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ON THE OCCURRENCE OF DISSIPATIVE FORCES IN THE SCATTERING OF TWO HEAVY IONS. I.

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Силы трения в реакциях между сложными ядрами. 1

На основе полумикроскопической модели для описания внутренних возбуждений в ядрах получено эффективное классическое уравнение для относительного движения, включающее силы трения.

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On the Occurrence of Dissipative Forces in the Scattering of Two Heavy Ions. I

Starting with a semi-microscopic model for the quantum description of colliding ions we deduce an effective classical equation for the relative motion in which the coupling between the relative motion and the intrinsic nucleonic degrees of freedom appears implicitly as dissipative and conservative forces. In our formulation we distinguish between processes (non-collective excitations) which have a short relaxation time and processes (low-lying collective excitations) which relaxation time is of the order of the collision time. In the first case a frictional-like term appears in the effective equation of motion while in the second case the effective force becomes nonlocal in time. A rigorous formulation of the problem in the Zubarev formalism is also given.

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1. INTRODUCTION

At present, one of the most complicated problems describing heavy ion reactions arises from the fact that there is a high probability for excitations of more than 100 MeV. At these excitation energies the level density is too large to use, e.g., the quantum coupled channel method for practical calculations. The simpler distorted wave Born approximation cannot be used because individual cross sections for certain partial waves are so large that the Born approximation fails to work. To overcome these difficulties, method of quantum statistical mechanics has been applied recently to the problem of heavy ion reactions. From the quantum many-body equation of motion of two colliding heavy ions a classical description of the motion of nuclei is derived by introducing dissipative forces, as, e.g., a frictional force [1-9].

For instance, Beck and Gross [1] and Gross [5] started with the general many-body Schrödinger equation for colliding heavy ions and deduced a classical equation of motion including a frictional force. The resulting classical model reproduces many general features of deep inelastic collisions between heavy ions [3,9].

A more detailed theoretical formulation of the same problem has been given by Hofman and Simens [8] who started with the linear response theory to construct a differential equation for the relative motion, in which the nucleonic degress of freedom appear implicitly as dissipative and conservative forces.

On the other hand, Kan and Griffin [4,6] started with a non-linear time-dependent Schrödinger equation to describe a quantum system with friction. In their formulation a priori a frictional potential proportional to the velocity potential of the wave function is added to the usual Hamiltonian. A classical dynamical model including dissipative forces also has been considered by Bondorf, Sobel and Soeker (see [7] and further references therein).

In spite of the fact that the origin and nature of the dissipative forces seem to be well understood there exist no quantitative estimates for the dissipative forces starting from a reasonable semi-microscopic model for the description of colliding ions. Here we intend to put forward such a formulation of the problem. Using a model familiar from the nuclear structure calculations to describe the intrinsic nucleonic degrees of freedom, some problems in deriving the effective equation of the relative motion containing dissipative forces are investigated.

Especially, if the effect of the coupling of the relative motion on the intrinsic nucleonic degrees of freedom is treated we should distinguish between processes which have a very short relaxation time τ and processes which relax only very slowly during the collision time t_{coll}.

In the first case ($\tau \ll t_{coll}$) it is possible to deduce a frictional force proportional to the velocity of the relative motion [8]. To this end it must be assumed that the considered nucleus is heated up during the collision process and that its temperature T changes very little during the relaxation time τ.

However, as we will see later not all dissipative processes can be expressed in such a simple form. Especially, those terms which give non-vanishing contributions to the dissipative force for $T=0$ must be worked out in detail, and terms which are not local in time must be discussed more rigorously. We hope to give some clarifying discussions in this paper.

Another problem is the treatment of processes with a long relaxation time ($\tau \gg T_{coll}$), e.g., the slow collective motion. Hofmann and Siemens [8] proposed that the slow collective motions should not be considered as friction, but must somehow be taken into account explicitly. In this work, we pay a special attention to the contribution of the slow collective motions to the dissipative processes which seem to be important in the (phenomenological) model calculation of nuclear friction [3,9].

To make the whole problem rather transparent, we consider collective (large relaxation time, $\tau \gg t_c$) and non-collective ($\tau \ll t_c$) excitation modes in a simple model. In section 2 we use the equation of motion to find the time dependence of the relative

coordinates and the operators describing the inner motion and to get an expression for dissipative force.

In Part II a more rigorous formulation of the problem is given by using the method of Zubarev-McLemnon to construct a non-equilibrium statistical operator which solves the Liouville equation. Concluding remarks can be found in Part II.

2. DERIVATION OF A CLASSICAL EQUATION OF RELATIVE MOTION OF TWO COLLIDING IONS

2.1. The Quantum Mechanical Description of the System

To simplify the subsequent derivations and discussions we do not take into account the intrinsic structure of one of the colliding ions that means, we consider the description of the inelastic collision of the mean field of the first ion with nucleons of the second ion producing excitations of the intrinsic degrees of freedom in the second ion. Effects which are connected with the possibility that nucleons of one nucleus could be scattered by the nucleons of the second nucleus are not taken into account in our considerations. Such an approach seems to be possible when the colliding ions do not overlap too much. The total Hamiltonian $\hat{H}$ suitable to our problem takes the form:

$$\hat{H} = \hat{H}_R + \hat{H}_0 + V_{int} , \qquad (2.1)$$

where $\hat{H}_R$ is the Hamiltonian of the relative motion, $\hat{H}_0$ describes the internal motion

of the nucleus and the coupling between the relative and internal motion is given by V_{int} . In more detail we have:

$$\hat{H}_R = -\frac{\hbar^2}{2M}\vec{\nabla}_R^2 + U(\vec{R}), \qquad (2.2)$$

where $U(\vec{R})$ is the empirical nucleus-nucleus potential given by the real part of the average optical model potential and M stands for the reduced mass. The Hamiltonian of the internal motion is written in the picture of elementary excitations described by phonons in the form:

$$\hat{H}_0 = \sum_j \omega_j b_j^+ b_j + \sum_{jk\ell} W_{jk\ell}(b_j^+ b_k^+ b_\ell + b_\ell^+ b_k b_j) + \dots , \qquad (2.3)$$

where $b_j^+(b_j)$ are the phonon creation (destruction) operators. Higher order terms are neglected in (2.3), however, if occasion arises we will have in mind that in (2.3) there is an infinite expansion series in terms of phonons. The interaction part takes the form

$$\hat{H}_{int} = \sum_j V_j(\vec{R})(b_j^+ + \bar{b}_j) + \sum_{jk} V_{jk}(\vec{R})(b_j^+ + \bar{b}_j)(b_k^+ + \bar{b}_k) + \qquad (2.4)$$

$$+ \sum_{jk\ell} V_{jk\ell}(\vec{R})(b_j^+ + \bar{b}_j)(b_k^+ + \bar{b}_k)(b_\ell^+ + \bar{b}_\ell) + \dots ,$$

where $\bar{b}_j$ denotes the time conjugate operator of b_j .

Due to the interaction part of $\hat{H}_0$ the phonon excitations have a finite life-time which we will denote by τ_j . If we consider the behaviour of different phonon modes during the collision process we must distinguish between two cases: modes with a life-time which is comparable or larger

than the collision time ($\tau_j \gtrsim t_{coll}$) and modes with a very short life-time ($\tau_j \ll t_{coll}$).

In the first case ($\tau_j \gtrsim t_{coll}$) we will call the modes collective ones. It will be proposed, that these collective modes are located in the discrete low-energy part of the eigenspectrum of $\hat{H}_0$.

In the second case ($\tau_j \ll t_{coll}$) we will call the modes non-collective ones. The energy spectrum of these non-collective modes has, besides the discrete part, large contributions from the continuous spectrum, too.

Because of the different relaxation properties the effect of V_{int} on these two types of excitations (collective and non-collective) must be considered separately.

Our aim is to obtain an effective classical equation for the relative motion of two ions. Such an equation of motion can be deduced rather easily by starting with the Heisenberg equation of motion for the operator $\vec{R}$:

$$\frac{\partial \vec{R}}{\partial t} = \frac{i}{\hbar}[H,\vec{R}] = -\frac{ih}{M}\vec{\nabla}_R \qquad (2.5)$$

$$\frac{\partial^2 \vec{R}}{\partial t^2} = \frac{i}{\hbar}[H,\dot{\vec{R}}] = -\frac{1}{M}\vec{\nabla}_R U(R) - \frac{1}{M}\vec{\nabla}_R V_{int}(\vec{R}) \qquad (2.6)$$

and averaging over the intrinsic degrees of freedom of one nucleus which structure is described in our model by phonon-excitations:

$$M\frac{\partial^2 \vec{R}}{\partial t^2} = \vec{F} = -\vec{\nabla}_R U(\vec{R}) - [\sum_j \vec{\nabla}_R V_j(\vec{R}) < b_j^+ + \bar{b}_j > +$$

$$+ \sum_{jk} \vec{\nabla}_R V_{jk}(\vec{R}) < (b_j^+ + \bar{b}_j)(b_k^+ + \bar{b}_k) > + \sum_{jk\ell} \vec{\nabla}_R V_{jk\ell}(\vec{R}) \times \qquad (2.7)$$

$$\times < (b_j^+ + \bar{b}_j)(b_k^+ + \bar{b}_k)(b_\ell^+ + \bar{b}_\ell) > = \vec{F}_1 + \vec{F}_2 \ ,$$

where the effective force $\vec{F}$ contains, besides the usual conservative force $\vec{F}_1 = -\vec{\nabla}_R U(\vec{R})$ terms ($\vec{F}2$) which are due to the coupling of the relative motion to the intrinsic degrees of freedom. It was shown [3,8,9] that $\vec{F}_2$ contains among further correction to the usual force a dissipative one which is connected with the irreversible energy transfer from the relative motion into the intrinsic motion of the nucleus. In other words, we have constructed a differential equation for the relative motion of the ions, in which the nucleonic degrees of freedom enter implicitly in terms of dissipative and conservative forces.

Note, that in (2.7), the function $V_{jk}(\vec{R}(t))$ which depends via $\vec{R}(t)$ on time, is still an operator in the space of the relative motion. To obtain the classical limit which we are interested in eq. (2.7) must be averaged over a wavepacket $|0>$. This wavepacket has to be so narrow that for any operator $\hat{O}(\vec{R},\vec{P})$ the following relation

$$<|\hat{O}(\vec{R},\vec{P})|> = \hat{O}(<|\vec{R}|>,<|\vec{P}|>) \qquad (2.8)$$

can be used.

To obtain the effective equation of motion in more detail it remains to solve the Heisenberg equation of motion for the operator $b^+(b)$ and to average the procedure. Of cource, this can be done in an approximate way only. In the next section we are dealing with the non-collective modes, while in section 2.3 the collective modes will be discussed.

2.2. The Treatment of the Non-Collective Modes

To obtain a more convenient expression for the force $\vec{F}$ and to extract the dissipative part we must know equation of motion for the operator $b_j^+(b_j)$. That can be done by using the corresponding Heisenberg equation of motion, which in our approach is given by:

$$ih\frac{db_j^+(t)}{dt} = \omega_j b_j^+ + \sum_{\ell k} W_{\ell kj}\, b_\ell^+ b_k^+ + 2\sum_{k\ell} W_{jk\ell}\, b_\ell^+ b_k + V_j^* + 2\sum_k V_{jk}(b_k^+ + \bar{b}_k) + 3\sum_{k\ell} W_{jk\ell}(b_k^+ + \bar{b}_k)(b_\ell^+ + \bar{b}_\ell).$$

The solution for the latter equation can be of an approximate nature only. We envisage it in the form:

$$b_j^+(t) = C_j(t)e^{-\frac{i}{h}\omega_j t}, \qquad (2.10)$$

where the function $C_j(t)$ in a perturbation treatment of the problem takes the form:

$$C_j(t) = b_j^+ - \frac{i}{h}\int_{-\infty}^{t} dt' e^{\frac{i}{h}\omega_j t'}\left(V_j^*(t') + 2\sum_k V_{jk}(t')\{k,t'\} + 3\sum_{k\ell} W_{jk\ell}(t')\{k,t'\}\{\ell,t'\} + \dots\right), \qquad (2.11)$$

where we have introduced the abbreviation

$$\{j,t\} = \left(e^{-\frac{i}{h}\omega_j t} b_j^+ - e^{\frac{i}{h}\omega_j t}\bar{b}_j\right).$$

Solving eq. (2.9) higher order terms of $\hat{H}_0$ have been neglected because in the lowest order these terms give no contribution to $\vec{F}_2$. By using (2.10) and (2.11) we find

$$(b_j^+(t) + \bar{b}_j(t)) = \{j,t\} - \frac{2}{h}\int_{-\infty}^{t} dt' \sin\frac{\omega_j}{h}(t-t')\{V_j^*(t') + 2\sum_{k'} V_{jk'}(t')\{k',t'\} + 3\sum_{k'\ell'} V_{jk'\ell'}(t')\{k',t'\}\{\ell',t'\} + \dots\} \qquad (2.12)$$

$$(b_j^+(t) + \bar{b}_j(t))(b_k^+(t) + \bar{b}_k(t)) = \{j,t\}\{k,t\} - \frac{2}{h}\int_{-\infty}^{t} dt' \sin\frac{\omega_k}{h}(t-t')\times\{j,t\}\{V_k^*(t') + \dots\} - \frac{2}{h}\int_{-\infty}^{t} dt' \sin\frac{\omega_j}{h}(t-t')\{V_j^*(t') + \dots\}\times\{k,t'\}. \qquad (2.13)$$

Now we can insert the expression (2.12)-(2.14) into (2.7) and perform the averaging procedure. In the sense of a perturbation treatment we take only the diagonal matrix elements. In general, a more complicated averaging procedure should give a similar result, if one assumes that after the short relaxation time τ_j the phases of the non-diagonal matrix elements are randomly distributed, so that their sum is negligible with respect to the diagonal ones. In so doing, the result for the first two terms (2.12) and (2.13) is:

$$\langle b_j^+(t) + \bar{b}_j(t)\rangle = \frac{2}{h}\int_{-\infty}^{0} d\tau \sin\frac{\omega_j}{h}\tau\{V_j^*(t+\tau) + 3\sum_{k'} V_{jk'k'}(t+\tau)(1+2n_{k'}(t+\tau))\} \qquad (2.15)$$

and

$$\langle (b_j^+(t) + \bar{b}_j(t))(b_k^+(t) + \bar{b}_k(t))\rangle = \delta_{jk}(1+n_j(t)) - \frac{8}{h}\int_{-\infty}^{0} d\tau \sin\frac{(\omega_j - \omega_k)}{h}\tau\{V_{jk}(t+\tau)(n_j(t+\tau) - n_k(t+\tau))\}, \qquad (2.16)$$

where $n_j(t')$ are occupation numbers of the non-collective states. In the simplest quasistatic approximation we can neglect the time dependence in $n_j(t')$ in the sense, that we are considering the nucleus heated after a certain time and n_j are determined by the temperature of nucleus. Or in other words, we look at the motion of the colliding ions for a rather short time Δt around a time t and neglect the change of the nucleus temperature in the time interval Δt when evaluating some important properties of the whole system of colliding ions. This quasistatic approach has been used by Hofman and Siemens /8/ and it permits one to express the dissipative part through a term proportional to the velocity $\dot{\vec{R}}$. That means that the force $\vec{F}_2$ becomes local in time. To calculate the effective force we have to insert (2.15) and (2.16) into (2.7) and if possible to perform the integration.

As proposed by Hofmann and Siemens the function $V_{jk}(t+\tau)$ in the second term of (2.16) can be expanded into a power series of τ around t. In sense of the quasistatic approach such an expansion is reasonable, because the relaxation time τ_j should be small in comparison with the collision time t_{coll}. The expansion reads

$$V_{jk}(\vec{R}(t+\tau)) = V_{jk}(\vec{R}(t)) + \dot{\vec{R}}\,\vec{\nabla}_R V_{jk}(\vec{R}(t))\,\tau + \left((\dot{\vec{R}}\,\vec{\nabla}_R V_{jk}) + (\dot{\vec{R}}\,\vec{\nabla}_R)^2 V_{jk}\right)\frac{\tau^2}{2} + \dots \qquad (2.17)$$

Inserting this expansion into (2.16) and performing the integration, we see that the

first term of (2.17) gives no rise to an effective force, because it is proportional to a δ-function of the type $\delta(\omega_j - \omega_k)$. A non-vanishing contribution to the force $\vec{F}_2$ can be expected only from the next terms of (2.17). Especially, the second terms inserted into (2.16) yield

$$-\frac{8}{h}\dot{\vec{R}}\sum_{jk}\frac{\partial V_{jk}(R(t))}{\partial R}[n_j(t)-n_k(t)]\int_{-\infty}^{0}\tau\,d\tau\sin\frac{(\omega_j-\omega_k)}{h}\tau = \qquad (2.18)$$

$$= 16\pi h\,\dot{\vec{R}}\sum_{jk}\vec{\nabla}_R V_{jk}(\vec{R}(t))[n_j(t)-n_k(t)]\delta'(\omega_j-\omega_k).$$

As is seen from (2.18) this term gives rise only to a dissipative force for finite temperatures $T(t)>0$. From the structure of the expression for the dissipative part proportional to $\dot{\vec{R}}$ it can be seen that the discrete spectrum in the lowest order gives no rise, because the argument of the δ-function is never zero. That means that we can expect contributions to the dissipative force, which is in our case a simple friction force, only if in the non-collective excitation there are admixtures from the continuous spectrum.

At the beginning of the process, if $T=0$, the main contribution to the dissipative force comes from (2.15),

$$\frac{2}{h}\int_{-\infty}^{0}d\tau\sin\left(\frac{\omega_j}{h}\tau\right)V_j^*(t+\tau) \qquad (2.19)$$

so that the nucleus will be heated up to a finite temperature in the initial state. However, this contribution cannot be represented by a term proportional to $\dot{\vec{R}}$, etc., because an expansion in a power series as is done above (2.18) makes no sense. In spite

of the fact that the integral could be calculated as soon as $V_j(t+\tau)$ is known, all single terms of the power series are equal to zero as long as the ground state is separated by a gap. After insertion of (2.15) into (2.7) the resulting force becomes not local in time but contains a dissipative part connected with the excitation of the internal degrees of freedom due to the time dependence of the interaction $V_j(t)$. Finally, it should be mentioned that the solution of the effective equation for the relative motion must be performed in a self-consistent way. That part of the force connected with (2.15) is not local in time and must be calculated from the whole history of the collision process. Also the part is coming from (2.16) which is proportional to $\dot{\vec{R}}$ but it depends on the temperature $T(t)$ via $n(T(t))$, which must be determined by the history of the collision process as will be pointed out in part II.

2.3. The Treatment of the Collective Modes

It has been mentioned above that the collective modes must be treated in a somewhat different way because their relaxation time is larger than the collision time. In this case it is necessary to work in an adiabatic representation of the collective motion. Then we can distinguish more directly between the dissipative and conservative part of the effective force.

To do this we neglect the higher order terms in H_0 and V_{int} and consider a Hamiltonian $\mathcal{H}$ of the form

$$\mathcal{H} = H_0' + V_{int}' = \sum_j \omega_j b_j^+ b_j + \sum_{jk} V_{jk}(b_j^+ + \bar{b}_j)(b_k^+ + \bar{b}_k) + \sum_j V_j (b_j^+ + \bar{b}_j). \tag{2.21}$$

Because higher orders in H_0 are assumed to be small it is reasonable to diagonalize the Hamiltonian by means of a canonical transformations:

$$b_j^+ = \sum_k (U_{jk}\,\beta_k^+ + V_{jk}\,\bar{\beta}_k) \tag{2.22}$$

As a result

$$\mathcal{H} = \sum_j \tilde{\omega}_j[R(t)]\beta_j^+\beta_j + \sum_j \tilde{V}_j[R(t)](\beta_j^+ + \bar{\beta}_j). \tag{2.23}$$

The transformation coefficients U_{jk} and V_{jk} are time-dependent via $R(t)$ and the resulting Hamiltonian $\mathcal{H}$ (2.23) emerges to have a time-dependent eigenspectrum.

The Heisenberg equation of motion for the operator $\beta_j^+(t)$ reads

$$ih\frac{d\beta^+(t)}{dt} = [H_0' + V_{int}', \beta^+(t)] = \tilde{\omega}_j(t)\beta_j^+(t) + V_j^*(t) \tag{2.24}$$

and its solution is given by

$$\beta_j^+(t) = a_j(t)\beta_j^+ - \frac{i}{h}a_j(t)\int_{-\infty}^{t} a_j(t')V_j^*(t')dt', \tag{2.25}$$

where we have introduced

$$a_j(t) = e^{-\frac{i}{h}\int_{-\infty}^{t}\omega_j(t')dt'}. \tag{2.26}$$

The effective force takes now the form

$$\vec{F} = -\vec{\nabla}_R U(\vec{R}) - \sum_j \vec{\nabla}_R \tilde{V}_j(\vec{R}(t))\langle\beta_j^+ + \bar{\beta}_j\rangle. \tag{2.27}$$

The averaging procedure in (2.27) is performed in the same manner as in Section 2.3. and we get

$$\langle \beta_j^+(t) + \bar{\beta}_j(t) \rangle = \frac{2}{h} \int_{-\infty}^{0} d\tau \sin\left(\frac{1}{h}\int_{0}^{\tau} \tilde{\omega}_j(t')dt'\right) \tilde{V}_j^*(t+\tau). \quad (2.28)$$

This expression inserted into (2.27) gives the lowest order contribution to the dissipative part of the effective force due to the consideration of the collective modes. The force connected with (2.27) contains an effect of memory and is determined by the whole trajectory $\mathbf{R}(t+\tau)$ in the past. This seems to be a reasonable result, because the relaxation τ_j is large. Note, that the dissipative force due to (2.28) does not depend on the temperature $\mathbf{T}(t)$. Thus, in the lowest order we obtain a non-vanishing contribution to the dissipative force due to the excitation of the collective modes for $\mathbf{T=0}$. Such a result could be expected because of the large relaxation time τ_j. In the course of collision the collective modes do not equilibrate and therefore it makes no sense to speak in this case about a temperature of the nucleus. For further clarifying discussions we refer to the next part.

As in the previous section it seems to be rather desirable to obtain an explicit expression for this type of force in dependence on $\vec{R}(t)$ and $\dot{\vec{R}}(t)$ at least approximately. This might be done in some special situation only.

One possibility consists in the following. We can again expand $\tilde{V}^*(t+\tau)$ in a power series of τ as done in (2.17) and at the

same time take in the sin of (2.28) the whole time-dependent function as an argument. Thus, the term proportional to $\dot{\vec{R}}$ is of the type

$$\frac{2}{h}\dot{\vec{R}}\,\tilde{V}_j^*(t)\int_{-\infty}^{0} \tau\, d\tau \sin\left(\frac{1}{h}\Omega_j(\tau)\right) d\tau \quad (2.29)$$

which after integrating (if possible) gives no δ-function or its derivatives.

Another possibility is connected with the use of special trial function for the time-dependence of $\mathbf{V}_j^*(t+\tau)$ and $\Omega_j(\tau)$. For instance, a special exponential or gaussian form for $\mathbf{V}_j^*(\mathbf{R}(t+\tau))$ and $\Omega_j(\mathbf{R}(t+\tau))$ permits one to calculate the integral. Of course, these possibilities proposed for extracting a dissipative part from (2.29) must be proved more rigorously at least by numerical calculations related to the special situations.

Up to now we have taken the Hamiltonian $\mathcal{H}$ in its simplest form neglecting the higher order terms. Therefore, we shall expect that the main effect leading to a dissipative force is taken into account by (2.29). Considering higher order terms in $\mathcal{H}$ a term will be obtained which is similar to that discussed in section 2.2. For instance, a term appears which is of the form

$$\int_{-\infty}^{0} \Psi(t+\tau)\sin\left(\frac{\omega_k+\omega_\ell-\omega_i}{h}\right)\tau\, d\tau[n_k(t+\tau)+n_\ell(t+\tau)-n_i(t+\tau)] \quad (2.30)$$

which can be discussed by the analogy with the term (2.18) in the previous section. If we assume that at least one of the energies ω_j belongs to the continuous spectrum, we can again find a connection of (2.30) with $\dot{\vec{R}}$. However, it must be provided that an expansion in a power series of τ is reasonable.

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