ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна



E4-97-53

E.A.Kuraev

# QED RADIATIVE CORRECTIONS TO THE PIONIUM LIFE TIME

Submitted to «Ядерная физика»

1997

# 1 Introduction

The precise measurement of the pionium life time [1] provide a crucial experiment to verify the (different) predictions of the Chiral Perturbation Theory (CHPT) concerning pion–pion interaction [2, 3]. Considering the conversion of two charged pions to two neutral due to strong interactions as the main mechanism[1] which determine the pionium lifetime $\tau$ the formula for it was derived many years ago [4]:

$$\tau_0^{-1} = \frac{16\pi\sqrt{2}}{9}(1-\frac{m_{\pi_0}}{m_{\pi_+}})^{\frac{1}{2}}(a_0-a_2)^2|\Psi_{n,0}(0)|^2[1+\frac{2}{9}m_{\pi_+}(m_{\pi_+}-m_{\pi_0})(2a_2+a_0)^2]^{-1}. \quad (1)$$

Right-hand side of (1) have a factorized form. One factor $|\Psi_{n,0}(0)|^2$ have a pure Coulomb interaction origin and describe the probability to form the pionium $s$-wave state. Another, $(a_0 - a_2)^2$, describes the low-energy pions-conversion process and the quantities $a_0, a_2$ are the scattering lengths in the states with the isotopic spin $0, 2$. Remaining factors have a kinematical origin.

The problem of calculation of corrections to $\tau^{-1}$ is rather subtle one. First, the isotopic-spin classification is violated by electromagnetic interactions, the electromagnetic interactions themselves provide the existence of pionium atom, and, finally, the influence of the strong interactions on the value of the wave function at zero distance is to be taken into account. So I propose such form of the corrected pionium lifetime:

$$\tau^{-1} = \tau_0^{-1}(1+\delta_\psi)(1+\delta)(1+\delta_a). \quad (2)$$

The quantities $\tau_0^{-1}$ is given in (1) and corresponds to the case when all corrections are switched off; $\delta_\psi$ include the corrections arising from modifications of Bethe–Salpiter equation for $\Psi(r)$ due to strong interactions [5], the quantity $\delta_a$ include the higher orders of CHPT contributions [2]. Main attention here will be paid to calculation of the pure QED correction (RC), $\delta$, in the lowest order of perturbation theory (PT).

Paper is organized as follows.In part 1 the recharged process

$$\pi^+(p_2) + \pi^-(p_1) \to \pi^0(q_1) + \pi^0(q_2) \quad (3)$$

for the case of free pions moving with the small relative velocity $v \approx 2\beta$, is considered using the approximation of the point–like pions interacting with the electromagnetic field. Calculating the virtual corrections we meet as

[1]The two quantum annihilation mechanism may be separated experimentally as well as it corresponds to photon energy (in the pionium center-of-mass frame) equal to pion mass. The background from four photon annihilation channel is of order $(\alpha/\pi)^2$, i.e. negligible.

usually the infrared, ultraviolet divergences, and the known Coulomb factor which describe the Coulomb interaction of the slow moving charged particles. It is shown that the last factor corresponds to small values of the loop momenta and is to be omitted when considering the Coulomb interaction corrected cross section of recharged process (3) (the similar problem arose in calculation of RC to parapositronium width calculation [7]). The infrared singularities are removed by the usual way when taking into account the emission of real soft photons. Some arguments are given to choice the ultraviolet cut-off parameter to be of order of $\rho$-meson mass, $\Lambda = m_\rho$. Within the same approach we also consider the realisitic case of bounded pions as components of pionium atom. In the second part we perform the calculations in frame of effective chiral lagrangian with vector meson dominance [6].

1.Consider first the RC to the cross section of the conversion process in frames of scalar electrodynamics when pions directly interact with the electromagnetic field. We will suppose the pions be real $p_i^2 = m_+^2 = m^2$, $q_i^2 = m_0^2$, $i = 1, 2$ and that they have a small relative velocity $v = 2\beta = 2\sqrt{1 - \frac{m^2}{\varepsilon^2}}$, where $\varepsilon$ is the energy of one of pions in the cms. The lowest order virtual correction in the case of point–like pion are determined by quantity $\delta_{virt}$:

$$\begin{aligned} \delta_{virt} &= \frac{2ReM_0M^{(1)*}}{|M_0|^2} = \frac{\alpha}{2\pi}(2a+b), \\ a &= \ln\frac{\Lambda^2}{m^2} + \ln\frac{m^2}{\lambda^2} - \frac{3}{4}, \\ b &= \int dk \frac{(2p_1-k)(-2p_2-k)}{(k^2-\lambda^2)(k^2-2p_1k)(k^2+2p_2k)}, \\ |k^2| &< \Lambda^2, \quad dk = \frac{d^4k}{i\pi^2}. \end{aligned} \tag{4}$$

In this equation $\Lambda, \lambda$ are the ultraviolet, infrared momentum cut-off's, $m$ is the renormalized pion mass. The quantity $a$ is related with the pion wave function renormalization:

$$G_\pi(k) = \frac{i}{k^2 - m_{bare}^2 - \Sigma(k)} \to \frac{iZ^f}{k^2 - m^2}, \quad Z^f = 1 + \frac{\alpha}{2\pi}a. \tag{5}$$

The standard calculation of the loop integral give

$$\begin{aligned} b &= \int_0^1 dx[\ln\frac{\Lambda^2}{p_x^2} - 1 + \frac{2m^2(1+\beta^2)}{(1-\beta^2)p_x^2}\ln\frac{p_x^2}{\lambda^2} - \frac{4m^2\beta^2}{p_x^2(1-\beta^2)}], \\ p_x^2 &= \frac{m^2}{1-\beta^2}[(1-2x)^2 - \beta^2] - i0. \end{aligned} \tag{6}$$

Using the I. Harris and L. Brown [7] result



$$\mathrm{Re} \int_0^1 \frac{dxm^2}{p_x^2} ln\frac{p_x^2}{\lambda^2} = -(1 - \frac{2}{3}\beta^2)(2 + \ln\frac{m^2}{\lambda^2}) - \frac{2}{9}\beta^2 + \frac{\pi^2(1-\beta^2)}{2\beta} + 0(\beta^3), \tag{7}$$

we obtain for contribution of loop integrals:

$$\begin{aligned} \delta_{virt} &= \frac{\alpha}{2\pi}[3\ln\frac{\Lambda^2}{m^2} - \frac{8}{3}\beta^2\ln\frac{m^2}{\lambda^2} \\ &- \frac{9}{2} - \frac{34}{9}\beta^2 + \frac{\pi^2}{\beta}(1+\beta^2)] + O(\frac{\alpha}{\pi}\beta^3). \end{aligned} \tag{8}$$

The contribution from emission of real photon (we may use here the soft photon emission approximation) gives:

$$\begin{aligned} \delta_{soft} &= -\frac{\alpha}{4\pi^3}\int \frac{d^3k}{\sqrt{\lambda^2+k^2}}(\frac{p_1}{p_1k} - \frac{p_2}{p_2k})^2, |k| < \delta\varepsilon = m - m_0. \\ \delta_{soft} &= \frac{\alpha}{\pi}\beta^2(\frac{8}{3}\ln\frac{2\delta\varepsilon}{\lambda} - \frac{20}{9}) + O(\frac{\alpha}{\pi}\beta^3). \end{aligned} \tag{9}$$

In the total sum $\delta_{virt} + \delta_{soft}$ the dependence on the photon mass $\lambda$ disappears. Keeping in mind that the effective velocities in pionium atom are of order of fine structure constant (in units of light velocity) we may neglect the contributions of order $\frac{\alpha}{\pi}\beta^2$.

Now we note that the term $\frac{\pi^2}{\beta}(1+\beta^2)$ in $\delta_{virt}$ (8) is the lowest order of expansion of the known Coulomb factor $J(x)$ which is to be included in matrix element module squared with charged particles in the initial state. This factor describe the Coulomb interaction of charged particles with the small relative velocity $v$. In the case of equal mass and opposite charges it have a form ($x = \frac{\alpha}{v}, v \approx 2\beta$):

$$J(x) = \frac{2\pi x}{1 - \exp\{-2\pi x\}} = 1 + \frac{\pi\alpha}{v}. \tag{10}$$

It arises from the region of small energies and 3-momenta of virtual loop photons $|k^0| \simeq m\alpha^2, |\vec{k}| \simeq m\alpha$.

Now we argue that the natural choice of the ultraviolet cut-off parameter $\Lambda$ is the typical vector meson mass, $\Lambda = m_\rho$. Really, at large values of loop momenta $|k|$ pion is to be considered as a quark-antiquark system. The conversion process a this level may be interpreted as an exchange by $u, d$ or $\bar{u}, \bar{d}$ quarks accompanied by (multi)gluon exchange. By simple power counting one may see that the virtual photon loop intergals are convergent. The natural scale of the loop momenta which provide its main contribution of order of typical hadron's mass, $|k| \simeq m_\rho$. This quantity play the role of the ultraviolet cut-off parameter, $\Lambda$. Really, the theoretical uncertainty of order

$\alpha/\pi$ in the RC will appear in such an approach as well as the confinement mechanism is purely investigated. Note that the choice $\Lambda = m_\rho$ crucially differs from the $\Lambda = M_Z$ which is natural for the case of semileptonic decays of pions [8, 9].

For the coulomb-corrected cross section of conversion process (6) $\sigma = J(x)\sigma_c$ we obtain:

$$\sigma_c = \sigma_0(1+\delta^f_{QED}), \delta^f_{QED} = \frac{\alpha}{\pi}[3\ln\frac{\Lambda}{m} - \frac{9}{4} + 0(1)], \Lambda \approx m_\rho. \qquad (11)$$

and $\sigma_0$ is the cross section without QED corrections. Main source of errors here due to uncertainty in the quantity $\Lambda$.

More exact results may be obtained in concrete models, such as the quark model. We will consider below the Effective Chiral Lagrangian with vector meson dominance, the particle analog of the quark model for this aim.

In the same frames of scalar QED we consider now the case of bound pions, $p_1^2 - m^2 = p_2^2 - m^2 = -\Delta$, where $\Delta \approx m^2\alpha^2$, i.e. the correction to pionium life time.

Again the virtual corrections will have a form as in previous case with the replacements

$$a \to a' = \ln\frac{\Lambda^2}{m^2} + 2\ln\frac{m^2}{\Delta} - \frac{3}{4}; b \to b' = \ln\frac{\Lambda^2}{m^2} + 1 + \int\frac{dx}{p_x^2}ln\frac{p_x^2}{\Delta} + 0(\beta^2). \quad (12)$$

In this case the emission of the real photons is forbidden. The Coulomb factor is to be removed to avoid the double counting as well as it is taken into account in the factor $|\Psi(o)|^2$ in (1). The resulting expression for the correction $\delta$ (2) have a form:

$$\delta_{QED} = \frac{\alpha}{\pi}(3\ln\frac{\Lambda}{m} - \frac{25}{4} + 0(1)), \Lambda = m_\rho. \qquad (13)$$

2.In frames of Effective Chiral Lagrangian with the vector meson dominance [6]:

$$L = \frac{1}{2}[m_\rho^2\rho_\mu^2 + (\partial_\mu\pi^+)^2 + (\partial_\mu\pi^-)^2] - \frac{e}{g}m_\rho^2 A_\mu\rho_\mu - ig\rho_\mu(\pi^+(\partial_\mu\pi^-) - \pi^-(\partial_\mu\pi^+)) + \ldots \qquad (14)$$

with irrelevant terms omitted, the direct interaction of pions to electromagnetic field absent.



The similar calculations[2] give the values of $Z_\pi$ for the cases of free pions and the bound ones:

$$Z_\pi^f = 1 + \frac{\alpha}{2\pi}[\ln\frac{m_\rho^2}{m^2} + \ln\frac{m^2}{\lambda^2} - \frac{7}{4} + \frac{m^2}{m_\rho^2}(\ln\frac{m_\rho^2}{m^2} - \frac{7}{6}) + 0((\frac{m}{m_\rho})^4)],$$

$$Z_\pi = 1 + \frac{\alpha}{2\pi}[\ln\frac{m_\rho^2}{m^2} + 2\ln\frac{m^2}{\Delta} - \frac{7}{4} + \frac{m^2}{m_\rho^2}(\ln\frac{m_\rho^2}{m^2} - \frac{7}{6}) + 0((\frac{m}{m_\rho})^4)]. \quad (15)$$

The final expressions for $\delta^f, \delta$ have a form:

$$\delta^f = \frac{\alpha}{2\pi}[3\ln\frac{m_\rho^2}{m^2} - \frac{15}{2} - \frac{m^2}{m_\rho^2}(\frac{16}{3}\ln\frac{m_\rho^2}{m^2} + \frac{98}{9}) + 0((\frac{m}{m_\rho})^4)],$$

$$\delta = \frac{\alpha}{2\pi}[3\ln\frac{m_\rho^2}{m^2} - \frac{23}{2} - \frac{m^2}{m_\rho^2}(\frac{16}{3}\ln\frac{m_\rho^2}{m^2} + \frac{98}{9}) + 0((\frac{m}{m_\rho})^4)]. \quad (16)$$

Numerically these quantities close to the case of scalar QED considered above. The contributions from Feynman graphs containing $\rho\pi\omega$ vertex turns out to be small (do not exceed 0.1%). We do not consider here the contributions arising from excited mesons in the intermediate state. It is, presumably also small (do not exceed 0.1%) due to smallness of their coupling to the pions. This question remains open.

The author is indebted to L.L. Nemenov who turns my attention to this problem and to J. Gasser, A. Smilga, M. Vysotsky, V. Novikov, M. Volkov, S. Gerasimov, A.V. Tarasov and A. Arbuzov for discussions.

# References

[1] L.G. Afanasjev et al., Phys. Lett. B308 (1993) 200;
L.L. Nemenov, Yad. Fiz. 15 (1972) 1047; ibid 16 (1972) 1047; ibid 41 (1985) 980.

[2] J. Gasser and H. Leutwyller, Ann. Phys. (N.Y.) 158 (1984) 142;
J. Bijnens et al., Nordita Preprint 95/77 N/P.

[3] M.Knecht and J.Stern The Second DAFNE Physics Handbook,1995,169.

[2] Technically it results as a factor $(m_\rho^2/(k^2 - m_\rho^2))^2$ in the integrand for $a$ and the similar change in calculation $Z_\pi$.Loop momentum integrals become ultraviolet convergent.

[4] S. Mandelstam, Proc. Roy. Soc. 233 (1954) 248;
S. Weinberg, Phys. Rev. Lett. 17 (1966) 616;
J.L. Uretsky and T.R. Palfrey, Phys. Rev. 121 (1961) 1798;
S. Deser et al., Phys. Rev. 96 (1954) 774.

[5] Z.K. Silagadze, JETP Lett. 60 (1994) 689;
V.E. Lubovitsky and A.G. Rusetsky, Phys. Lett. B389 (1996) 181.

[6] Y. Brihaye, N.K. Pak and P. Rossi, Phys. Lett. 164B (1985) 111.

[7] G. Adkins, Ann. of Phys. 146 (1983) 78;
V.N. Baier and V.S. Fadin, ZhETP 57 (1969) 225;
I. Harris and L. Brown, Phys. Rev. 105 (1957) 1656.

[8] A. Sirlin, Nucl. Phys. B196 (1982) 83.

[9] R. Decker and M. Finkemeier, Nucl. Phys. B438 (1995) 17.

Received by Publishing Department
on February 21, 1997.