes the scalar field interactions

$$V^{soft}_{R_p} = \sum_{i=scalars} m_i^2 |\phi_i|^2 + h_L A_L H_1 \tilde{L} \tilde{E}^c + h_D A_D H_1 \tilde{Q} \tilde{D}^c + \\ + h_U A_U H_2 \tilde{Q} \tilde{U}^c + \mu B H_1 H_2 + \text{H.c.} \tag{6}$$

and the "soft" gaugino mass terms

$$\mathcal{L}_{GM} = -\frac{1}{2}\left[M_1 \tilde{B}\tilde{B} + M_2 \tilde{W}^k \tilde{W}^k + M_3 \tilde{g}^a \tilde{g}^a\right] - \text{H.c.} \tag{7}$$

As usual, $M_{3,2,1}$ denote the "soft" masses of the $SU(3) \times SU(2) \times U(1)$ gauginos $\tilde{g}, \tilde{W}, \tilde{B}$ while m_i stand for the masses of the scalar fields. The gluino $\tilde{g}$ soft mass M_3 coincides in this framework with its physical mass denoted hereafter as $m_{\tilde{g}} = M_3$.

As mentioned in the introduction, we concentrate on the phenomenology of the trilinear R-parity violating terms LQD^c in the superpotential (4) and perform a comprehensive analysis of their contribution to the $0\nu\beta\beta$-decay.

These terms lead to the following $\Delta L = 1$ lepton-quark operators

$$\mathcal{L}_{\lambda} = \lambda'_{ijk}[\tilde{\nu}_{iL} \bar{d}_k P_L d_j + \tilde{d}_{jL} \bar{d}_k P_L \nu_i + \tilde{d}^*_{kR} \bar{d}_j P_R \nu^c_i - \tilde{e}_{iL} \bar{d}_k P_L u_j \\ - \tilde{u}_{jL} \bar{d}_k P_L e_i - \tilde{d}^*_{kR} \bar{u}_j P_R e^c_i] + \text{H.c.} \tag{8}$$

Here, as usual $P_{L,R} = (1 \mp \gamma_5)/2$.

Starting from this fundamental Lagrangian, one can derive the low-energy effective Lagrangian [5], which describes the quark-level $0\nu\beta\beta$-transition $dd \to uu + 2e^-$. Integrating out the heavy degrees of freedom, we come up with the formula:

$$\mathcal{L}_{qe} = \frac{G_F^2}{2m_p} \bar{e}(1+\gamma_5)e^c \left[\eta^{PS} J_{PS} J_{PS} - \frac{1}{4}\eta^T J_T^{\mu\nu} J_{T\mu\nu}\right]. \tag{9}$$

These $\Delta L_e = 2$ lepton-number violating effective interactions are induced by heavy SUSY particles exchange. An example of the Feynman diagram contributing to $\mathcal{L}_{qe}$ is given in Fig. 1. A complete list of the relevant diagrams can be found in [5]. Compared to Ref. [5] we have properly taken into account in the Lagrangian (9) the contribution of the color octet currents.

The color-singlet hadronic currents in Eq. (9) are

$$J_{PS} = J_P + J_S, \quad J_P = \bar{u}^{\alpha}\gamma_5 d_{\alpha}, \quad J_S = \bar{u}^{\alpha} d_{\alpha}, \quad J_T^{\mu\nu} = \bar{u}^{\alpha}\sigma^{\mu\nu}(1+\gamma_5) d_{\alpha}. \tag{10}$$

where α is the color index and $\sigma^{\mu\nu} = (i/2)[\gamma^\mu, \gamma^\nu]$.

The effective lepton-number violating parameters η^{PS}, η^T depend on the fundamental parameters of the $R\!\!\!/_p$ MSSM and can be written in the form

$$\eta^{PS} = \eta_{\chi\tilde{e}} + \eta_{\chi\tilde{f}} + \eta_\chi + \eta_{\tilde{g}} + 7\eta'_{\tilde{g}}, \tag{11}$$
$$\eta^T = \eta_\chi - \eta_{\chi\tilde{f}} + \eta_{\tilde{g}} - \eta'_{\tilde{g}}, \tag{12}$$

where we denoted

$$\eta_{\tilde{g}} = \frac{\pi\alpha_s}{6}\frac{\lambda'^2_{111}}{G_F^2 m_{\tilde{d}_R}^4}\frac{m_P}{m_{\tilde{g}}}\left[1 + \left(\frac{m_{\tilde{d}_R}}{m_{\tilde{u}_L}}\right)^4\right] \tag{13}$$
$$\eta_\chi = \frac{\pi\alpha_2}{2}\frac{\lambda'^2_{111}}{G_F^2 m_{\tilde{d}_R}^4}\sum_{i=1}^4 \frac{m_P}{m_{\chi_i}}\left[\epsilon^2_{Ri}(d) + \epsilon^2_{Li}(u)\left(\frac{m_{\tilde{d}_R}}{m_{\tilde{u}_L}}\right)^4\right] \tag{14}$$
$$\eta_{\chi\tilde{e}} = 2\pi\alpha_2\frac{\lambda'^2_{111}}{G_F^2 m_{\tilde{d}_R}^4}\left(\frac{m_{\tilde{d}_R}}{m_{\tilde{e}_L}}\right)^4 \sum_{i=1}^4 \epsilon^2_{Li}(e)\frac{m_P}{m_{\chi_i}}, \tag{15}$$
$$\eta'_{\tilde{g}} = \frac{\pi\alpha_s}{12}\frac{\lambda'^2_{111}}{G_F^2 m_{\tilde{d}_R}^4}\frac{m_P}{m_{\tilde{g}}}\left(\frac{m_{\tilde{d}_R}}{m_{\tilde{u}_L}}\right)^2, \tag{16}$$
$$\eta_{\chi\tilde{f}} = \frac{\pi\alpha_2}{2}\frac{\lambda'^2_{111}}{G_F^2 m_{\tilde{d}_R}^4}\left(\frac{m_{\tilde{d}_R}}{m_{\tilde{e}_L}}\right)^2 \sum_{i=1}^4 \frac{m_P}{m_{\chi_i}}[\epsilon_{Ri}(d)\epsilon_{Li}(e) + \tag{17}$$
$$+ \ \epsilon_{Li}(u)\epsilon_{Ri}(d)\left(\frac{m_{\tilde{e}_L}}{m_{\tilde{u}_L}}\right)^2 + \epsilon_{Li}(u)\epsilon_{Li}(e)\left(\frac{m_{\tilde{d}_R}}{m_{\tilde{u}_L}}\right)^2\Bigg].$$

In Eqs. (13)-(17) we used the standard notations $\alpha_2 = g_2^2/(4\pi)$ and $\alpha_s = g_3^2/(4\pi)$ for the $SU(2)_L$ and $SU(3)_c$ gauge coupling constants. We also denoted the gluino $\tilde{g}$ and the neutralinos χ_i masses as $m_{\tilde{g}}$ and m_{χ_i} respectively. The Majorana neutralinos χ_i are linear combinations of the gaugino and higgsino fields

$$\chi_i = \mathcal{N}_{i1}\tilde{B} + \mathcal{N}_{i2}\tilde{W}^3 + \mathcal{N}_{i3}\tilde{H}_1^0 + \mathcal{N}_{i4}\tilde{H}_2^0. \tag{18}$$

Here $\tilde{W}^3$ and $\tilde{B}$ are the neutral $SU(2)_L$ and $U(1)$ gauginos while $\tilde{H}_1^0$, $\tilde{H}_2^0$ are the higgsinos which are superpartners of the two neutral Higgs boson fields H_1^0 and H_2^0 with weak hypercharges $Y = -1, \ +1$, respectively.

The matrix $\mathcal{N}_{ij}$, introduced in Eq. (18), rotates the 4×4 neutralino mass matrix M_χ to the diagonal form $Diag[m_{\chi_i}]$. We define these matrix as in Ref. [11].

Neutralino couplings are defined as [11]

$$\epsilon_{Li}(\psi) = -T_3(\psi)\mathcal{N}_{i2} + \tan\theta_W\,(T_3(\psi) - Q(\psi))\,\mathcal{N}_{i1}, \tag{19}$$
$$\epsilon_{Ri}(\psi) = Q(\psi)\tan\theta_W\mathcal{N}_{i1}. \tag{20}$$

Here Q and T_3 are the electric charge and the weak isospin of the fields $\psi = u, d, e$.

3 Hadronization of $R\!\!\!/_p$ SUSY quark-lepton interactions

The next step deals with reformulation of the quark-lepton interactions in Eq. (9) in terms of the effective hadron-lepton interactions. This is necessary for the subsequent nuclear structure calculations.

There are the two possibilities of hadronization of the effective Lagrangian $\mathcal{L}_{qe}$ in Eq. (9). One can place the four quark fields in the two initial neutrons and two final protons separately. This is the conventional 2N-mode of $0\nu\beta\beta$-decay shown in Fig. 2(a). Then $nn \to pp + 2e^-$ transition is directly induced by the underlying quark subprocess

$$dd \to uu + 2e^-. \tag{21}$$

In this case the nucleon transition is mediated by the exchange of a heavy supersymmetric particle like the gluino $\tilde{g}$ with the mass $m_{\tilde{g}} \gtrsim 100$GeV. Therefore, the two decaying neutrons are required to come up very closely to each other what is suppressed by the nucleon-nucleon short range repulsion.

Another possibility is to incorporate quarks involved in the underlying $R\!\!\!/_p$ SUSY transition in Eq. (21) not into nucleons but into two virtual pions [6] or into one pion as well as into one initial neutron and one final proton. Now the $nn \to pp + 2e^-$ transition is mediated by the charged pion-exchange between the decaying neutrons, as shown in Fig. 2(b,c). This is what we call the one- and two-pion modes of $0\nu\beta\beta$-decay. Since the interaction region extends to the distances $\sim 1/m_\pi$ this mode is not suppressed by the short range nucleon-nucleon repulsion. An additional enhancement of the π-modes comes from the hadronization of the $R\!\!\!/_p$ SUSY quark-lepton vertex operator in Eq. (9) as discussed below. In Ref. [6] it was shown that the two-pion mode absolutely dominates over the 2N-mode. In what follows, we are arguing that it dominates over the one-pion mode as well.

The effective hadronic Lagrangian taking into account both the nucleon (p, n) and π-meson degrees of freedoms in a nucleus can be written as follows:

$$\mathcal{L}_{he} = \mathcal{L}_{2N} + \mathcal{L}_{2\pi} + \mathcal{L}_{1\pi} + \mathcal{L}_s = \frac{G_F^2}{2m_p}\,\bar{p}\Gamma^{(i)}n \cdot \bar{p}\Gamma^{(i)}n \cdot \ \bar{e}(1+\gamma_5)e^c - \tag{22}$$
$$-\frac{G_F^2}{2m_p}\,m_\pi^2\left[m_\pi^2 a_{2\pi}\left(\pi^-\right)^2 - a_{1\pi}\bar{p}\,i\gamma_5 n \cdot \pi^-\right] \cdot \ \bar{e}(1+\gamma_5)e^c + g_s\,\bar{p}\,i\gamma_5 n\,\pi^+.$$

Here $\mathcal{L}_s$ stays for the standard pion-nucleon interaction with the coupling $g_s = 13.4 \pm 1$ known from experiment. The lepton-number violating terms $\mathcal{L}_{2N}$, $\mathcal{L}_{1\pi}$, $\mathcal{L}_{2\pi}$ generate the conventional two-nucleon mode, the one and two pion-exchange modes of the $0\nu\beta\beta$-decay respectively. The corresponding diagrams are presented in Fig. 2.

The two-nucleon mode term $\mathcal{L}_{2N}$ with different operator structures $\Gamma^{(i)}$ had been considered in [4, 5] within the $R\!\!\!/_p$ MSSM. As was already mentioned this term gives the sub-dominant contribution to $0\nu\beta\beta$-decay in comparison with the pion terms [6]. Therefore, in this paper we concentrate on the effect of the pion-exchange contribution generated by the terms $\mathcal{L}_{1\pi}$ and $\mathcal{L}_{2\pi}$ in Eq. (22).

The basic parameters $a_{2\pi}$ and $a_{1\pi}$ of the Lagrangian $\mathcal{L}_{he}$ in Eq. (22) can be approximately related to the parameters of the quark-lepton Lagrangian $\mathcal{L}_{qe}$ in Eq. (9), using the on-mass-shell "matching conditions" [6]

$$< \pi^+, 2e^-|\mathcal{L}_{qe}|\pi^- > \ = \ < \pi^+, 2e^-|\mathcal{L}_{2\pi}|\pi^- >, \tag{23}$$
$$< \pi^+, p, 2e^-|\mathcal{L}_{qe}|n > \ = \ < \pi^+, p, 2e^-|\mathcal{L}_{1\pi}|n >. \tag{24}$$

In order to solve these equations we apply the widely used factorization and vacuum dominance approximations [15] for the matrix elements of the products of the two quark currents. Then we obtain, taking properly into account the combinatorial and color factors:

$$\langle\pi^+|J_{PS}J_{PS}|\pi^-\rangle \approx \frac{5}{3}\langle\pi^+|J_P|0\rangle\langle 0|J_P|\pi^-\rangle,$$

$$\langle\pi^+|J_T^{\mu\nu}J_{T\mu\nu}|\pi^-\rangle \approx -4\langle\pi^+|J_P|0\rangle\langle 0|J_P|\pi^-\rangle, \quad (25)$$

$$\langle p|J_{PS}J_{PS}|n\pi^-\rangle \approx \frac{5}{3}\langle p|J_P|n\rangle\langle 0|J_P|\pi^-\rangle, \quad (26)$$

$$\langle p|J_T^{\mu\nu}J_{T\mu\nu}|n\pi^-\rangle \approx -4\langle p|J_P|n\rangle\langle 0|J_P|\pi^-\rangle. \quad (27)$$

Here we applied the equalities

$$< 0|J_S|\pi > = < 0|J_T^{\mu\nu}|\pi(p_\pi) > = 0. \quad (28)$$

The scalar matrix element vanishes due to the parity arguments, the tensor one vanishes due to $J_T^{\mu\nu} = -J_T^{\nu\mu}$ and impossibility of constructing an antisymmetric object having only one external 4-vector p_π.

We also use the following relationships for the hadronic matrix elements:

$$\langle 0|\bar{u}\gamma_5 d|\pi^-\rangle = i\sqrt{2}f_\pi\frac{m_\pi^2}{m_u + m_d} \equiv im_\pi^2 h_\pi, \quad (29)$$

$$\langle p|\bar{u}\gamma_5 d|n\rangle = F_P\langle p|\bar{p}\gamma_5 n|n\rangle. \quad (30)$$

where $f_\pi = 0.668\ m_\pi$. For the nucleon pseudoscalar constant F_P we take its bag model value $F_P \approx 4.41$ from Ref. [16].

In this approximation we solve the "matching conditions" in Eqs. (23)-(24) and determine the coefficients in Eq. (22)

$$a_{k\pi} = c_{k\pi}\frac{3}{8}[\eta^T + \frac{5}{3}\eta^{PS}], \quad (31)$$

with

$$c_{1\pi} = \frac{8}{3}h_\pi F_P \approx 132.4, \quad c_{2\pi} = \frac{4}{3}h_\pi^2 \approx 170.3. \quad (32)$$

Here we accepted the conventional values of the current quark masses $m_u = 4.2$ MeV, $m_d = 7.4$ MeV. The large ratio of the pion mass to the small current quark masses provide an additional enhancement factor of the pion mechanism as mentioned before the Eq. (22). Thus, we have obtained the approximate hadronic "image" $\mathcal{L}_{he}$ of the fundamental quark-lepton Lagrangian $\mathcal{L}_{qe}$ given in Eq. (9).

4 Nuclear matrix elements of $\not{R}_p$ SUSY induced $0\nu\beta\beta$-transition

Starting from the Lagrangian $\mathcal{L}_{he}$ in Eq. (22) it is straightforward to calculate the $0\nu\beta\beta$-nuclear matrix element

$$< (A, Z+2), 2e^-|S-1|(A,Z) > = < (A, Z+2), 2e^-|Texp[i\int d^4x\mathcal{L}_{he}(x)]|(A,Z) > \quad (33)$$

The nuclear structure is involved via the initial (A,Z) and the final (A, Z+2) nuclear states having the same atomic weight A, but different electric charges Z and Z+2. The nucleon-level diagrams, which correspond to the leading order contributions to the amplitude in Eq. (33), are given in Fig. 2.

The standard framework for the calculation of this nuclear matrix element is the non-relativistic impulse approximation (NRIA) [17].

The final result for the half-life of $0\nu\beta\beta$-decay in $0^+ \to 0^+$ channel with the two outgoing electrons in the S-state, regarding all the three above-described possibilities of hadronization, reads

$$[T_{1/2}(0^+ \to 0^+)]^{-1} = \quad (34)$$

$$= G_{01}\left|\eta^T \cdot \mathcal{M}_{\tilde{q}}^{2N} + (\eta^{PS} - \eta^T)\cdot\mathcal{M}_{\tilde{f}}^{2N} + \frac{3}{8}(\eta^T + \frac{5}{3}\eta^{PS})\left(\frac{4}{3}M^{1\pi} + M^{2\pi}\right)\right|^2.$$

Here G_{01} is the standard phase space factor tabulated for various nuclei in Ref. [18]. The nuclear matrix elements $\mathcal{M}_{\tilde{q},\tilde{f}}^{2N}$ governing the sub-dominant two-nucleon mode were presented in Ref.[5]. As was already mentioned its contribution can be safely neglected. The one- and the two-pion modes nuclear matrix elements $M^{1\pi}$ and $M^{2\pi}$ we write down in the form

$$\mathcal{M}^{k\pi} = \left(\frac{m_A}{m_p}\right)^2\frac{m_p}{m_e}\alpha^{k\pi}\left(M_{GT}^{k\pi} + M_T^{k\pi}\right) \quad (35)$$

Here, $m_A = 850$MeV is the mass scale of the nucleon form factor.

The structure coefficients $\alpha^{k\pi}$ in Eq. (35) are related to the coefficients $c_{k\pi}$ introduced in Eq. (32) and have the following explicit form

$$\alpha^{1\pi} = -6g_s h_\pi\rho F_P \approx -4.4\cdot 10^{-2}, \quad (36)$$

$$\alpha^{2\pi} = g_s^2 h_\pi^2\rho \approx 2.0\cdot 10^{-1}, \quad (37)$$

where

$$\rho = \frac{1}{36f_A^2}\left(\frac{m_\pi}{m_p}\right)^4\left(\frac{m_p}{m_A}\right)^2, \quad (38)$$

with $f_A \approx 1.261$ being the axial nucleon coupling.

The two types of the Gamow-Teller and tensor nuclear matrix elements are given by the expressions

$$M_{GT}^{k\pi} = \langle 0_f^+|\sum_{i\neq j}\tau_i^+\tau_j^+\sigma_i\cdot\sigma_j F_1^{(k)}(x_\pi)\frac{R}{r_{ij}}|0_i^+\rangle, \quad \text{with} \quad k = 1,2 \quad (39)$$

$$M_T^{k\pi} = \langle 0_f^+|\sum_{i\neq j}\tau_i^+\tau_j^+[3(\sigma_i\cdot\hat{\mathbf{r}}_{ij})(\vec{\sigma_j}\cdot\hat{\mathbf{r}}_{ij}) - \sigma_i\cdot\sigma_j]F_2^{(k)}(x_\pi)\frac{R}{r_{ij}}|0_i^+\rangle. \quad (40)$$

where

$$x_\pi = m_\pi r_{ij}, \quad \mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j, \quad r_{ij} = |\mathbf{r}_{ij}|, \quad \hat{\mathbf{r}}_{ij} = \frac{\mathbf{r}_{ij}}{r_{ij}}, \quad (41)$$

$\mathbf{r}_i$ is a coordinate of the ith nucleon and $R = r_0 A^{1/3}$ is the mean nuclear radius, with $r_0 = 1.1 fm$.

The structure functions $F_1^{(k)}(x_\pi)$ and $F_2^{(k)}(x_\pi)$ have their origin in the integration of the pi-meson propagators and take the following form:

$$F_1^{(1)}(x) = e^{-x}, \quad F_2^{(1)}(x) = (3+3x+x^2)\frac{e^{-x}}{x^2}, \tag{42}$$

$$F_1^{(2)}(x) = (x-2)e^{-x}, \quad F_2^{(2)}(x) = (x+1)e^{-x}. \tag{43}$$

We calculate the nuclear matrix elements within the proton-neutron renormalized Quasiparticle Random Phase Approximation (pn-RQRPA) [9, 10]. This nuclear structure method has been developed from the proton-neutron QRPA (pn-QRPA) approach, which has been frequently used in the $0\nu\beta\beta$-decay calculations. The pn-RQRPA is an extension of the pn-QRPA by incorporating the Pauli exclusion principle for the fermion pairs. The limitation of the conventional pn-QRPA is traced to the quasiboson approximation (QBA), which violates the Pauli exclusion principle. In the QBA one neglects the terms coming from the commutator of the two bifermion operators by replacing the exact expression for this commutator with its expectation value in the uncorrelated BCS ground state. In this way the QBA implies the two-quasiparticle operator to be a boson operator. The QBA leads to too strong ground state correlations with increasing strength of the residual interaction in the particle-particle channel what affects the calculated nuclear matrix elements severely. To overcome this problem the Pauli exclusion principle has to be incorporated into the formalism [9], [10] in order to limit the number of quasiparticle pairs in the correlated ground state. The commutator is not anymore boson like, but obtains corrections to its bosonic behavior due to the fermionic constituents. The pn-RQRPA goes beyond the QBA. The Pauli effect of fermion pairs is included in the pn-RQRPA via the renormalized QBA (RQBA) [9], [10], i.e. by calculating the commutator of two bifermion operators in the correlated QRPA ground state. The RQBA was applied to the $2\nu\beta\beta$-decay in Ref. [9] and to the $0\nu\beta\beta$-decay for the first time in Ref. [10]. Now it is widely recognized that the QBA is a poor approximation and that the pn-RQRPA offers the advantages over pn-QRPA. Let us stress that there is no collapse of the pn-RQRPA solution for a physical value of the nuclear force and that the nuclear matrix elements have been found significantly less sensitive to the increasing strength of the particle-particle interaction in comparison with QRPA results [2]. Thus, the pn-RQRPA provides significantly more reliable treatment of the nuclear many-body problem for the description of the $0\nu\beta\beta$ decay.

For numerical treatment of the $0\nu\beta\beta$-decay matrix elements given in Eqs. (39) and (40) within the pn-RQRPA we transform them by using the second quantization formalism to the form containing the two-body matrix elements in the relative coordinate. One obtains [2]:

$$< \mathcal{O}_{ij} >= \sum_{\substack{pnp'n' \\ J^\pi m_i m_f \mathcal{J}}} (-)^{j_n+j_{p'}+J+\mathcal{J}} (2\mathcal{J}+1) \begin{Bmatrix} j_p & j_n & J \\ j_{n'} & j_{p'} & \mathcal{J} \end{Bmatrix} \times$$
$$< p,p';\mathcal{J}|f(r_{ij})\tau_i^+\tau_j^+\mathcal{O}_{ij}f(r_{ij})|n,n';\mathcal{J}> \times$$
$$< 0_f^+ \parallel \widetilde{[c_{p'}^+\tilde{c}_{n'}]_J} \parallel J^\pi m_f >< J^\pi m_f|J^\pi m_i >< J^\pi m_i \parallel [c_p^+\tilde{c}_n]_J \parallel 0_i^+ > . \tag{44}$$

$\mathcal{O}_{ij}$ represents the coordinate and spin dependent part of the two body transition operators of the $0\nu\beta\beta$-decay nuclear matrix elements in Eqs. (39) and (40). The short-range

correlations between the two interacting nucleons are taken into account by the correlation function

$$f(r) = 1 - e^{-\alpha r^2}(1-br^2) \quad \text{with} \quad \alpha = 1.1 \text{ fm}^2 \quad \text{and} \quad b = 0.68 \text{ fm}^2. \tag{45}$$

The one-body transition densities and the other details of the nuclear structure model are given in [2, 9, 10].

The calculated nuclear matrix elements for the $0\nu\beta\beta$-decay of various isotopes within the pn-RQRPA are presented in Table 1. The considered single-particle model spaces both for protons and neutrons have been as follows: i) For A=76, 82 the model space consists of the full $2-4\hbar\omega$ major oscillator shells. ii) For A=96, 100, 116 we added to the previous model space $1f_{5/2}$, $1f_{7/2}$, $0h_{9/2}$ and $0h_{11/2}$ levels. iii) For A=128, 130, 136 the model space comprises the full $2-5\hbar\omega$ major shells. iv) For A=150 the model space extends over the full $2-5\hbar\omega$ shells plus $0i_{11/2}$ and $0i_{13/2}$ levels.

The single particle energies were obtained by using Coulomb corrected Woods Saxon potential. The interaction employed was the Brueckner G-matrix which is a solution of the Bethe-Goldstone equation with the Bonn one-boson exchange potential. Since the considered model space is finite, we have renormalized pairing interactions by the strength parameters d_{pp} and d_{nn} [19] to the empirical gaps defined by Moeller and Nix [20]. The particle-particle and particle-hole channels of the G-matrix interaction of nuclear Hamiltonian H have been renormalized with parameters g_{pp} and g_{ph}, respectively. The nuclear matrix elements listed in the Table 1 have been obtained for the $g_{ph} = 0.80$ and $g_{pp} = 1.0$

The following note is in order: According to our numerical analysis, variations of the nuclear matrix elements presented in Table 1 do not exceed 20% within the physical region of the nuclear structure parameter g_{pp} $(0.8 \leq g_{pp} \leq 1.2)$.

As seen from the Table 1 $M^{2\pi}$ is significantly larger than $M^{1\pi}$. It is partially due to the mutual cancellation of the $M^{1\pi}_{GT}$ and $M^{1\pi}_T$ in the construction of $M^{1\pi}$ in Eq. (35) (see Table 1) and due to the suppression of the structure coefficient $\alpha^{1\pi}$ in comparison with $\alpha^{2\pi}$. Thus, we conclude that the two-pion mode contribution to $0\nu\beta\beta$-decay (Fig. 2(c)) dominates both over the one-pion (Fig. 2(b)) and the two-nucleon contributions (Fig. 2(a)). The dominance of the two-pion mode over the two-nucleon one was previously proven in Ref. [6].

5 Constraints on $\not{R}_p$ SUSY from $0\nu\beta\beta$-experiments. Comparative analysis

Having all the quantities in the $0\nu\beta\beta$-decay half-life formula (34) specified we are ready to extract the limits on the $\not{R}_p$ parameters from non-observation of the $0\nu\beta\beta$-decay.

The experimental lower bound $T_{1/2}^{exp}$ for the half-life of a certain isotope Y provides the following constraint on the effective $\not{R}_p$ SUSY parameters

$$\eta_{SUSY} \equiv \frac{3}{8}(\eta^T + \frac{5}{3}\eta^{PS}) \leq \eta_{SUSY}^{exp} = \frac{10^{-7}}{\zeta(Y)}\sqrt{\frac{10^{24}\text{years}}{T_{1/2}^{exp}}}, \tag{46}$$

Here we introduced the SUSY sensitivity $\zeta(Y)$ of a $0\nu\beta\beta$-decaying isotope Y

$$\zeta(Y) = 10^5 \left|\frac{4}{3}M^{1\pi} + M^{2\pi}\right| \sqrt{G_{01}}. \tag{47}$$

The quantity $\zeta(Y)$ is an intrinsic characteristic of an isotope Y depending only on the nuclear matrix elements $M^{1\pi}$, $M^{2\pi}$ and on the phase space factor G_{01}. The large numerical values of the SUSY sensitivity ζ defined in (47) correspond to those isotopes within the group of $\beta\beta$-decaying nuclei which are the most promising candidates for searching SUSY in the $0\nu\beta\beta$-decay.

The numerical values of $\zeta(Y)$ calculated in the pn-RQRPA are presented in the Table 1 and displayed in Fig. 3 in the form of a histogram. It is seen that the most sensitive isotope is ^{150}Nd, then follows ^{100}Mo.

It is understood that the SUSY sensitivity ζ cannot be the only criterion for selecting an isotope for the $0\nu\beta\beta$-experiment. Other microscopic and macroscopic properties of the isotope are also important for building a $0\nu\beta\beta$-detector.

The current experimental situation in terms of the accessible half-life and the corresponding upper limit on the effective SUSY parameter η_{SUSY} is presented in Table 2. We conclude that the best upper limit on the $\not R_p$ SUSY parameter η_{SUSY} has been established by the Heidelberg-Moscow experiment [1]. We denote this limit as $\eta^{exp}_{SUSY}(H-M)$. For comparison in the bottom row of the Table 2 we show the lower half-life limits $T^{exp}_{1/2}(\eta^{H-M}_{SUSY})$, which must be reached by $0\nu\beta\beta$-experiments with the other nuclei to reach this presently best constraint $\eta_{SUSY} \leq \eta^{exp}_{SUSY}(H-M)$ on the $\not R_p$ SUSY. The result of this comparison is illustrated in Fig. 3.

Using the values of η^{exp}_{SUSY} from the Table 2 one can easily calculate the corresponding constraints on the λ'_{111} parameter. There are two types of the constraints for each value of η^{exp}_{SUSY} parameter

$$\lambda'_{111} \leq 1.8\sqrt{\eta^{exp}_{SUSY}}\left(\frac{m_{\tilde{q}}}{100\text{GeV}}\right)^2\left(\frac{m_{\tilde{g}}}{100\text{GeV}}\right)^{1/2}, \tag{48}$$

$$\lambda'_{111} \leq 12.5\sqrt{\eta^{exp}_{SUSY}}\left(\frac{m_{\tilde{e}}}{100\text{GeV}}\right)^2\left(\frac{m_{\chi}}{100\text{GeV}}\right)^{1/2}. \tag{49}$$

These formulas are derived from Eqs. (11)-(17), applying a widely used ansatz of the universal squark $\tilde{q}$ mass $m_{\tilde{q}}$ at the weak scale $m_{\tilde{u}} \approx m_{\tilde{d}} \approx m_{\tilde{q}}$. This approximation is well motivated by the constraints from the flavor changing neutral currents. Formula (49) takes into account only the lightest neutralino contribution with the mass m_χ. We also assume absence of spurious compensations between terms of different nature such as the $\tilde{g}-q-\tilde{q}$ and $\chi-e-\tilde{e}$. The running QCD coupling constant $\alpha_s(Q)$ has been taken at the scale $Q = 1$GeV with the normalization on the world average value $\alpha_s(M_Z) = 0.120$ [29]. The second limit in Eq. (49) is derived with the additional assumptions that the lightest neutralino is B-ino dominant and that $m_{\tilde{q}} \geq m_{\tilde{e}}/2$. Both these assumptions are phenomenologically reasonable, although they must not be always correct.

The best constraint from the Heidelberg-Moscow experiment [1] is

$$\lambda'_{111} \leq 1.3\cdot 10^{-4}\left(\frac{m_{\tilde{q}}}{100GeV}\right)^2\left(\frac{m_{\tilde{g}}}{100GeV}\right)^{1/2}, \tag{50}$$

$$\lambda'_{111} \leq 9.3\cdot 10^{-4}\left(\frac{m_{\tilde{e}}}{100GeV}\right)^2\left(\frac{m_{\chi}}{100GeV}\right)^{1/2} \tag{51}$$

These limits are very strong and, as it was already pointed out in Ref. [5]-[7], lie beyond the reach of the near future accelerator experiments (though, accelerator experiments are potentially sensitive to the other couplings than λ'_{111}).

To constrain the size of λ'_{111} itself one needs additional assumptions on the masses of the SUSY-partners. If the values of these masses would be around their present experimental lower limits $\sim$ 100GeV [29], one could constrain the coupling to

$$\lambda'_{111} \leq 1.3\cdot 10^{-4}. \tag{52}$$

A conservative bound can be set by assuming all the SUSY-masses being at the "SUSY-naturalness" bound of $\sim$ 1TeV, leading to

$$\lambda'_{111} \leq 0.04. \tag{53}$$

This completes our analysis. The other details concerning the experimental prospects for searching for $\not R_p$ SUSY in $0\nu\beta\beta$-experiments can be inferred directly from Table 2 and Fig. 3.

6 Conclusion

In summary, we have analyzed the general case of the pion realization for the short-ranged $\not R_p$ SUSY mechanism of $0\nu\beta\beta$-decay taking into account both the one-pion and two-pion modes. We have shown that the two-pion mode $\not R_p$ SUSY contribution to $0\nu\beta\beta$-decay dominates over the one-pion mode contribution. Previously [6] we had proven that the two-pion mode dominates over the conventional two-nucleon one. We also pointed out that non-observation of $0\nu\beta\beta$-decay casts severe limitations on the $\not R_p$ SUSY extensions of the standard model of electroweak interaction. Although a complicated nuclear many-body problem needs to be solved the limits are so stringent that they overcome the uncertainties in the nuclear and hadronic matrix elements, leading to limits that are much stronger than those from accelerator and the other non-accelerator experiments. We gave the list of nuclear matrix elements for all the experimentally interesting isotopes and presented the so called SUSY sensitivities of these isotopes. These characteristic might be helpful for planning future searches for SUSY in $0\nu\beta\beta$-decay. On this basis we compared the present status of the various $0\nu\beta\beta$-experiments and their abilities to detect the SUSY signal.

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TABLES

TABLE I. Nuclear matrix elements for the pion-exchange R-parity violating SUSY mode of $0\nu\beta\beta$-decay for the experimentally most interesting isotopes calculated within the renormalized pn-QRPA. G_{01} is the integrated kinematical factors for $0^+ \rightarrow 0^+$ transition [18]. $\zeta(Y)$ denotes according to Eq. (47) the sensitivity of a given nucleus Y to the SUSY signal.

Nucleus	$\mathcal{M}^{1\pi}_{GT}$	$\mathcal{M}^{1\pi}_{T}$	$\mathcal{M}^{2\pi}_{GT}$	$\mathcal{M}^{2\pi}_{T}$	$\mathcal{M}^{1\pi}$	$\mathcal{M}^{2\pi}$	G_{01} $\times 10^{15} y$	$\zeta(Y)$
^{76}Ge	1.30	-1.02	-1.34	-0.65	-18.2	-601	7.93	5.5
^{82}Se	1.23	-0.87	-1.26	-0.57	-23.9	-551	35.2	10.8
^{96}Zr	0.77	-1.11	-0.85	-0.67	22.1	-458	73.6	11.8
^{100}Mo	1.43	-1.73	-1.52	-1.05	19.4	-776	57.3	18.1
^{116}Cd	0.92	-0.78	-0.94	-0.47	-9.3	-423	62.3	10.8
^{128}Te	1.25	-1.57	-1.40	-0.99	21.4	-720	2.21	3.3
^{130}Te	1.10	-1.48	-1.26	-0.93	25.1	-660	55.4	14.9
^{136}Xe	0.61	-0.84	-0.74	-0.54	15.5	-387	59.1	9.0
^{150}Nd	1.85	-2.70	-2.07	-1.68	56.4	-1129	269.	55.6

TABLE II. The present state of the $R\!\!\!/_p$ SUSY searches in $\beta\beta$-decay experiments. $T^{exp}_{1/2}$(present) is the best presently available lower limit on the half-life of the $0\nu\beta\beta$-decay for a given isotope. η^{exp}_{SUSY} is the corresponding upper limit on the $R\!\!\!/_p$ SUSY parameter. $T^{exp}_{1/2}(\eta^{H-M}_{SUSY})$ is the calculated half-life of $0\nu\beta\beta$-decay assuming $\eta_{SUSY} = \eta^{H-M}_{SUSY}$ with η^{H-M}_{SUSY} being the best limit deduced from the Heidelberg-Moscow ^{76}Ge experiment [1].

Nucleus	^{76}Ge	^{82}Se	^{96}Zr	^{100}Mo	^{116}Cd
Ref.	[1]	[21]	[22]	[23]	[24]
$T^{exp}_{1/2}$(present)	1.1×10^{25}	2.7×10^{22}	3.9×10^{19}	5.2×10^{22}	2.9×10^{22}
η^{exp}_{SUSY}	5.5×10^{-9}	5.6×10^{-8}	1.4×10^{-6}	2.4×10^{-8}	5.4×10^{-8}
$T^{exp}_{1/2}(\eta^{H-M}_{SUSY})$	1.1×10^{25}	2.9×10^{24}	2.4×10^{24}	1.0×10^{24}	2.9×10^{24}

Nucleus	^{128}Te	^{130}Te	^{136}Xe	^{150}Nd
	[25]	[26]	[27]	[28]
$T^{exp}_{1/2}$(present)	7.7×10^{24}	8.2×10^{21}	4.2×10^{23}	1.2×10^{21}
η^{exp}_{SUSY}	1.1×10^{-8}	7.4×10^{-8}	1.7×10^{-8}	5.2×10^{-8}
$T^{exp}_{1/2}(\eta^{H-M}_{SUSY})$	3.1×10^{25}	1.5×10^{24}	4.1×10^{24}	1.1×10^{23}

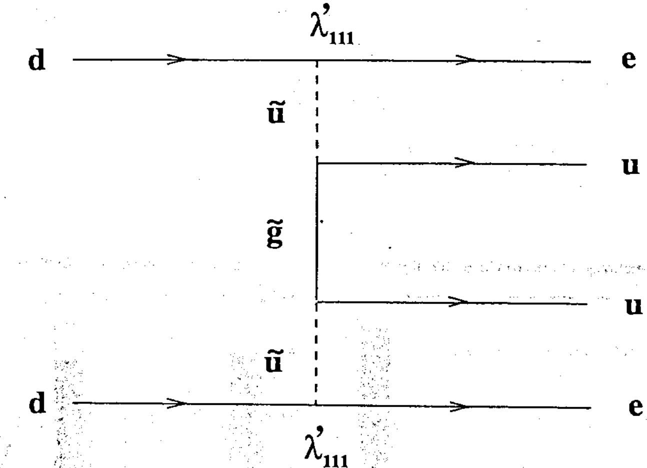

Fig. 1: An example of the supersymmetric contribution to $0\nu\beta\beta$-decay with the gluino $\tilde{g}$ and two squarks $\tilde{u}$ in the intermediate state.

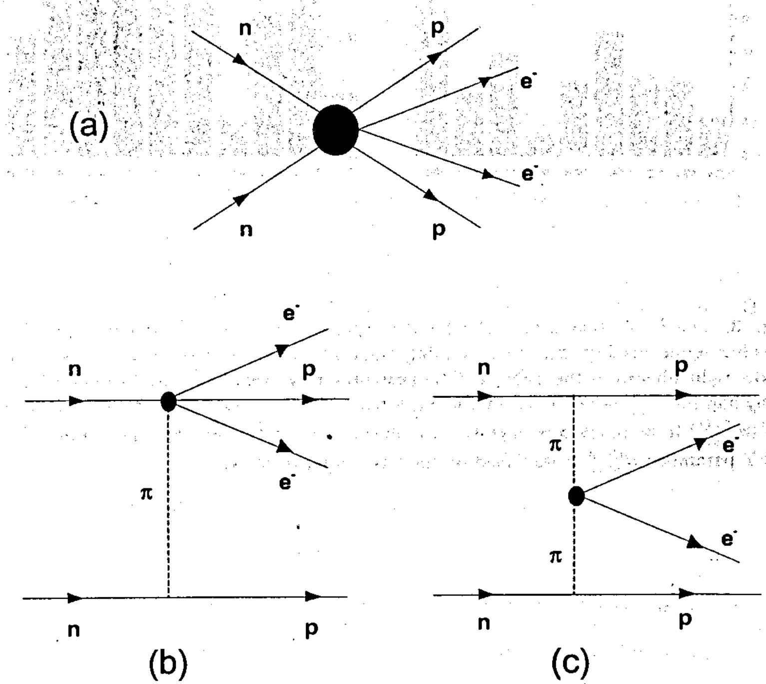

Fig. 2: The hadronic-level diagrams for the short-ranged SUSY mechanism of $0\nu\beta\beta$-decay. (a) the conventional two-nucleon mode, (b) the one-pion exchange mode, (c) the two-pion exchange mode.

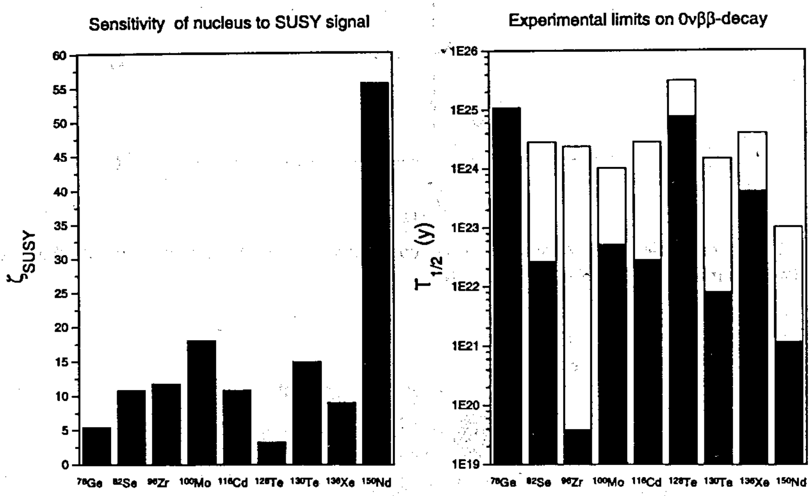

Fig. 3: The SUSY sensitivity $\zeta(Y)$ for the experimentally interesting nuclei (on the left). This histogram displays the corresponding numerical values in the Table 1. The histogram on the right illustrates the Table 2. The best presently available lower limits on the $0\nu\beta\beta$-decay half-life $T_{1/2}^{exp}$ are denoted by the black bars. The open bars indicate the half-life limits $T_{1/2}^{exp}(\eta_{SUSY}^{H-M})$ to be reached by a given experiment to reach the presently best limit on the $R\!\!\!/_p$ SUSY parameter η_{SUSY}^{H-M} established by the ^{76}Ge experiment [1].

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