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DYNAMIC APPROACH TO FUSION OF MASSIVE NUCLEI

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Динамический подход к слиянию массивных ядер

В рамках динамической модели исследована роль входного канала в реакциях слияния–деления, ведущих к образованию изотопов одного и того же сверхтяжелого элемента. Расчеты выполнены для реакций $^{48}Ca + ^{244}Pb$ и $^{74,76}Ge + ^{208}Pb$, в которых может образоваться сверхтяжелый элемент $Z = 114$. Показано, что в этих реакциях имеются ограничения на значения энергий пучка, при которых вероятность захвата достаточно велика. В сочетании с ограничением, следующим из величины внутреннего барьера слияния, это позволяет установить оптимальное значение энергии пучка для данной комбинации: налетающее ядро — мишень.

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Dynamic Approach to Fusion of Massive Nuclei

The role of the entrance channel in the fusion–fission reactions leading to nearly the same superheavy compound nucleus is studied in the framework of dynamic model. The calculations are done for $^{48}Ca + ^{244}Pu$ and $^{74,76}Ge + ^{208}Pb$ reactions which could lead to formation of superheavy element $Z = 114$. It is shown that for these reactions there is an energy window for the values of the bombarding energy at which a capture probability is sufficiently large. Together with the restriction coming from the intristic barrier for fusion, it helps to find an optimal value of the bombarding energy for a given projectile — target combination.

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1 Introduction

The cross section for the production of the superheavy elements depends on the choice of the projectile-target combination and the bombarding energy $E_{c.m.}$. The optimal choice is determined by the requirements to have a larger fusion cross section and larger survival probability of a compound nucleus relative to fission. For a given projectile-target combination, a larger value of the bombarding energy is needed to overcome the reaction barrier which is determined by the nucleus-nucleus potential and the dynamic barriers if they exist. However, the excitation energy of the compound nucleus increases with the bombarding energy. It decreases the survival probability relative to fission of a nucleus produced in a reaction and therefore puts a restriction on the upper value of a bombarding energy. To determine the optimal value of $E_{c.m.}$ it is necessary to analyse a dependence of a partial fusion cross section which is proportional to a capture probability, on a bombarding energy. To do it, we require in dynamic model to describe the initial stage of a heavy ion collision. Such a model has been developed in our earlier papers [1,2] and it is the aim of the present paper to apply this model to calculate the capture probability. The latter quantity is determined by the dynamic aspects of the reaction mechanism and by the depth of the pocket in the nucleus-nucleus interaction potential. We have calculated a nucleus-nucleus interaction potential using a double–folding procedure with the Migdal's effective forces [3]. As the examples we consider below the following reactions: ^{48}Ca + ^{244}Pu and ^{74,76}Ge + ^{208}Pb.

2 Basic formalism

The cross section of production of the evaporation residues (σ_{er})

$$\sigma_{er}(E) = \sum_{l=0}^{\infty}(2l+1)\sigma_l^{fus}(E,l)W_{sur}(E,l) \tag{1}$$

is determined by the partial fusion cross section ($\sigma_l^{fus}(E)$),

$$\sigma_l^{fus}(E) = \sigma_l^{capture}(E)P_{CN}(E,l), \tag{2}$$

$$\sigma_l^{capture}(E) = \frac{\lambda^2}{4\pi}\mathcal{P}_l^{capture}(E), \tag{3}$$

where λ is a wavelength, $P_{CN}(E,l)$ is a factor taking into account a decrease of the fusion probability due to dinuclear system break up before fusion, $\mathcal{P}_l^{capture}(E)$

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is the capture probability which depends on the collision dynamics and determines the amount of partial waves leading to capture. The cross section of production of the evaporation residues σ_{er} depends as well as on the probability $(W_{sur}(E,l))$ that the compound nucleus survives during the deexcitation cascade at the bombarding energy E.

To calculate capture probability $\mathcal{P}_l^{capture}(E)$ we shall use a dynamic approach developed in [1,2]. In this approach, the system of equations is derived to describe the radial motion of colliding nuclei and an evolution of their intrinsic states during the heavy ion collision. The relative motion coordinate $\mathbf{R}(t)$ and the velocity $\dot{\mathbf{R}}\,(t)$ are determined by solving the equations of motion

$$\mu(\mathbf{R}(t))\,\ddot{\mathbf{R}}_k + \sum_j \gamma_{kj}[\mathbf{R}(t)]\,\dot{\mathbf{R}}_j\,(t) = -\frac{\partial W(\mathbf{R}(t))}{\partial \mathbf{R}_k} \tag{4}$$

where

$$\mu(\mathbf{R}) = mA_TA_P/(A_T+A_P) + \delta\mu(\mathbf{R}),$$

$\delta\mu(\mathbf{R})$ is the dynamic contribution to the reduced mass μ, $\gamma_{kj}[\mathbf{R}(t)]$ is the friction tensor, $W(\mathbf{R}) = V(\mathbf{R}) + \delta V(\mathbf{R})$ is the nucleus-nucleus interaction potential and $\delta V(\mathbf{R})$ is the dynamic contribution to a nucleus-nucleus potential which is due to the rearrangement of the densities of the interacting nuclei during reaction. To calculate $\delta\mu(\mathbf{R})$, $\gamma_{kj}[\mathbf{R}(t)]$ and $\delta V(\mathbf{R})$, it is necessary to find the occupation numbers of the single particle states. Since the excitation energy of the interacting nuclei changes significantly during the course of the collision, it is necessary to take into account the time dependence of the occupation numbers. An evolution of the occupation numbers has been defined by a numerical solution of the von Neumann equation for the single particle density matrix $\hat{\tilde{n}}$ with the Hamiltonian $\hat{H}$ which takes the following form in the second quantized representation

$$\hat{H}(\mathbf{R}(t),\xi) = \sum_P \varepsilon_P a_P^+ a_P + \sum_T \varepsilon_T a_T^+ a_T + \sum_{i,i'} V_{ii'}(\mathbf{R}(t)) a_i^+ a_{i'} + V_{res}. \tag{5}$$

Here ξ is the short notation for the relevant intrinsic variables, the third term on the r.h.s. of the Eq. (5) can be written as

$$\sum_{i,i'} V_{ii'}(\mathbf{R}(t)) a_i^+ a_{i'} = \sum_{P,P'} \Lambda_{PP'}^{(T)}(\mathbf{R}(t)) a_P^+ a_{P'} + \sum_{T,T'} \Lambda_{TT'}^{(P)}(\mathbf{R}(t)) a_T^+ a_{T'} + \sum_{T,P} g_{PT}(\mathbf{R}(t))(a_P^+ a_T + \text{h.c.}), \tag{6}$$

where $P \equiv (n_P, j_P, l_P, m_P)$ and $T \equiv (n_T, j_T, l_T, m_T)$ are the sets of quantum numbers characterizing the single particle states in the noninteracting projectile and the target nuclei, respectively. The last term on the r.h.s. of Eq. (5) represents the residual interaction. Since an explicit allowance for the residual interaction is very complicated it is customary to take into account a two–particle collision integral in the linearized form (τ–approximation) [1,2,4-6]

$$i\hbar \frac{\partial \hat{\tilde{n}}(t)}{\partial t} = [\hat{\mathcal{H}}(\mathbf{R}(t)), \hat{\tilde{n}}(t)] - \frac{i\hbar}{\tau}[\hat{\tilde{n}}(t) - \hat{\tilde{n}}^{eq}(\mathbf{R}(t))], \tag{7}$$

where $\tilde{n}^{eq}(\mathbf{R}(t))$ is the local quasi-equilibrium distribution, *i.e.* a Fermi distribution with the temperature $T(t)$ corresponding to the excitation energy at the internuclear distance $\mathbf{R}(t)$. All formulae needed to calculate $\gamma_{kj}[\mathbf{R}(t)]$ and $\delta V(\mathbf{R})$ are given in [2,4,6].

The nuclear part of a nucleus-nucleus potential $V(\mathbf{R}(t))$ is calculated using the double-folding procedure between the effective nucleon-nucleon forces $f_{eff}[\rho(x)]$ suggested by Migdal [3] and the densities of the interacting nuclei taken in the Woods–Saxon form

$$\rho_K^{(0)}(\mathbf{r}, \mathbf{R}_K(t), \theta_K, \beta_2^{(K)}) = \left[1 + \exp\left(\frac{|\mathbf{r} - \mathbf{R}_K(t)| - R_{0K}(1 + \beta_2^{(K)} Y_{20}(\theta_K))}{a}\right)\right]^{-1}, \tag{8}$$

where $\mathbf{R}_K$ are the center of mass coordinates and R_{0K} are the half density radii of interacting nuclei $K = 1,2$; $\beta_2(K)$ are the quadrupole deformation parameters determined by the $B(E2)$ to the first-excited 2^+ state (its value is taken from [7]) and θ_K are the axial symmetry axes orientations relative to $\mathbf{R}(t)$. Thus, we have a possibility to consider fusion at different mutual orientations of the interacting nuclei.

The competition between complete fusion and quasifission of a dinuclear system formed after capture and its further evolution are described using the method developed in [8]. This method is based on the assumption that dinuclear system formed in the collision of two nuclei evolve to fusion by increasing its mass asymmetry. It means that the mass asymmetry degree of freedom $\eta = (A_T - A_P)/(A_T + A_P)$ is the main dynamic variable. The internuclear distance $R(t)$ takes the value corresponding to the location of the minimum of the nucleus-nucleus interaction potential for every given value of η. The evolution of the system along mass asymmetry degree of freedom is described by a driving potential $U(Z,l)$ which is calculated as

$$U(Z,l) = B_1 + B_2 + U_{12}(R_m) - B_0. \tag{9}$$

Here, B_1 and B_2 are the binding energies of the nuclei in a dinuclear system, $U_{12}(R_m)$ is the value of the nucleus–nucleus interaction potential at the minimum, B_0 is the binding energy of the compound nucleus (the binding energies B_i are obtained from [9] and from [10] particularly for the superheavy elements). Therefore, a dinuclear system to be fused should overcome the intrinsic barrier (B^*_{fus}) which is determined by the difference between the values of a driving potential located at the Businaro–Gallone point ($\eta = \eta_{BG}$) and the initial point corresponded to reaction under consideration. For the reactions considered below, the initial value of η is smaller than η_{BG}. The quasifission, which is in competition with the fusion is considered as a motion in the nucleus-nucleus interaction potential $W(R)$. Thus, for quasifission, it is necessary to overcome a barrier of $W(R)$. The competition of fusion and quasifission is taken into account by the factor $P_{CN}(E,l)$, which is calculated using the following relation derived from the statistical model arguments

$$P_{CN} = \frac{\rho(E^* - B^*_{fus})}{\rho(E^* - B^*_{fus}) + \rho(E^* - B^*_{qf})}. \qquad (10)$$

Here $\rho(E^* - B^*_K)$ is the level density

$$\rho(E^* - B^*_K) = \frac{g(\epsilon_F)K_{rot}}{2\sqrt{g_1(\epsilon_F)g_2(\epsilon_F)}} \frac{\exp\left[2\pi\sqrt{g(\epsilon_F)(E^* - B^*)/6}\right]}{\left[\frac{3}{2}g(\epsilon_F)(E^* - B^*)\right]^{\frac{1}{4}}(E^* - B^*_K)\sqrt{48}}. \qquad (11)$$

In Eq. (10), B^*_{fus} is the barrier of the driving potential $U(Z,l)$, which should be overcome on the way from the initial value of η to $\eta = 1$. The B^*_{qf} is the barrier of the nucleus-nucleus interaction potential which should be overcome if dinuclear system decays in two fragments, E^* is an excitation energy of the compound nucleus which is equal to difference between $E_{c.m.}$ and the minimum of nucleus–nucleus potential ($E^* = E\mathrm{c.m.} - U(R_m)$), $g_{1,2}(\epsilon_F)$ are the single particle level densities of the fragments of the dinuclear system and $g(\epsilon)$ is their sum, K_{rot} is a factor taking into account rotation of a dinuclear system

$$K_{rot} = \frac{\sqrt{6(E^* - B^*)/g(\epsilon_F)}}{\pi} J_\perp, \qquad (12)$$

where $J_\perp$ is the rigid body moment of inertia for rotation around the axis perpendicular to the line connecting the centers of fragments.

3 Results and discussion

We consider below the following reactions which are discussed now as possible ways to search for superheavy element with $Z = 114$. They are ^{48}Ca + ^{244}Pu (I) and ^{74,76}Ge + ^{208}Pb (II,III).

Basing on the dynamic model developed in [1] (which is described concisely in the preceding section) we have calculated the capture cross section $\sigma_l^{capture}(E)$ for the reactions under consideration. The results are shown in Fig. 1. It is seen that, for these reactions there is an energy window for the values of the bombarding energy at which a capture cross section is large enough to have a physical interest. The lower limit for the bombarding energy (E_{min}) is defined by a total nucleus-nucleus interaction potential $W(R) = V(R) + \delta V(R)$. Note that E_{min} is somewhat larger than the value of the entrance Coulomb barrier, because of the kinetic energy loss due to friction. So, E_{min} is determined by a dynamic calculation. The upper limit (E_{max}) comes from an incomplete dissipation of the relative kinetic energy. Thus, the values of E_{min} and E_{max} are determined by the depth of the pocket in the potential $W(R)$ (Fig. 2) and by dissipative forces. If a bombarding energy is larger than E_{max} the dissipative forces could not provide a complete dissipation of the relative kinetic energy and dinuclear system decays into two fragments instead of being fused. As it is seen from Fig. 1, reaction with the lighter projectile (I) has a larger value of the capture cross section than other two reactions (II) and (III). The reason is that for ^{48}Ca + ^{244}Pu reaction the pocket of the nucleus–nucleus interaction potential is deeper and wider than for ^{74,76}Ge + ^{208}Pb (see Fig. 2). The potentials presented in Fig. 2 are calculated taking into account a deformation of the interacting nuclei assuming the tip-tip orientation of the colliding projectile and target nuclei. For other orientations of the colliding nuclei the potential is more flat and the depth of the pocket is smaller. Moreover, in these cases an entrance barrier and the minimum of the pocket of $W(R)$ have larger absolute energies than in the case of the tip-tip orientation. Therefore, an excitation energy of a compound nucleus will be larger than in the last case. An excess of the excitation energy will decrease the survival probability of the evaporation residues. Thus, in the fusion of massive nuclei their mutual orientation strongly influences not only the capture cross section but also the probability that the compound nucleus survives during deexcitation.

The existence of the window for the bombarding energy has a crucial influence on the fusion process. From one side a larger bombarding energy will be needed to overcome an intrinsic barrier (B^*_{fus}) to form a compound nucleus. From other side an increase of the bombarding energy decreases the capture probability starting from some values of the bombarding energy because the friction force is not strong enough to provide a complete dissipation of the kinetic energy.

To analyse a fusion process further, we need in a dynamic model which describes an evolution of a dinuclear system to compound nucleus. Below, we shall use a model developed in [8]. According to this model a dinuclear system evolves to compound nucleus by increasing its mass asymmetry. It means that driving potential (9) plays the main role in the fusion dynamics and a dinuclear system should overcome the Businaro Gallone point to be fused. The driving potentials for the reactions which we analyse are presented in Figs. 3-5. The values of the barriers which should be overcome to get a compound nucleus (B^*_{fus}) depend on the compound system and the reaction choice which determines the initial value of the mass asymmetry. These are equal to 6 MeV for ^{48}Ca + ^{244}Pu (Fig. 3) and 28 MeV for ^{74,76}Ge + ^{208}Pb (Figs. 4 and 5). To overcome the barrier, a dinuclear system should have the corresponding excitation energy. However, the possible values of the excitation energy which are defined by the amount of a dissipated energy are restricted by the framework of the energy window for bombarding energies leading to capture. The possible values of the excitation energy can be estimated and the results are shown in Fig. 6. For ^{48}Ca + ^{244}Pu reaction the excitation energy can take the values from 19 MeV up to 41 MeV which are larger than the barrier B^*_{fus} of the driving potential. In the case of ^{74,76}Ge + ^{208}Pb reactions, the excitation energy E^* takes the values between 6 MeV and 16 MeV. This value is lower than the value of B^*_{fus}=28 MeV for these reactions but it is larger than the quasifission barrier which is about (3 – 5) MeV (Fig. 2). An increase of the beam energy in order to obtain an adequate excitation energy does not help because dinuclear system can not be formed. The corresponding value of the beam energy will exceed E_{max}. Thus, according to our calculations of a capture cross section and the model of fusion suggested in [8], the compound nucleus can not be formed with a measurable cross section in the ^{74,76}Ge + ^{208}Pb reactions. However, it is not excluded that a dinuclear system can prefer the trajectory in the $R - \eta$ plane for fusion different from that suggested in [8] or other mechanism of the compound nucleus formation like cluster transfer [11] might play an important role.

The other question concerns the probability that the excited compound nucleus formed in a fusion process survives during deexcitation. An increase of an excitation

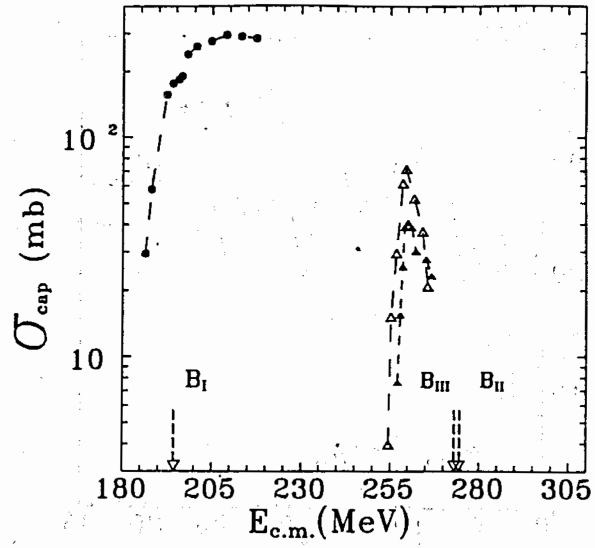

Figure 1: The calculated capture cross section as a function of the beam energy for the ^{48}Ca + ^{244}Pu (I) (full circles), ^{74}Ge + ^{208}Pb (II) (full triangles), and ^{76}Ge + ^{208}Pb (III) (open triangles) reactions; B_i is the Bass barrier for the reaction (i), i=I, II, and III.

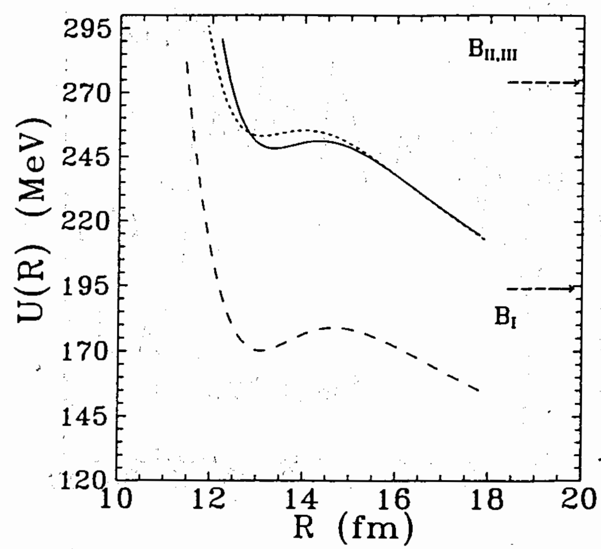

Figure 2: The nucleus-nucleus interaction potential calculated for ^{76}Ge + ^{208}Pb (solid curve), ^{74}Ge + ^{208}Pb (dotted curve) and ^{48}Ca + ^{244}Pu (dashed curve) reactions; B_i is the Bass barrier for the reaction (i), i=I, II, and III.

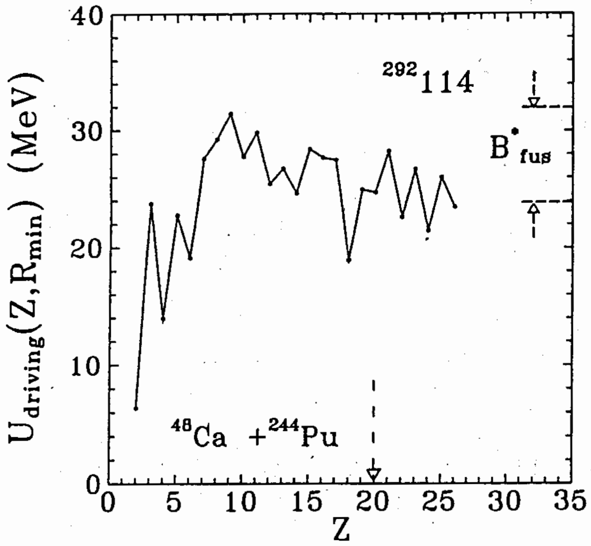

Figure 3: The driving potential for the superheavy element $^{292}114$. The arrow indicates an initial charge asymmetry which corresponds to the $^{48}Ca + ^{244}Pu$ reaction.

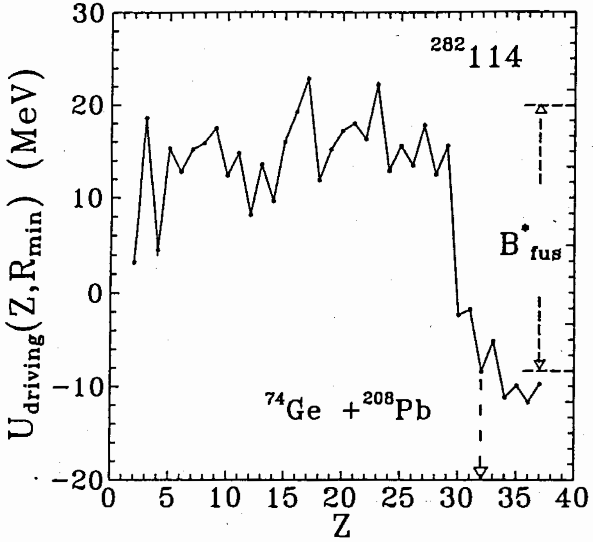

Figure 4: The driving potential for the superheavy element $^{282}114$. The arrow indicates an initial charge asymmetry which corresponds to $^{74}Ge + ^{208}Pb$ reaction.

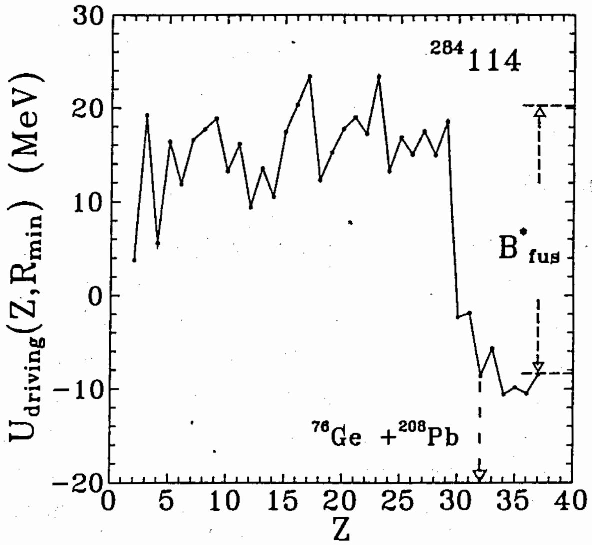

Figure 5: The driving potential for the superheavy element $^{284}114$. The arrow indicates an initial charge asymmetry which corresponds to $^{76}Ge + ^{208}Pb$ reaction.

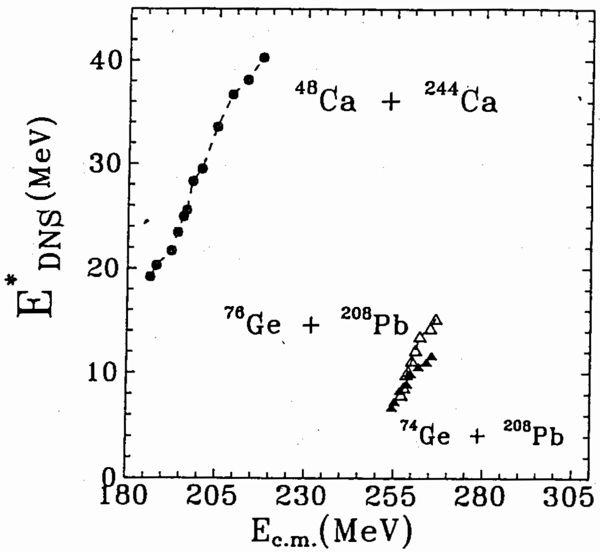

Figure 6: The excitation energy of a dinuclear system formed after capture of nuclei in reactions: $^{48}Ca + ^{244}Pu$ (I) (full circles), $^{74}Ge + ^{208}Pb$ (II) (full triangles), and $^{76}Ge + ^{208}Pb$ (III) (open triangles) as a function of the beam energy in the center of mass system.

energy decreases the influence of the shell effects on stability of a compound nucleus and decrease the fusion probability. However, this question is not analysed in the present paper.

4 Conclusion.

We have analysed the partial fusion cross sections for the reactions with massive nuclei leading to compound nucleus with $Z = 114$: ^{48}Ca + ^{244}Pu and ^{74,76}Ge + ^{208}Pb. The main attention is paid to the calculations of the capture probability, which is a characteristic feature of an initial stage of the collision. It is shown that for the considered reactions, there is an energy window for the bombarding energy at which the capture cross section is large enough to have a physical interest. This result puts a strong limitations on the choice of the bombarding energy for a given reaction. However, from other side, the excitation energy should be large enough to overcome an intrinsic barrier for the fusion [8]. Thus, both restrictions can be used to obtain an optimal choice of the projectile-target combination and of the bombarding energy.

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