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NEW POSSIBILITIES OF STUDYING PROPERTIES OF DEFORMED NUCLEI AT INTERMEDIATE AND HIGH EXCITATION ENERGIES

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Новые возможности исследования свойств деформированных ядер при средних и высоких энергиях возбуждения

Проведено дальнейшее развитие версии квазичастично-фононной модели деформированных ядер (КФМЯ), в которой используется обобщенный оператор фонона, содержащий произвольное число компонент различной мультипольности электрического и магнитного типов. Для этого случая обобщен метод силовых функций. Рассмотрены статистические моменты энергетического распределения различных компонент волновых функций, полученные с использованием энергетически-взвешенных правил сумм. Проведено их сравнение с рассчитанными численно усредненными характеристиками фрагментации. Предложена процедура расчета свойств A-нечетного ядра с учетом полного набора фононных состояний соответствующего ему четно-четного остова.

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Malov L.A. E4-97-362
New Possibilities of Studying Properties of Deformed Nuclei at Intermediate and High Excitation Energies

The version of the quasiparticle-phonon nuclear model of deformed nuclei (QPNM) has further been developed which makes use of the generalised phonon operator containing an arbitrary number of components of different multipolarity of the electric and magnetic types. The strength function method has been generalised for this case. Statistical moments of the energy distribution of different components of the wave functions, which were obtained with the use of energy-weighted sum rules, are considered. They are compared with the numerically calculated characteristics of fragmentation. A procedure of calculating properties of an odd-A nucleus is proposed which takes into account a complete set of phonon states of the relevant doubly even core.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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Introduction

Papers [1,2] give a thorough analysis of specific features of deformed nuclei which manifest themselves while studying their structure at intermediate and high energies within the microscopic approach [3-5]. What is implied are excitations of an order of the nucleon binding energy and higher: 3 MeV $\leq E \leq 20 \div 30$ MeV.

In particular, it is important to take into account that phonon excitations of nuclei specified by a certain projection of the K angular momentum onto the symmetry axis of a nucleus and parity π may be generated by different multipole and spin-multipole interactions. A correct description in this case can be achieved by introducing a phonon operator with a definite momentum projection $\mu = K$ [1,2,6-8] but containing components with a different multipolarity λ.

The use of an approach like that allowed one to develop a version of the quasiparticle-phonon nuclear model (QPNM) for a unique consideration of states of the electric and magnetic types [1,2]. These papers differ by the way of introducing a generalised phonon operator and the form of its representation though their final results are very close. Moreover, in [1] tensor forces and multipole pairing have been taken into account.

To overcome additional difficulties that arise at the considered excitation energies and are due to a large density of nonrotational states and complexity of their structure, the strength function (SF) method, earlier

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developed in [9,10], has been improved in [2]. However, it was generalised only to odd-A nuclei. In this paper, we succeeded in generalising it to a more complicated case. Here we present other examples of further development of QPNM:

1.The SF method is developed for the version of QPNM with a generalized phonon.

2.Averaged characteristics (statistic moments) of the fragmentation of simple configurations are calculated.

3.The method is worked out for computing an odd-A nucleus with the allowance for a complete set of states of the phonon spectrum of the corresponding core(i.e. without using the conventional cut-off of the phonon space).

The basic idea of the studies and their purpose is to avoid some intermediate computations, sometimes either unnecessary or rather detailed, and to directly determine nuclear characteristics observed experimentally by using both the general mathematical properties and peculiarities of QPNM equations.

1 Development of the SF method for a generalized version of QPNM (doubly even nuclei)

In ref. [2], the SF method was generalized to the case of odd-A nuclei. It was also extended to the case when averaging was made over both initial and final states of a nucleus, i.e. a two-dimensional SF was introduced. The application of this method to doubly even nuclei has certain mathematical peculiarities due to the secular equation

$$|\mathcal{F}^{\mu}_{\ell\ell'}(\omega_{\mu i})| \equiv \mathcal{F}^{\mu}(\omega_{\mu i}) = 0 \tag{1}$$

being nonsymmetric because of neutron and proton variables. It defines the energy spectrum $\omega_{\mu i}$ of phonons for the system of RPA equations of a doubly even nucleus [1,2]

$$\begin{pmatrix} \kappa_{+}^{(\ell\mu)} X_{\ell\ell'}^{N\mu i} - \delta_{\ell\ell'} & \kappa_{-}^{(\ell\mu)} X_{\ell\ell'}^{Z\mu i} \\ \kappa_{-}^{(\ell\mu)} X_{\ell\ell'}^{N\mu i} & \kappa_{+}^{(\ell\mu)} X_{\ell\ell'}^{Z\mu i} - \delta_{\ell\ell'} \end{pmatrix} \begin{pmatrix} \mathcal{D}_{\ell'}^{N\mu i} \\ \mathcal{D}_{\ell'}^{Z\mu i} \end{pmatrix} \equiv \|\mathcal{F}^{\mu}_{\ell\ell'}(\omega_{\mu i})\| \cdot \mathcal{D}_{\ell'}^{\tau\mu i} = 0. \tag{2}$$

Here i is the number of a phonon with the projection μ; $\kappa_{\pm}^{(\ell\mu)} = \kappa_0^{(\ell\mu)} \pm \kappa_1^{(\ell\mu)}$ are constants of multipole and spin-multipole forces; the quantities $\mathcal{D}_{\ell}^{\tau\mu i}$ determine the phonon wave function [2]; $\tau = \{N, Z\}$ for neutron and proton variables. The quantities $X_{\ell\ell'}^{\tau\mu i}$ [1,2] are elements of the matrix $\mathcal{F}^{\mu}_{\ell\ell'}(\omega_{\mu i})$ with dimension $2N_\ell \times 2N_\ell$, where N_ℓ is the number of multipoles of the electric ($\ell = \lambda$) and magnetic ($\ell = \lambda L$) types with the projection μ onto the nucleus symmetry axis.

Using the general properties of a homogeneous system of equations and the normalization condition of a phonon, we can connect its norm with the derivative of the secular equation (1) $\dot{\mathcal{F}}^{\mu}(\omega_{\mu i}) \equiv \partial\mathcal{F}^{\mu}(\omega_{\mu i})/\partial\omega_{\mu i}$, or in a more general form, one obtains

$$\mathcal{D}_{\ell}^{\tau\mu i}\mathcal{D}_{\ell'}^{\tau'\mu i} = \frac{8}{\dot{\mathcal{F}}^{\mu}(\omega_{\mu i})} \sum_{n=0,1} (-1)^{n_\ell^\tau + n_{\ell'}^{\tau'} + n} \kappa_{(n)}^{(\ell\mu)} M_{n_\ell^\tau + n, n_{\ell'}^{\tau'}}, \tag{3}$$

where $M_{n,n'}$ is the minor of matrix $\mathcal{F}^{\mu}_{\ell\ell'}(\omega_{\mu i})$, $\kappa_{(0,1)}^{(\ell\mu)} \equiv \kappa_{+,-}^{(\ell\mu)}$. Relation (3) allows us to write expressions for cross sections of different processes with excitation of a phonon $\omega_{\mu i}$ in a special form, where the dependence on $\dot{\mathcal{F}}^{\mu}(\omega_{\mu i})$ is shown explicitly. For instance, the reduced probability of $E\lambda(M\lambda)$ transition from the ground state to an excited state $I^\pi K$ with $K = \mu$ and energy $\omega_{\mu i}$ assumes the form

$$B\Big(E\lambda(M\lambda); 0^+_{g.s.} \to (I^\pi K)_i\Big) \equiv |\mathcal{M}^{\lambda\mu i}|^2 = \frac{\mathcal{P}^{\lambda\mu}(\omega_{\mu i})}{\dot{\mathcal{F}}^{\mu}(\omega_{\mu i})}, \tag{4}$$

where

$$\mathcal{P}^{\lambda\mu}(\omega_{\mu i}) = \left| \begin{array}{c|cc} (e_{eff}^{N\lambda\mu})^2 \tilde{X}_{\lambda\lambda}^{N\mu i} + (e_{eff}^{Z\lambda\mu})^2 \tilde{X}_{\lambda\lambda}^{Z\mu i} & e_{eff}^{N\lambda\mu}\tilde{X}_{\lambda\ell'}^{N\mu i} & e_{eff}^{Z\lambda\mu}\tilde{X}_{\lambda\ell'}^{Z\mu i} \\ \hline e_{eff}^{N\lambda\mu}\kappa_{+}^{\ell\mu}\tilde{X}_{\lambda\ell'}^{N\mu i} + e_{eff}^{Z\lambda\mu}\kappa_{-}^{\ell\mu}\tilde{X}_{\lambda\ell'}^{Z\mu i} & \|\mathcal{F}^{\mu}_{\ell\ell'}\| & \\ e_{eff}^{Z\lambda\mu}\kappa_{+}^{\ell\mu}\tilde{X}_{\lambda\ell'}^{Z\mu i} + e_{eff}^{N\lambda\mu}\kappa_{-}^{\ell\mu}\tilde{X}_{\lambda\ell'}^{N\mu i} & & \end{array} \right| \tag{5}$$

is the bordered determinant whose first row and first column contain the amplitudes $\tilde{X}^{\tau\mu i}_{\lambda\ell}$ of the corresponding $E\lambda$ or $M\lambda$ transitions, and $e^{\tau\lambda\mu}_{eff}$ is the effective charge.

It is not difficult to solve the homogeneous system (2) and calculate (4) for an arbitrary root i. However, if the number of roots is very large (for instance, in calculating giant resonances, it sometimes amounts to several thousand), it may become a technical difficulty. Moreover, a thorough calculation turns out to be excess if for comparison with an experiment only integral information is required and there is no necessity to calculate it in detail for each value of i.

Therefore, instead of a traditional approach with a solution of the eigenvalue problem, one can formulate a final goal of finding a strength function of a sought quantity depending on energy ω [9,10] having determined it as

$$b(E\lambda(M\lambda)\mu,\omega) = \sum_i B\Big(E\lambda(M\lambda);\omega_{\mu i}\Big)\rho(\omega-\omega_{\mu i}). \tag{6}$$

Now we average (4), in accordance with (6), using the Lorentz function normalized to 1

$$\rho(\omega) = \frac{1}{2\pi}\cdot\frac{\Delta}{\omega^2+\Delta^2/4}. \tag{7}$$

Then, SF can easily be determined with the use of the residue theorem. Computing the integral along the closed contour of infinite radius and the residues at poles $\omega^* = \omega \pm \mathbf{i}\Delta/2$, as well as at 0 (if a 'spurious' state is present) and at ∞ (when calculating higher statistic moments), we obtain the following expression for SF of the reduced probability of electromagnetic transition and its moments n

$$b_n(E\lambda(M\lambda)\mu,\omega) \approx \frac{1}{\pi}Im\left\{\frac{\mathcal{P}^{\lambda\mu}(\omega^*)}{\mathcal{F}^{\mu}(\omega^*)}\omega^{*n}\right\}_{\omega^*=\omega+\mathbf{i}\Delta/2} + \sum_{\omega=0,\infty} res\frac{\mathcal{P}^{\lambda\mu}(\omega)}{\mathcal{F}^{\mu}(\omega)}\omega^n\rho(\omega), \tag{8}$$

which are related to the photoabsorption cross sections [11].

The calculations performed for deformed nuclei on the basis of the results expounded here allow a common analysis of the role of different interactions and a more exact determination of the relation of electric and magnetic transitions.

Similar expressions take place for SF describing the fragmentation of two-quasiparticle components of the phonon wave function and other quantities.

2 Calculation of statistical moments of fragmentation of simple configurations in odd-A nuclei

It has been shown in [9,11] that application of the QPMN allowed one to study in more detail the fragmentation of different nuclear characteristics (spectroscopic factors, reaction cross sections and others) and of some individual states of different complexity. Fig. 1 exemplifies calculations of the fragmentation of one-quasiparticle components in ^{153}Sm. On the upper horizontal axis the relevant quasiparticle energy ε_ρ is shown (all energies are given relatively the ground state energy η_g). As it is seen from this figure and has been shown in [9,11] the fragmentation has rather a complicated nature and depends on a nucleus, quantum characteristics of a state, energy interval considered and other quantities.

The overall analysis of this picture can be made by calculating the set of statistical moments of the obtained distribution [10]. This kind of calculations allows one to determine the distribution centroid, its dispersion and other higher moments.

However, the above statistical characteristics of the fragmentation can be obtained without numerical calculations presented in Fig.1. For this it is sufficient to calculate certain commutators [11,12] of the model Hamiltonian [2] with the operators that correspond to the components of the wave function of the nuclear system considered (one-quasiparticle, quasiparticle$\nu\otimes$phononμi or others) and use the complete space of its in-

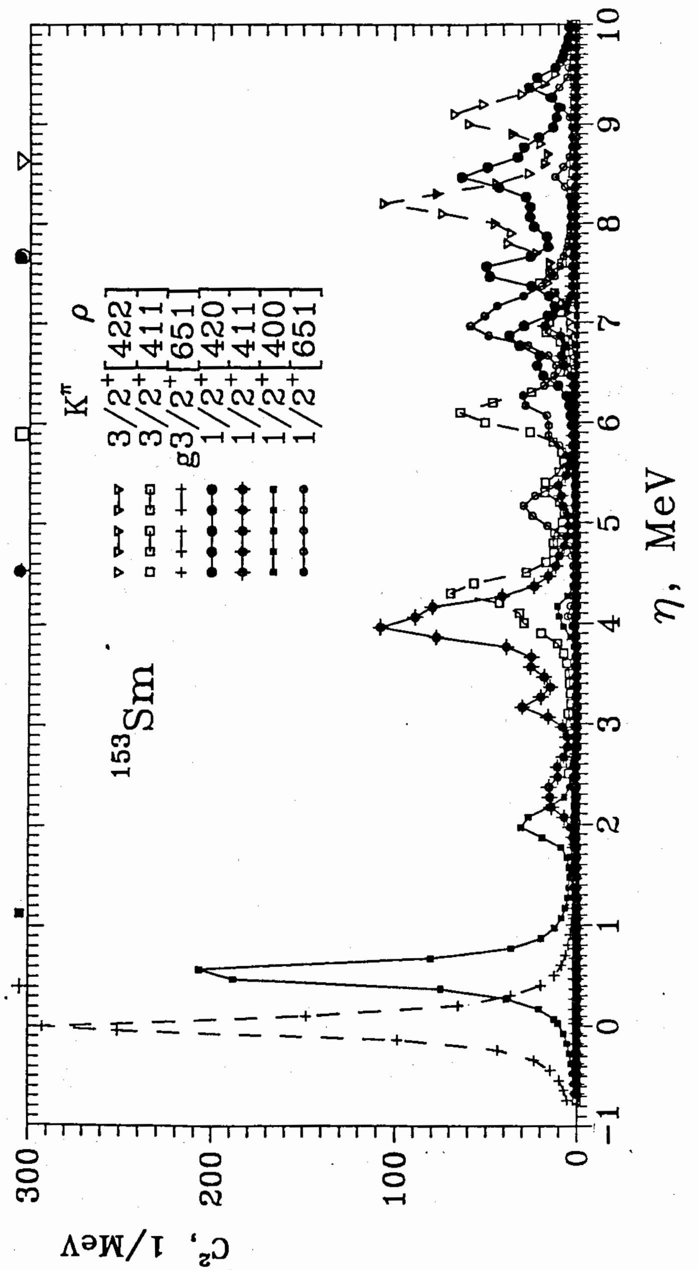

Fig.1. Fragmentation of single-particle states ρ in ^{153}Sm.

termediate states. Multiplicity of the above commutators depends on the number of the sought statistical moment of distribution of a considered component.

Thus, for the distribution centroid of the strength C_ρ^2 of one-quasiparticle component with the Nilsson quantum numbers denoted by ρ, we get

$$\bar{\eta}_\rho = \sum_j \eta_j (C_\rho^j)^2 = 1/2 \sum_{\sigma\sigma'} <0|[\alpha_{\rho\sigma}, [H, \alpha^+_{\rho\sigma'}]]|0> = \varepsilon_\rho. \tag{9}$$

Here, $\alpha^+_{\rho\sigma'}$ is the quasiparticle creation operator, and η_j is the state j energy of an odd-A nucleus. Averaging is made over the quasiparticle vacuum. It is seen from (9) that in spite of the complexity of the intermediate state j the distribution centroid C_ρ^2 is determined by the quasiparticle energy ε_ρ [13].

For its dispersion [14] we get

$$\sigma_\rho^2 = \sum_j \eta_j^2 (C_\rho^j)^2 - \left[\sum_j \eta_j C_\rho^j\right]^2 = 1/2 \sum_{\sigma\sigma'} <0|[\alpha_{\rho\sigma}, [H, [H, \alpha^+_{\rho\sigma'}]]]|0>$$

$$= \sum_{\nu n \mu i} (\Gamma^{n\mu i}_{\rho\nu})^2. \tag{10}$$

Here

$$\Gamma^{n\mu i}_{\rho\nu} = \frac{1}{2\sqrt{2}} \sum_\ell f^{n\ell\mu}_{\rho\nu} v_{\rho\nu} \mathcal{D}^{\tau\mu i}_\ell \tag{11}$$

determines the strength of interaction of the quasiparticle ρ with the phonon μi; $f^{n\ell\mu}_{\rho\nu}$ are matrix elements of the multipole and spin-multipole forces; $v_{\rho\nu}$ is the combination of the coefficients of the Bogoliubov transformation.

The distribution centroid of the strength $(D_{\nu\mu i})^2$ of the quasiparticle$\nu\otimes$phononμi state

$$\bar{\eta}_{\nu\otimes\mu i} = 1/2 \sum_{\sigma\sigma'} <0\left|[\alpha_{\nu\sigma} Q_{\mu i}, [H, \alpha^+_{\nu\sigma'} Q^+_{\mu i}]]\right|0> = \sum_j \eta_j \left(D^j_{\nu\mu i}\right)^2 = \varepsilon_\nu + \omega_{\mu i}. \tag{12}$$

Analogous results are obtained for statistical moments of the fragmentation of other components of the wave function of nuclei.

The table shows some results of numerical verification of the validity of these relations for ^{185}W with the use of the SF method with different averaging functions: with the Lorentz function (in brackets) and its square [15].

ρ	ε_ρ, MeV	$\bar{\eta}, MeV$	$\sum\Gamma^2, MeV^2$	σ_ρ^2, MeV^2
510↑	0.9	0.9(0.96)	0.15	0.15(0.33)
521↓	2.88	2.88(2.88)	0.36	0.35(0.51)
651↑	3.57	3.59(3.57)	1.16	1.17(1.28)

It is seen from the table that higher moments ($n \geq 2$) can be calculated with the Lorentz function with noticeable errors which can however be determined by the method described in [10].

3 Calculation for odd-A nuclei taking account of the complete basis of phonon states

In the QPNM, phonons of a doubly even core are calculated microscopically; therefore, all characteristics of the relevant odd-A nucleus are determined without introducing any new free parameters. Obviously, this is one of the advantages of the model.

To find a solution of the eigenvalue problem for an odd-A nucleus, one has to diagonalize the matrix whose dimension depends on the number of one-quasiparticle states ρ with given K^π. The number of these states is about $\sim 5 \div 7$, and the elements of the relevant matrix have the form [2,3]

$$V_{\rho\rho'} = \sum_{\nu n\mu i} \frac{\Gamma_{\rho\nu}^{\tau n\mu i}\Gamma_{\rho'\nu}^{\tau n\mu i}}{\varepsilon_\nu + \omega_{\mu i} - \eta}. \tag{13}$$

It is natural that for numerical calculations one has to introduce truncation in the number of phonons i taken into account for each its projection μ so that the space of basis states of an odd-A nucleus could be reduced without loosing accuracy. While for the study of excitations of

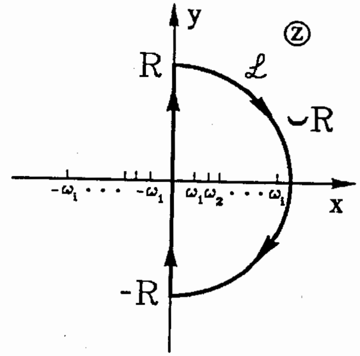

Fig. 2. The contour of integration in the complex plane.

an odd-A nucleus at energies $\sim 1 \div 2$ MeV it is sufficient to take into account phonons with $i = 5 \div 10$ and energy $\omega_{\mu i} \leq 2 \div 3$ MeV, for intermediate and high excitation energies $\sim 5 \div 10$ MeV one has to take into account $50 \div 100$ and more phonons for each value of μ. As a result, the dimension of the space of basis states of an odd-A nucleus increases by an order of 1-2 and amounts to $10^4 - 10^5$, which makes calculations difficult and complicates the analysis of the obtained results even with the use of the SF method.

However, using expressions (3) and (11), one can show that $V_{\rho\rho'} \sim \sum_{\mu i} 1/\dot{\mathcal{F}}^\mu(\omega_{\mu i})$, and then summation in (13) over i can be substituted by integration over the contour $\mathcal{L}$ shown in Fig. 2. Having calculated the integrals over the imaginary axis and the right semicircumference with the radius $R \longmapsto \infty$ and having found the residues at the poles of the integrand, we finally get

$$V_{\rho\rho'} = \frac{1}{4\pi}\sum_{\nu n\mu}(\eta - \varepsilon_\nu)\int_0^\infty Re\left\{\frac{\mathcal{P}_{\rho\rho'}^{\nu n\mu}(\mathbf{i}y)}{\mathcal{F}^\mu(\mathbf{i}y)[\mathbf{i}y - (\eta - \varepsilon_\nu)]}\right\}dy +$$
$$+\frac{1}{8}\sum_{\nu n\mu\ell}\kappa_+^{(\ell\mu)} f_{\rho\nu}^{n\ell\mu} v_{\rho\nu} f_{\rho'\nu}^{n\ell\mu} v_{\rho'\nu} - \frac{1}{4}\sum_{\nu n\mu}\frac{\mathcal{P}_{\rho\rho'}^{\nu n\mu}((\eta - \varepsilon_\nu)^2)}{\mathcal{F}^\mu((\eta - \varepsilon_\nu)^2)}. \tag{14}$$

Here, $\mathcal{P}_{\rho\rho'}^{\nu n\mu}(\omega)$ is the bordered determinant which differs from (5) by the first row and column that are expressed through the matrix elements $f_{\rho\nu}^{n\ell\mu}$. A similar method has been used in ref. [16] in calculating SF moments.

Analogous results have been obtained for other quantities used in calculating the properties of odd-A nuclei. At present, a possibility to generalize a similar procedure to doubly even nuclei in the two-phonon approximation is considered.

In conclusion, we should like to note that improvement of the QPNM, expounded in this paper, allows one to simplify calculations of the properties of deformed nuclei and analyze experimental data at the level satisfying the requirements of modern experiment.

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