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IN THE SCATTERING
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**ON THE OCCURRENCE
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OF TWO HEAVY IONS. II.**

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In this paper we will show how a more general formulation of our problem can be given. In order to describe the time evaluation of the quantum mechanical system characterized by the Hamiltonian

$$\hat{H} = \hat{H}_R + \hat{H}_0 + V_{int} \quad (1)$$

given in part I, we will use, instead of the wave function, the statistical operator $\rho(t)$ which satisfies the Liouville-von Neuman equation

$$\frac{\partial}{\partial t} \rho(t) + \frac{i}{\hbar} [H, \rho(t)] = 0. \quad (2)$$

A formal solution of this equation can be found using the method of Zubarev^{1/} to construct the nonequilibrium statistical operator (NSO) $\rho(t)$. To this end we have to specify a set of parameters P_m which characterize the state of the system during the collision process, i.e., during a time interval of the order of the collision time t_{coll} . We will choose the following parameter set

$$P_m = \{H_R, N, \vec{P}, H'_0, n_i\}, \quad (3)$$

where the index i runs over all the "collective" modes with $\tau_i \gtrsim t_{\text{coll}}$. because during the collision process the state of the system is characterized by the mean values of the occupation numbers n_i . On the other hand, in $H'_0 = \sum_j h\omega_j n_j$ the index j runs over all "non-collective" modes with $\tau_j \ll t_{\text{coll}}$. Because of the short relaxation time these modes equilibrate very rapidly during the collision process, and the state of the system is characterized only by the mean value of H'_0 , and the concept of a temperature as a parameter can be introduced which determines the mean occupation number $\langle n_j \rangle$ (Bose distribution) on the noncollective modes.

Further observables which enter into P_m are the total energy and momentum of the relative motion of ions and the particle number N of the considered nucleus. We assume, that all other fluctuations equilibrate very rapidly, in the opposite case additional parameters have to be introduced into (3.3) which further characterize the state of the system.

The NSO can be obtained from the solution of the equation^{/10/}

$$\frac{\partial}{\partial t} \ln \rho(t) + \frac{i}{h} [H, \ln \rho(t)] = -\epsilon (\ln \rho(t) - \ln \rho_g(t)), \quad (4)$$

which coincides in the limit $\epsilon \rightarrow 0$ with the Liouville-von Neumann equation (2). In (4) the infinitesimal source in the right-hand

side is introduced to select the retarded solution by destroying the time reflection symmetry of (1). Given the mean values $\langle P_m \rangle_t$ at time t , the quasiequilibrium statistical operator $\rho_g(t)$ is determined by the maximum of information entropy under given additional conditions

$$\langle P_m \rangle_g \equiv \text{Sp} \{ \rho_g(t) P_m \} = \langle P_m \rangle_t \quad (5)$$

so that the expression

$$\rho_g(t) = \exp \{ -S(t) \} \quad (6)$$

with

$$S(t) = \ln \text{Sp} \exp \{ -\sum_m P_m F_m(t) \} + \sum_m P_m F_m(t) \quad (7)$$

follows.

The parameters F_m , which arise from accounting (5) by Lagrangian multipliers, have the meaning of the thermodynamic conjugate parameters corresponding to P_m (3) and are given by

$$F_m = \{ \beta_R, -\beta_R (\mu - \frac{M}{2} V^2), -\beta_R \vec{V}(t), \beta(t), F_i(t) \}. \quad (8)$$

The physical meaning of these parameters is known from thermodynamics. Especially, in (8) the chemical potential μ and the parameter β_R are not of interest for us, because at present we are not interested in effects connected with fluctuations of the particle number of the total energy of the system,

and these observables are considered as constants of motion. $\vec{V}(t)$ has the meaning of ion velocity $\vec{V}(t) = \frac{1}{M} \langle \vec{P} \rangle_t$, the parameter $\beta(t)$ can be considered as the temperature of the subsystem described by H'_0 and the parameters $F_i(t)$ can be considered as generalized temperatures which determine the mean occupation probabilities $\langle n_i \rangle_t$.

From (4) we have for $\rho(t)$ the formal solution

$$\rho(t) = \exp\left\{-S(t) + \int_{-\infty}^0 dt' e^{\epsilon t'} U^+(t', 0) \dot{S}(t+t') U(t', 0)\right\}, \quad (9)$$

where $U(t, 0)$ describes the time evolution given in accordance with the Heisenberg picture as

$$i\hbar \frac{\partial}{\partial t} U(t, 0) = H(t) U(t, 0), \quad U(t, t) = 1. \quad (10)$$

The entropy production $\dot{S}(t)$ introduced in (9) is defined by the equation

$$\frac{d}{dt} S(t) = \frac{\partial}{\partial t} S(t) + \frac{i}{\hbar} [H, S(t)]. \quad (11)$$

We would like to mention that the formal solution (9) of the Liouville-von Neumann equation (2) is correct not only near the thermal equilibrium, but also as the solution of a variational problem for $\rho_g(t)$ it should work far away from the equilibrium state where non-linear effects might occur. Near to the thermal equilibrium the usual linear response formalism (see, e.g., /2/) can be

obtained if one restricts the considerations only to mechanical perturbations as given, e.g., by the external field /1/.

If we assume that the entropy production \dot{S} is small, then the exponent in (9) can be expanded and we obtain

$$\rho(t) = e^{-S(t)} \left\{ 1 + \int_{-\infty}^0 dt' e^{\epsilon t'} \int_0^1 dr e^{rS(t)} U^+(t', 0) \dot{S}(t+t') U(t', 0) e^{-rS(t)} \right\}. \quad (12)$$

Now, we can calculate the effective force as the average of the force operator (2.6), part I:

$$\vec{F} = \frac{i}{\hbar} [H, P] = \vec{V}_R(U(R)) + V_{int}(R)$$

with $\rho(t)$ (12). A clear distinction between the non-dissipative (conservative) and the dissipative part connected with the production of entropy is possible. The dissipative part of the force we are interested in is given by

$$\vec{F}_{diss.} = \text{Sp} \left\{ \int_{-\infty}^0 dt' e^{\epsilon t'} \int_0^1 dr e^{(r-1)S(t)} U^+(t', 0) \dot{S}(t+t') U(t', 0) e^{-rS(t)} \vec{F} \right\}. \quad (13)$$

This formula (13) can be considered as a very general expression for the calculation of the dissipative force, which is connected with irreversible ($S \neq 0$) transfer of energy of the relative motion of ions into the intrinsic degrees of freedom of the nucleus.

We will now consider some approximations to reproduce the discussion of dissipative force of part I in the framework of the formalism given here. In the lowest order of V_{int} ,

the operator $U(t,0)$ commutes with the entropy operator $S(t)$ (7). Furthermore, if we neglect the time dependence of $F_m(t)$ in $S(t)$ (11) (the quasistatic approximation) the integration over t' in (13) can be performed with taking into account the relation

$$e^{\tau S} \frac{i}{h} [V_{int}, S] e^{-\tau S} = \frac{d}{d\tau} \frac{i}{h} e^{\tau S} V_{int} e^{-\tau S} \quad (14)$$

and we obtain the result

$$\vec{F}_{diss.}^{static}(t) = -\frac{1}{h^2} \int_{-\infty}^0 dt' e^{\epsilon t'} \text{Sp} \{ e^{-S(t)} [V_{int}(t'), [V_{int}(t), \vec{P}]] \}, \quad (15)$$

which is in correspondence with that obtained in part I on the basis of a concrete model. It should be mentioned, that another expression for (15) can be given if we replace, in the lowest order of V_{int} , and in the quasistatic approximation, $U^+(t',0)S(t+t')U(t,0)$ in (13) by $\beta_R \vec{V}(t) \vec{F}(t')$

$$\vec{F}_{diss.}^{static}(t) = \beta_R \text{Sp} \{ e^{-S(t)} \int_{-\infty}^0 dt' e^{\epsilon t'} \int_0^{\beta_R} d\tau \vec{F}(t'-i\tau) \vec{F} \} \vec{V}(t), \quad (16)$$

which has been obtained by Hofmann and Siemens^{2/}. This formula shows a close similarity to the force-force correlation functions used in the theory of electric resistivity^{3/} or friction coefficients in liquids. The applicability of the quasistatic approximation to our special model has been discussed in detail in part I. Especially, a contribution arises from excitation which belongs to

the continuous part of the spectrum of H_0 only at a finite temperature. In the case of low-lying collective states and also to obtain a finite temperature $\beta^{-1}(t)$ at the beginning of the reaction the quasistatic approximation cannot be used. All these effects can be treated in a rather elegant manner if, instead of (13), we consider kinetic equations for the occupation numbers n_j, n_i introduced in (2.15), (2.16) in part I. Similar to the calculation for (15) we obtain

$$\langle \dot{n}_j \rangle = -\frac{1}{h^2} \int_{-\infty}^0 dt' e^{\epsilon t'} \text{Sp} \{ e^{-S(t)} [V_{int}(t'), [V_{int}(t), n_j]] \}, \quad (17)$$

where $V_{int}(t)$ depends also explicitly on t via $R(t)$. For instance, if we use only the first term of the expression (2.4) (part I) for $V_{int}(t)$, we find

$$\langle \dot{n}_j \rangle^{(1)} = \frac{1}{h^2} \int_{-\infty}^0 dt' e^{\epsilon t'} (e^{\frac{i}{h} \omega_j (t-t')} + e^{\frac{i}{h} \omega_j (t'-t)}) V_j(R(t')) V_j(R(t)), \quad (18)$$

which corresponds to (2.11), (2.28) in part I. We point out, that in the case $R = \vec{V} = 0$ and in the quasistatic approximation (18) goes to zero because in these cases the integral can be taken directly and is proportional to $\delta(\omega_j)$.

It should be mentioned that in the case of long-lived modes the adiabatic form of the Hamiltonian (e.g., Hamiltonian \mathcal{H} in part I) has to be used. Otherwise the non-diagonal elements of the density matrix which are connected with the non-dissipative motion do not thermalize during the collision

time and must be added explicitly to $\{P_m\}$.

From expression (18) we have also a non-vanishing contribution to the effective force at the beginning of the reaction, if the temperature $T = \beta^{-1} = 0$. One has to treat the collective modes separately by kinetic equations like (18) provided that there are only weak contributions from the continuous spectrum. Otherwise their effect must also be taken into account in (16).

The non-collective modes thermalize and are described by a common temperature $T = \beta^{-1}$ corresponding to the mean value of $\langle H'_0 \rangle$. In order to obtain the time dependence of the temperature we have to consider H_0 as a function of time t . This can be done by means of (18) for the non-collective modes taking $j \neq 1$ and if we assume that the states of the continuous spectrum belong to the non-collective modes. All the energy picked up by these dissipative effects is distributed among the intrinsic degrees of freedom according to $T(t) = \beta^{-1}(t)$ and is given by

$$\sum_{j \neq i} h\omega_j \langle \dot{n}_j \rangle + F_{\text{diss.}}^{\text{quasistatic}} \dot{R}(t) = C(R) \frac{d}{dt} T(t). \quad (19)$$

The coefficient $C(R)$ is the specific heat of the nucleus which can be evaluated if the density of the states is given. This equation (19) describes the set on of the quasistatic friction process if the temperature increases. However, eq. (19) must be solved in a selfconsistent way, because $F_{\text{diss.}}^{\text{static}}$ depends also on the temperature T . Note, that the collective modes described by

the kinetic equation (17) can also thermalize at times which are, however, large with respect to the collision time t_{coll} . In real situation of ion collision, at the beginning of the reaction also the energy transfer from the collective modes to H_0 due to a small coupling should be taken into account in (19) to establish a finite temperature if these modes are highly excited.

The use of Zubarev's non-equilibrium statistical operator presented here gives the possibility to consider more involved processes. This can be done for instance evaluating $\rho(t)$ in better approximations or by introducing more parameters to characterize the state of the system. The very interesting case of particle exchange during the collision should be treated in this formalism. Further advances of the formalism can be expected if its applicability to strong non-equilibrium states, which, of course, occur in nuclear collisions, will be investigated.

It seems to be desirable to get an expression for the dissipative part due to excitation processes described by (17), (18) in terms of $R(t)$ and $\dot{R}(t)$. However, because the quasistatic approach does not work, no expressions local in time can be found. That means that the dissipative force depends on the whole history of the collision process described by $R(t')$ with $t' < t$. It is clear that the memory effects are important if the relaxation time τ becomes large in comparison with t_{coll} . Approximate expressions, however, can be found if the dependence of, e.g., $V_{ij}(R(t'))$ on time is assumed to be for instance of an exponential

or gaussian type. Then, the calculation of $\langle h_j \rangle$ can be simplified. This concerns mainly the first stage of the collision at which the heating of the nucleus is important.

CONCLUSION

On the basis of a concrete model which describes the coupling of the internal degrees of freedom to the relative motion the arising dissipative forces acting on the classical relative motion of the two ions have been studied. An independent investigation of low-lying collective modes which have a large lifetime with respect to the collision time t_{coll} and non-collective modes having a rather short lifetime produces a rather useful method in evaluating these dissipative forces.

In the case of the non-collective modes which relax rather fast in the course of the collision the quasistatic approach indeed permits one to extract, only for finite temperatures, a frictional-like force proportional to the velocity of the relative motion, however, the whole dissipative force contains further terms which are partly nonlocal in time. Moreover, one has to pay attention to the fact, that there exist terms in the whole dissipative force which give contributions also at temperature $T=0$.

In the case of collective modes it turns out that the resulting effective force becomes nonlocal in time, i.e., it depends on the whole history of the collision process.

Because of the large lifetime of the collective states in this case the quasistatic approach does not work, and, therefore, it is impossible to extract a frictional-like term from the whole dissipative force.

By using a more general Zubarev formalism in calculating the non-equilibrium statistical operator it has been shown how to select out the dissipative forces which are simply connected with the production of entropy. This formalism permits one also to treat the collective modes. To this end a special kinetic equation (17) for the calculation of the mean excitation probability of the collective states in the course of the collision has been derived. All the energy, picked up by the dissipative effects which are due to collective excitations could be considered by expression (19) to obtain a self-consistent equation for $T(t)$. Involved processes connected with particle exchange and strong non-equilibrium states should be treated in this formalism.

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