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The separation method was proposed by Moszkovski and Scott [1] for qualitative estimates of the Brueckner reaction matrix elements. The idea of the method is to divide the potential $v(r)$ of the nucleon-nucleon interaction into the short-range v^s and long-range v^l parts:

$$v(r) = v(r)\theta(d-r) + v(r)\theta(r-d) \quad v^s(r) + v^l(r). \qquad (1)$$

Here $r = r_1 - r_2$, r_1 and r_2 coordinates of interacting particles, $\theta(x)$ the Heviside function. $v^s(r)$ may have the hard core. For the states with even relative orbital moments one can choose the separation parameter d in such a way that the Brueckner reaction matrix can be approximated by $v^l(r)$: that is the main merit of the method. But at the same time its two drawbacks are immediately apparent. Firstly, the method is non-applicable to odd states; secondly, the separation parameter depends on the quantum numbers of the relative motion, hence, the non-diagonal matrix elements turn out to be indefinite.

The above mentioned difficulties are eliminated automatically if one uses the fixed d. We can show that all the advantages of the method are preserved. The equation for the elements of the reaction matrix G is as follows:

$$G_{a\beta} = v_{a\beta} + \sum_{\nu} \frac{\langle a|G|\nu\rangle Q_{\nu}\langle\nu|v|\beta\rangle}{\omega - E_{\nu}}, \qquad (2)$$

or in the operator form:

$$G = v + G\frac{Q}{\omega - H_0}v. \qquad (2a)$$

Here $\langle r_1, r_2|\nu\rangle \equiv \phi_{\nu}(r_1, r_2)$ is the antisymmetrized wave function of two independent particles, described by the model Hamiltonian H_0 (oscillator, for example); E_{ν}, the energy of the pair of particles in a ν state; Q, the operator excluding the occupied states from the intermediate ones; ω, the starting energy, which is a parameter, when G -matrix elements are calculated off the energy shell, and is equal to E_a on the energy shell.

Let us remove from (2) the "unpleasant" short-range part of the potential v^s introducing for it the scattering matrix of the two free particles interacting by v^s :

$$t_{a\beta} = v^s_{a\beta} + \sum_{\nu} \frac{\langle a|v^s|\nu\rangle\langle\nu|t|\beta\rangle}{E_{\beta} - E_{\nu}}, \qquad (3)$$

or in the operator form:

$$t|\beta\rangle = v^s(1 + \frac{1}{E_\beta - H_0}t)|\beta\rangle. \qquad (3a)$$

While iterating (3) it is not difficult to notice that all the integrals over r, on the right-hand side, are from 0 to d. Therefore, it is possible to say that the operator t is different from zero, as v^s, only in the interval $0 \le r \le d$.

Let us muliply (2a), from the right side, by $(1 + \frac{1}{E_\beta - H_0}t)|\beta\rangle$:

$$G(1 + \frac{1}{E_\beta - H_0}t)|\beta\rangle = v(1 + \frac{1}{E_\beta - H_0}t)|\beta\rangle +$$

$$+ G\frac{Q}{\omega - H_0}v(1 + \frac{1}{E_\beta - H_0}t)|\beta\rangle.$$

If we break v into v^s and v^ℓ, and use equation (3a), we will find out the required relation between G, t and v^ℓ:

$$G[1 + \frac{1}{E_\beta - H_0}t - \frac{Q}{\omega - H_0}(t + v^\ell + v^\ell \frac{1}{E_\beta - H_0}t)]|\beta\rangle =$$

$$= (t + v^\ell + v^\ell \frac{1}{E_\beta - H_0}t)|\beta\rangle. \qquad (4)$$

Define now the correlated wave function ψ by the relation:

$$t\phi_{\nu}(r_1, r_2) = v^s \psi_{\nu}(r_1, r_2). \tag{5}$$

From (3a) and (5) it is obvious that:

$$v^s \left| \psi_\beta - \phi_\beta - \frac{1}{E_\beta - H_0} t|\beta\rangle \right| = 0. \tag{6}$$

For $r \leq d$, v^s is not identically equal to zero, thus:

$$\psi_\beta = \phi_\beta + \frac{1}{E_\beta - H_0} t|\beta\rangle, \quad r \leq d. \tag{7}$$

By multiplying (7), from the left side, by the operator $(E_\beta - H_0)$ we obtain the differential equation for ψ:

$$(E_\beta - H_0)\psi_\beta = t\phi_\beta, \quad r \leq d, \tag{8}$$

or

$$(E_\beta - H_0)\psi_\beta = v^s \psi_\beta, \quad r \leq d. \tag{8a}$$

In the interval $r > d$, where $v^s = 0$, we can assume the square bracket in (6) equal to everything and to zero as well. Using that statement we get immediately that:

$$\psi_\beta = \phi_\beta, \quad r > d \tag{9}$$

and hence,

$$\frac{1}{E_\beta - H_0} t|\beta> = 0, \qquad r > d. \tag{10}$$

Since the interaction influences only relative coordinates, it would be better to pass to them in the wave functions also. For the oscillator functions this can be reached by the Moshinsky transformation:

$$\phi_\beta(r_1, r_2) = \sum_{n,N} C^{\beta}_{nN} \phi_n(r) \phi_N(R),$$

where $n = (n, \ell)$, $N = (N, L)$ quantum numbers of the relative and centre of mass motions. Assume, as usual, that the operator t does not influence the wave function of the centre of mass motion, i.e.,

$$t\phi_\beta(r_1, r_2) = \sum_{n,N} C^{\beta}_{nN} \phi_N(R) t\phi_n(r) =$$

$$= \sum_{n,N} C^{\beta}_{nN} \phi_N(R) v^s \psi_n(r). \tag{11}$$

If we have written (11), we suppose, therefore, that

$$\psi_\beta(r_1, r_2) = \sum_{n,N} C^{\beta}_{nN} \phi_N(R) \psi_n(r).$$

The differential equation for the correlated wave function of a relative motion

$$[E_{n\ell} - H_0(r) - v^s(r)] \psi_{n\ell}(r) = 0, \quad r \le d \tag{12}$$

can be deduced from (8). The boundary condition at $r = r_c$ (r_c is the radius of the potential hard core) is evident: $\psi_{n\ell}(r_c) = 0$. At $r = d$ it is naturally to suppose $\psi_{n\ell}(d) = \phi_{n\ell}(d)$. Then $\psi_{n\ell}(r)$ will be continuous from 0 to ∞. If there is a tensor component in the interaction, then (12) splits into two (at $\ell = 0,1$) or three (at $\ell > 1$) connected equations for the components of ψ : $u_{n\ell}$ and $w_{n,\ell\pm 2}$. Boundary conditions are chosen as previously, in such a way that for $r \geq d$ $u_{n\ell}$ and $w_{n,\ell\pm 2}$ may coincide with corresponding model wave functions:

$$u_{n\ell}(r_c) = 0, \qquad u_{n\ell}(d) = \phi_{n\ell}(d),$$

$$w_{n\,\ell\pm 2}(r_c) = 0, \qquad w_{n,\ell\pm 2} = \phi_{n\mp 1,\ell\pm 2}(d).$$

Let us substitute (7) and (8) into (4):

$$G\left[1 - \frac{Q}{\omega - H_0}(t + v^{\ell})\right]|\beta\rangle + G|\psi_\beta - \phi_\beta\rangle = (t + v^{\ell})|\beta\rangle . \qquad (13)$$

With the notation $\langle\phi_\nu|\psi_\beta\rangle = b_{\nu\beta}$ we write (13) in detail as follows:

$$\sum_\nu G_{\alpha\nu}\left[b_{\nu\beta} - Q_\nu \frac{(t + v^{\ell})_{\nu\beta}}{\omega - E_\nu}\right] = (t + v^{\ell})_{\alpha\beta} . \qquad (14)$$

To calculate the matrix elements $t_{\alpha\beta}$ one can use equation (8):

$$t_{\alpha\beta} = \langle a|(E_\beta - H_0)\psi_\beta\rangle =$$

$$= \left[\phi_a \frac{\partial \psi_\beta}{\partial r} - \frac{\partial \phi_a}{\partial r}\psi_\beta\right]_{r=d} + (E_\beta - E_a)\overline{\langle a|\psi_\beta\rangle}. \qquad (15)$$

The bar above the matrix element shows that the integration over r is made from r_c to d.

One can write (14) symbolically, as an equation for matrixes:

$$G \cdot B = A .$$

One can get G-matrix elements by inverting the matrix B. Such an operation is quite feasible as the terms in the sum are decreasing rapidly. Sometimes it is convenient to use an iteration procedure. So, to first order in v it gives:

$$G^{(1)}_{\alpha\beta} = t_{\alpha\beta} + v^{\ell}_{\alpha\beta} .$$

After the second iteration, we get:

$$G^{(2)}_{\alpha\beta} = V_{\alpha\beta} + \langle a|V|\chi_\beta\rangle + \sum_{\nu>\mu} \frac{V_{a\nu} V_{\nu\beta}}{\omega - E_\nu},$$

where $V = t + v^{\ell}$, $\chi_\beta = \phi_\beta - \psi_\beta$, μ occupied states. And in detail:

$$G^{(2)}_{\alpha\beta} = t_{\alpha\beta} + v^{\ell}_{\alpha\beta} + \langle \alpha | t | \chi_{\beta} \rangle -$$

$$- \sum_{\mu} \frac{t_{\alpha\mu} v^{\ell}_{\mu\beta} + v^{\ell}_{\alpha\mu} t_{\mu\beta}}{\omega - E_{\mu}} + \sum_{\nu > \mu} \frac{t_{\alpha\nu} t_{\nu\beta} + v^{\ell}_{\alpha\nu} v^{\ell}_{\nu\beta}}{\omega - E_{\nu}} .$$

Let us examine the diagonal matrix element $G^{(1)}_{\alpha\alpha}$:

$$G^{(1)}_{\alpha\alpha} = (\phi_{\alpha} \frac{\partial \psi_{\alpha}}{\partial r} - \frac{\partial \phi_{\alpha}}{\partial r} \psi_{\alpha})_{r=d} + v^{\ell}_{\alpha\alpha}$$

$$= \sum_{nn'N} C^{\alpha}_{nN} C^{\alpha}_{n'N} \{ \phi_{n}(d) [\frac{\partial \psi_{n'}}{\partial r} - \frac{\partial \phi_{n'}}{\partial r}]_{r=d} + v^{\ell}_{nn'} \} . \quad (16)$$

By selecting, following Scott and Moszcovski, new d_n for each state n, one can eliminate the first term in (16) for even ℓ states. Then we have obtained the known expression

$$G^{(1)}_{\alpha\alpha} \simeq v^{\ell}_{\alpha\alpha} .$$

However, it contains the error due to the fact that one does not know how to calculate the non-diagonal matrix elements $v^{\ell}_{nn'}$. At a constant d we avoid the difficulty, and

the first term in (16) gives us an idea on the value of the mentioned error*. Besides, the selection of the value of the separation parameter d may now be connected with some additional conditions. For example, in the estimation of a binding energy, we may consider d as a variational parameter.

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2. A.Osman. Nuovo Cim., 31A, 487 (1976).

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*Recently the paper by Osman has appeared [2]. There he adds a pseudopotential v^p to an interaction to keep d constant. From (16) it is easi to see that matrix elements $v^p_{nn'}$ must coincide with $c_n(d)\left[\frac{\partial \phi_{n'}}{\partial r} - \frac{\partial \psi_{n'}}{\partial r}\right]_{r=d}$. The calculations done with the Hamada-Johnston potential on the oscillator basis give for the $^1S_0$-state: v^p_{nn} =0.0, -23.1; -55.1; -89.6; -136.3 MeV for n =0,1,2,3,4, respectively (d =1.02 fm, the oscillator parameter ν =1.76 fm).