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EFFECT OF RESIDUAL INTERACTION IN THE PARTICLE-PARTICLE CHANNEL ON LOWLYING EXCITATIONS IN SPHERICAL NUCLEI

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In the past few years alongside with more exact theoretical description of nuclear spectra within the traditional models (e.g., the "pairing + multipole- multipole interaction" model) there appeared a number of works dealing with the new components of effective nuclear forces. In particular, more thorough investigations are made for the forces in the particle-particle channel. The monopole forces in this channel are responsible for the existence of pairing correlations in atomic nuclei. These forces are widely studied /1/. Now the theorists investigate other components. In the deformed nuclei the influence of new forces in the particle-particle channel on the properties of one-phonon states was studied in ref. /2/. The form of new forces and their constants were deduced on the basis of the gauge invariance of the bare internucleonic interaction. This method was first proposed in ref. /3/. The role of the particle-particle forces in the formation of the characteristics 2_1^+ and 3_1^- levels of spherical nuclei was investigated in the framework of the theory of finite Fermi-systems. There are some other papers concerning this subject /5-8/.

Since it is unknown such a characteristic of nuclear excitations which originates only due to new forces in the particle-particle channel, the determination of their constants encounters certain difficulties. So, the authors of paper /4/ had to change the strength of the particle-particle interaction for various excitations. Attempts were made to determine the value of new constants on the basis of a correct descriptions of (p,t) reaction probabilities with the excitation of 2^+_1 states [5,6]. Note, the authors of paper [5] assert that only introducing the quadrupole pairing one can interpret the experiment on the above reactions. In paper [2,8] the values of quadrupole pairing constants were extracted from the constants of usual monopole pairing. The authors of papers [3,4,8] consider the $n-p$ interaction in the particle-particle channel to be absent, whereas in ref. [5,6] the following relation was assumed for the constants of these forces: $k_{nn}=k_{pp}=k_{np}$.

In the present paper we study the influence of the multipole-multipole interaction in the particle-particle channel on the energies and electromagnetic characteristics of 2^+ and 3^- excitation of spherical nuclei. For the constants of these forces we have used the above assumption of authors of refs. [5,6]. The case $k_{np}=0$ was studied in ref. [9].

The model Hamiltonian has the following form (such a form was used in ref. [5,6]).

$$H=\sum_{jm} E_j a^+_{jm} a_{jm} - \frac{G}{4} P^+ P - \frac{1}{2}\sum_{\lambda\mu} \kappa_\lambda Q^+_{\lambda\mu} Q_{\lambda\mu} + k_\lambda P^+_{\lambda\mu} P_{\lambda\mu} \qquad (1)$$

with the following notations:

$$P^+ = \sum_{jm} (-)^{j-m} a^+_{jm} a^+_{j-m}$$

$$P^+_{\lambda\mu} = \sum_{\substack{j_1 m_1\\ j_2 m_2}} \langle j_1 m_1 | (ir)^\lambda Y_{\lambda\mu} | j_2 m_2 \rangle a^+_{j_1 m_1} a^+_{j_2 m_2}$$

$$Q^+_{\lambda\mu} = \sum_{\substack{j_1 m_1\\ j_2 m_2}} \langle j_1 m_1 | (ir)^\lambda Y_{\lambda\mu} | j_2 m_2 \rangle a^+_{j_1 m_1} a_{j_2 m_2}$$

The equation for the energies of one-phonon states of doubly even nucleus in RPA can be deduced either directly by the variational principle using the usual definition of the phonon [1] or having specified general equation from ref. [10] for the multipole-multipole residual interaction. As a result we derive the secular equation:

$$\mathcal{F}(\omega_{\lambda i}) \equiv \det \begin{vmatrix} F_{\lambda i} - \frac{1}{\kappa_\lambda} & X^+_{\lambda i} & \omega_{\lambda i} X^-_{\lambda i} \\ X^+_{\lambda i} & G^+_{\lambda i} - \frac{1}{k_\lambda} & \omega_{\lambda i} G_{\lambda i} \\ \omega_{\lambda i} X^-_{\lambda i} & \omega_{\lambda i} G_{\lambda i} & G^-_{\lambda i} - \frac{1}{k_\lambda} \end{vmatrix} = 0 \qquad (2)$$

Let us interpret the definitions introduced in (2):

$$F_{\lambda i} = \sum_{jj'} \frac{(f^\lambda_{jj'} u^+_{jj'})^2 \epsilon_{jj'}}{\epsilon^2_{jj'} - \omega^2_{\lambda i}}; \quad G^+_{\lambda i} = \sum_{jj'} \frac{(f^\lambda_{jj'} v^-_{jj'})^2 \epsilon_{jj'}}{\epsilon^2_{jj'} - \omega^2_{\lambda i}}$$

$$G^-_{\lambda i} = \sum_{jj'} \frac{(f^\lambda_{jj'} v^+_{jj'})^2 \epsilon_{jj'}}{\epsilon^2_{jj'} - \omega^2_{\lambda i}}; \quad X^+_{\lambda i} = \sum_{jj'} \frac{(f^\lambda_{j'j})^2 u^+_{jj'} v^+_{jj'}}{\epsilon^2_{jj'} - \omega^2_{\lambda i}} \epsilon_{jj'}$$

$$X^-_{\lambda i} = \sum_{jj'} \frac{(f^\lambda_{jj'})^2 v^+_{jj'} u^+_{jj'}}{\epsilon^2_{jj'} - \omega^2_{\lambda i}} \epsilon_{j'j}; \quad G_{\lambda i} = \sum_{jj'} \frac{(f^\lambda_{jj'})^2 v^+_{jj'} v^-_{jj'}}{\epsilon^2_{jj'} - \omega^2_{\lambda i}} \epsilon_{jj'} \qquad (3)$$

In (3) $f^\lambda_{jj'}$ denotes the reduced single-particle matrix element of the operator of the multipole moment, $\epsilon_{jj'}$, $\omega_{\lambda i}$ are the energies of the two-quasiparticle and one-phonon states, respectively.

$$u^+_{jj'} = u_j v_{j'} + u_{j'} v_j ; \quad v^\pm_{jj'} = u_j u_{j'} \pm v_j v_{j'}$$

u_j, v_j are the Bogolubov transformation coefficients. It is easy to see that for $k_\lambda = 0$ eq. (2) converts to the well-known equation for the energy of a usual phonon generated by the multipole-multipole forces in the particle-hole channel. If $\kappa_\lambda = 0$ we derive the equation for the phonon generated by the forces in the particle-particle channel.

$$(G^{+}_{\lambda i} - \frac{1}{k_{\lambda}})(G^{-}_{\lambda i} - \frac{1}{k_{\lambda}}) - \omega^{2}_{\lambda i}\, G^{2}_{\lambda i} = 0$$

In the general case the contribution of one or another channel to the phonon structure is determined by the relative value of appropriate constants. We do not present the expressions for the amplitudes of two-quasiparticle components composing the phonon as they are very cumbersome (see refs. /6,9/).

As is known, the energies of 2^{+}_{1} levels and probabilities of its discharge in doubly even spherical nuclei are satisfactorily reproduced with the quadrupole-quadrupole forces in the particle-hole channel. However, one has to choose κ_2 especially in each nucleus /11,12/ . Besides,, the structure of 2^{+}_{1} levels is very collective /12/ in most of the nuclei and this results in a very large value of anharmonic corrections. One may hope that the inclusion of the quadrupole pairing will destroy to a certain extent the coherent structure of 2^{+}_{1} states and will decrease its collectivity.

The concrete calculations are performed for some isotopes of Te and Cd. The parameters of the Saxon-Woods single-particle potential and the values of the correlation functions and chemical potentials are taken from ref. /13/. In calculations we have taken into account the single-particle levels from the bottom of the potential approximately to 15 *MeV* energy in the quasidiscrete spectrum. The dimension of single-particle basis could allow one to use $e_{eff} = 0$.

Since the correct description of the experimental energy of the 2^{+}_{1} level is the starting point in the determination of constants, we have investigated the form of the function $\omega_{2^{+}_{1}}(\kappa_2, k_2) = E(2^{+}_{1})_{exp}$. *Figures 1* and *2* represent the curves for the isotopes of Te and Cd , respectively. Their form is practically the same in all nuclei, the only difference is in the absolute value of constants. It is known, that the sensibility of $\omega_{2^{+}_{1}}$ to κ_2 (in the case $k_2 = 0$) is large and even a small

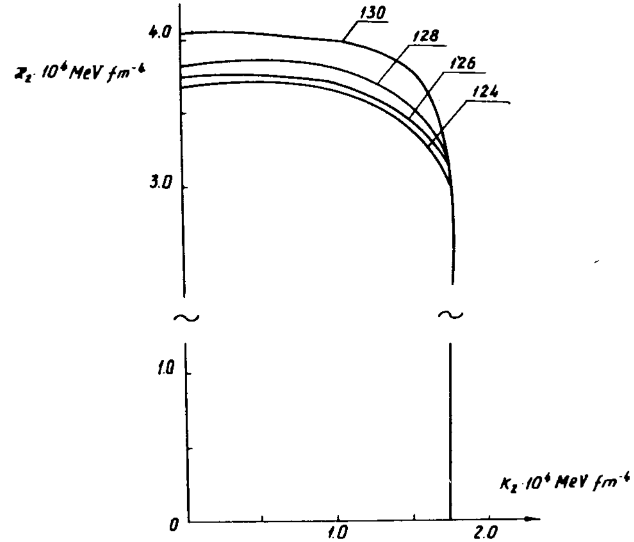

Fig. 1. The curves $\omega_{2^{+}_{1}}(\kappa_2, k_2) = E(2^{+}_{1})_{exp}$ *for isotopes* $^{124-130}$Te.

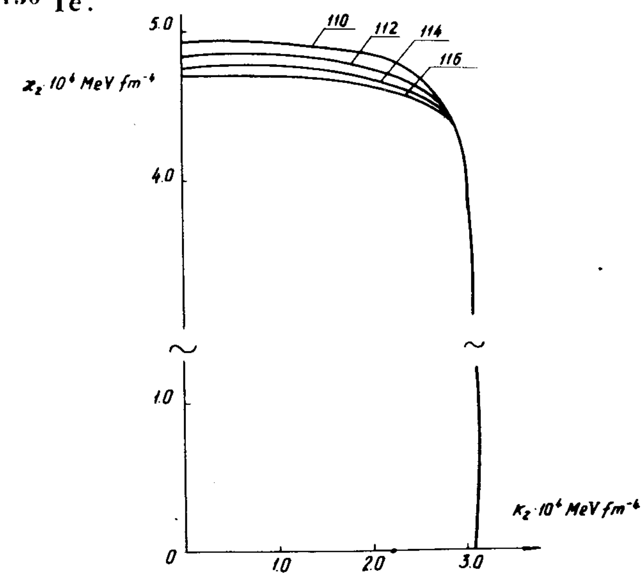

Fig. 2. The curves $\omega_{2^{+}_{1}}(\kappa_2, k_2) = E(2^{+}_{1})_{exp}$ *for isotopes* $^{110-116}$Cd.

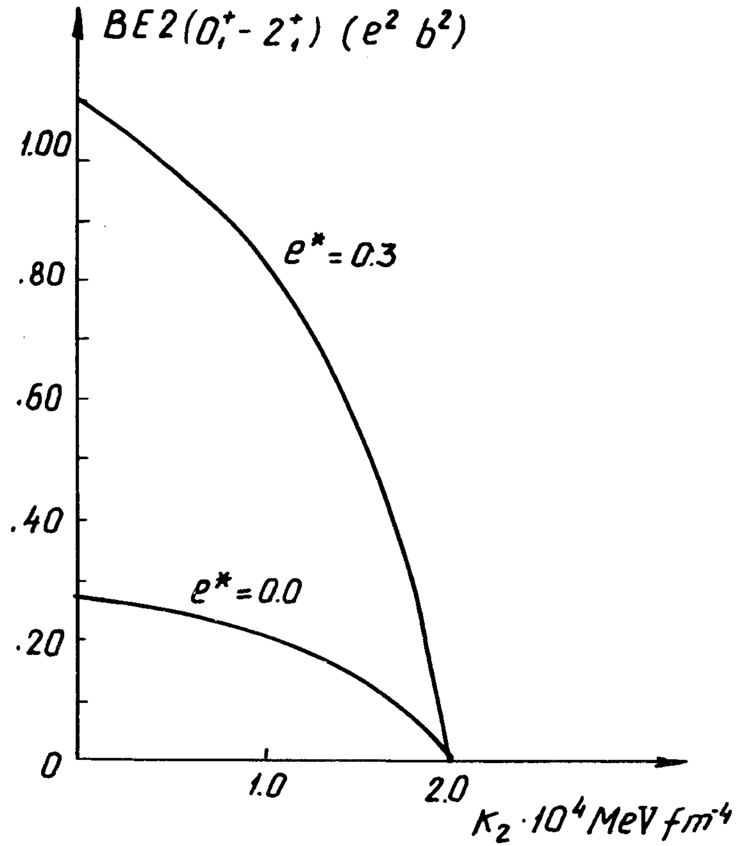

Fig. 3. B(E2)$_{theor.}$ *as a function of* k_2, κ_2 *for two values* e_{eff} (e_{eff} =0.3, 0.0) *in* ^{126}Te. *Constants* κ_2, k_2 *change along the appropriate curve on fig. 1.*

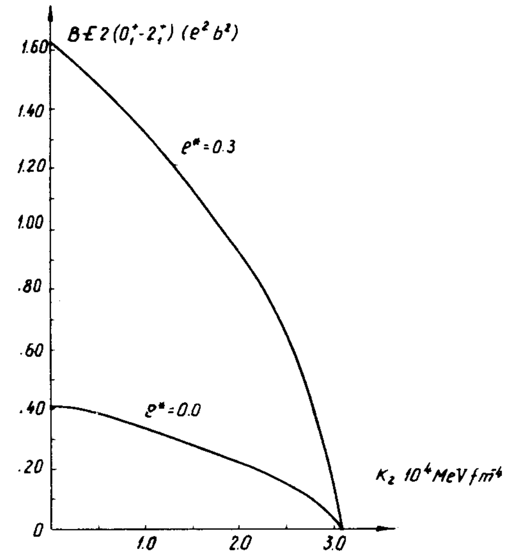

Fig. 4. B(E2)$_{theor.}$ *as a function of* k_2, κ_2 *for two values* e_{eff} (e_{eff} =0.3, 0.0) *in* ^{114}Cd. *Constants* κ_2, k_2 *change along the appropriate curve on fig. 2.*

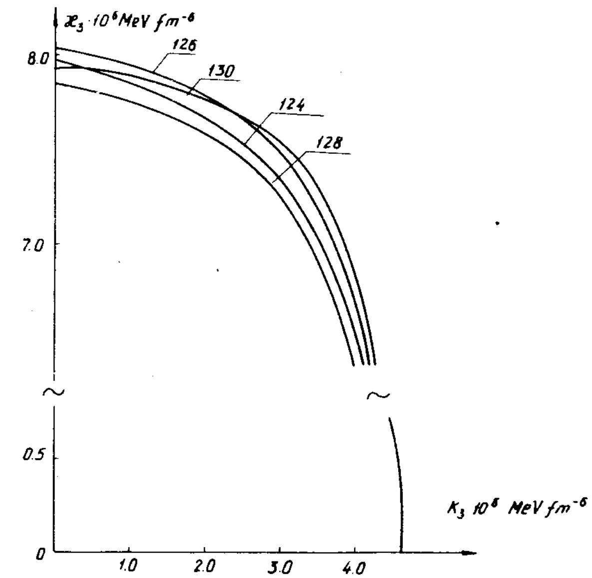

Fig. 5. The curves $\omega_{3_1^-}(\kappa_3, k_3) = E(3_1^-)_{exp}$ *for isotopes* $^{124-130}$Te .

deviation of constants may result in considerable errors in the energies of levels [12]. Due to this fact one can not satisfactorily describe the energies of κ_2 states by the constant value of κ_2 even in different isotopes of one element. The quadrupole pairing leads to the convergence of values κ_2 for different isotopes and the decrease of the sensibility of $\omega_{2_1^+}$ to its value. Having chosen the values κ_2 and k_2 from the banding region of curves in *figs. 1* and *2*, we can describe by the same values the energies of 2_1^+ levels in different isotopes with the deviations not larger than 150 *keV*. The calculation with the constant κ_2 (when $k_2 = 0$) indicates a very large deviation $\omega_{2_1^+}$ from the experiment in proportion to removal from the isotope which caused the choice of the κ_2. These results are presented in *Tables 1* and *2* (for the isotopes of Te and Cd). The constants for Te and Cd noticeably differ, as is seen from the values given in *Tables 1* and *2*. However, the ratio κ_2 / k_2 is approximately the same both for the tellurium and cadmium isotopes. In the first case it is equal to 2.14, in the second , to 1.9. Note, also, that the equality

$$\frac{\kappa_2(Cd)}{\kappa_2(Te)} \approx \frac{k_2(Cd)}{k_2(Te)} \approx \left[\frac{A(Cd)}{A(Te)}\right]^{7/3}$$ is fulfilled with accuracy 15%. Certainly, to make the conclusions more definitely one should perform investigations in a more wide mass range.

Now, we analyze the probabilities of E2-transitions. The change of the constants along the curves in *figs.1,2* (i.e., such a change when $\omega_{2_1^+}(\kappa_2, k_2) = E(2_1^+)_{exp}$) leads to a rapid decrease of the quantity B(E2). This is demonstrated in *figs. 3,4* (for ^{126}Te and ^{112}Cd, respectively). Since $B(E2)_{theor.}$ is less than the experimentally determined B(E2) already at $k_2 = 0$, then at the values of κ_2 and k_2 from *Tables 1,2* this difference becomes rather noticeable and is compensated only by increasing e_{eff} (for Te $e_{eff} = 0.2$, for Cd $e_{eff} = 0.3$). For the used

Table 1

The results of calculation with constant values of κ_2, k_2 in isotopes $^{124-130}$Te

Nucleus	$E(2_1^+)$ MeV Exp.	$E(2_1^+)$ MeV $\kappa_2=3.45\times10^{-4}$ MeV.fm^{-4}, $k_2=1.61\times10^{-4}$ MeV.fm^{-4}	$E(2_1^+)$ MeV $\kappa_2=3.8\times10^{-4}$ MeV.fm^{-4}, $k_2=0$	$B(E2,\ 0^+_{g.s.}\to 2_1^+)\ e^2 b^2$ Exp.	$B(E2,\ 0^+_{g.s.}\to 2_1^+)\ e^2 b^2$ Theory $k_2\neq 0$, $e_{eff}=0.2$	$B(E2,\ 0^+_{g.s.}\to 2_1^+)\ e^2 b^2$ Theory $k_2=0$, $e_{eff}=0$
^{124}Te	0.602	0.45	0.16	0.60	0.75	1.27
^{126}Te	0.666	0.57	0.41	0.49	0.45	0.49
^{128}Te	0.743	0.72	0.83	0.39	0.31	0.23
^{130}Te	0.840	0.91	1.27	0.30	0.20	0.14

Table 2

The results of calculations with constant values of κ_2, k_2 in isotopes $^{110-116}$Cd

Nucleus	$E(2_1^+)$ MeV			$B(E2,\ 0^+_{g.s.} \rightarrow 2_1^+)\,e^2b^2$		
	Exp.	$\kappa_2=4.66\times10^{-4}$ MeV . fm^{-4} $k_2=2.46\times10^{-4}$ MeV . fm^{-4}	$\kappa_2=4.81\times10^{-4}$ MeV . fm^{-4} $k_2=0$	Exp.	Theory $k_2\neq0$ $e_{eff}=0.3$	Theory $k_2\neq0$ $e_{eff}=0.0$
^{110}Cd	0.658	0.73	1.01	0.46	0.42	0.21
^{112}Cd	0.617	0.64	0.78	0.51	0.48	0.29
^{114}Cd	0.558	0.53	0.44	0.56	0.58	0.54
^{116}Cd	0.514	0.42	-	0.62	0.74	-

number of the single-particle levels such a value e_{eff} seems to be very large. However, it provides better agreement with experiment than at κ_2 = const. and $k_2=0$ (see *Tables*) though in both cases the changes of B(E2) from isotope to isotope are correctly described. Note, that the authors of all the papers concerning the study of electromagnetic transitions with the particle-particle channel encountered the necessity to increase the effective charge. Thus, in paper /4/ at a sufficiently wide single-particle basis (three major shells) the effective charges for protons and neutrons were taken the following: $e^Z_{eff} = 0.3$, $e^N_{eff} = 1.0$. But even on this condition in the Cd isotopes $B(E2)_{theor}$ is less than $B(E2)_{exp}$ by a factor of 2-2.5. If one uses the values of the constants from ref. /5/, the quantity B(E2) at $e_{eff}=0$ will appear to be by one order of magnitude less than the experimental one. Large effective charges were used in paper /2/. The cause of the decrease of $B(E\lambda)$ while increasing of k_2 is of two kinds. First there is destroyed the coherence in the expression for the transition probability which takes place due to the coincidence of the E2-transition operator with the quadrupole-quadrupole interaction operator in the particle-hole channel. Second, the collectivity of the 2_1^+ state is weakened with increasing k_2 (this is specified quantitatively by decreasing $\mathcal{F}'(\omega)$). Note, also, that the decrease of $\mathcal{F}'(\omega)$ results in the weakening of the interaction of quasiparticles with phonons, or, by the other words, the decrease of anharmonic corrections.

Besides, we have calculated the quadrupole moment of the first 2^+ levels. It is true that these calculations are of methodical interest only, since to interpret the experimental quantity $Q_2(2_1^+)$ it is necessary to take into account the anharmonicity of nuclear vibrations. However, in ref. /14/ attempt was made to explain the value of $Q_2(2_1^+)$ without such corrections. In this calculations the particle-particle channel was also taken into account. Our results do not confirm those of ref. /14/ and point out a small value of $Q_2(2_1^+)$ Obviously, it is due to extreme roughness of calculations of this paper.

The results for the 3^- states are the same as for the 2^+ states in the main features. In *fig. 5* we have plot the curves $\omega_{3^-_1}(\kappa_3, k_3) = E(3^-_1)_{exp}$ for $^{124-130}Te$. As compared with *fig. 1* these curves intersect, and the distance between the curves for different isotopes is larger. However, the energies of 3^-_1 levels in different isotopes can be described at constant values k_3, κ_3 with the same accuracy as for the 2^+_1 levels. The quantity $B(E3)$ is less sensitive to the inclusion of the particle-particle interaction as $B(E2)$ and coincides with the experimental value when $e_{eff} \approx 0.1 \div 0.2$.

It seems we can make the following conclusions from the present results. The inclusion of the new particle-particle forces improves the description of the energies and reduced transition probabilities of 2^+_1 levels in doubly even spherical nuclei. We can calculate these characteristics with the constant values of k_2 and κ_2 for different isotopes of a given element. However, it is necessary to use the larger value of e_{eff}. The influence of octupole-octupole forces in particle-particle channel is less significant. Further investigations should take into account the interaction of quasiparticles with phonons and try to describe the so-called "two-phonon" states.

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