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UNIFIED MODEL FOR FISSION IN THE FRAMEWORK OF THE FOUR PROJECTORS THEORY

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1. INTRODUCTION

After the appearance of the paper [1], the macroscopic-microscopic approach, which has made so many contributions to our understanding of nuclear fission, nuclear masses and various types of collective properties of the nuclei, is now being applied to the dynamical description of the fission or to the heavy-ion reactions leading to the synthesis of compound nuclei.

In the last ten years the number of the scientific papers devoted to this problem seems to be growing exponentially for a number of reasons: a) the increasing experimental interest (the search of the superheavy nuclei, the search of other nuclear density quasistable states and so on), b) the applicability of different methods belonging to the different branches of physics, c) the gathering of the different nuclear models into the super-unified model, describing not only statically but dynamically also the motion of the two complex nuclei coming into collision or going on from the mother nucleus, d) in connection with the points b) and c) we have to choose the important degrees of freedom and formulate the equations of motion (inertias, and forces both conservative and nonconservative).

The aim of the present paper is to apply the Franz [2,3] theory of decaying nuclear systems to the fission process using the four projectors [4-8,54] theory and the fragmentation dynamics in the nucleon-nucleon collision [9-13].

An explicit expression for the fission width (hence, the fission half-lives) is obtained. The fission width corresponding to the particular fission mode is a product of the corresponding penetrability factor [14–21] with a preformation factor (i.e., the probability that the fission products are already formed). The last factor replaces [22] the usual characteristic frequency factor (number of assaults) estimated by a statistical model.

From our theory results that the preformation factor depends linearly on the fission yield as a function of the mass and charges of the fragments. In our treatment the most important degrees of freedom to get the preformation factor are the collective mass-fragmentation [9] and charge-fragmentation [10] degrees of freedom.

2. *FRANZ THEORY*

The idea of Franz [2,3] is that what is common to all unstable nuclear systems is that the production and the measurements are macroscopic manipulations, and thus for instance it is impossible to fix the time of the beginning of the measurement microscopically. Therefore we have to work with the statistical operators describing both the prepared states of the unstable nuclear system and the space of localizing the nuclear system. The first operator is the operator describing the density of the incoherent, normalized to unity prepared states $|\lambda_k>$ of the still undecayed nuclei:

$$\rho = \sum_k |\lambda_k > \omega(k) < \lambda_k| \,, \qquad (1)$$

where $\omega(k)$ is the weight of the corresponding $|\lambda_k>$ state and the second operator

$$W(t) = \int^R dx\, e^{iHt} |x><x| e^{-iHt} \qquad (2)$$

projects onto the nuclear spatial domain defined so that all the spatial coordinates of the nucleons $|x| \leq R$ (the nuclear radius of the mother nucleus). As an observable determining the decay law is assumed to be the ratio [4,8,54]:

$$P(t) = \frac{\text{trace}\{\rho W(T+t)\}}{\text{trace}\{\rho W(T)\}} = \frac{N(T+t)}{N(T)} \qquad (3)$$

of the number $N(T+t)$ of the undecayed nuclear systems at the time $T+t$ to the number $N(T)$ of the undecayed nuclear systems at the time T. The time T denotes the macroscopic time in which one is sure that the decay products (fission fragments) are inside the spatial domain R. As a nuclear system we understand the system of the A-nucleons belonging at the beginning to the mother nucleus.

Many authors assume different models for the so-called prepared states that are going to decay. These states must describe the localized nondecayed instable nuclear systems for a macroscopic time T. It is natural to assume that such states are superpositions of bound states [6,8,35] embedded in the continuum (BSEC) in their different forms (doorway-, hallway-states, etc.), generated by the asymptotic channel Hamiltonian that describe the motions of every pair of fission fragments (decay products). This assumption is a natural application of the causality principle in the quantum mechanics. These BSEC-states give birth to the poles [23–26] in the second Riemann sheet by including the residual interactions.

3. *ASYMPTOTIC CHANNEL HAMILTONIANS OF THE FISSION FRAGMENTS*

The collective potential describing the fission fragments in the decay process is obtained in the framework of the microscopic-macroscopic model [9–21,37–42]. It contains three terms

$$V_{coll} = V_{LDM} + \Delta V_{sc} + \Delta V_{pc} \,. \qquad (4)$$

The first term (V_{LDM}) is frequently calculated by means of the liquid drop model or the droped model [40,41] with some corrections due to the finite range of the nuclear forces [37,42]:

$$V_{LDM} = V_{LDM}^{SURF} + V_{LDM}^{COUL} = V_0^{SURF} \iint \rho(\vec{r}) \frac{\exp\{-\frac{|\vec{r}-\vec{r}'|}{a}\}}{|\frac{\vec{r}-\vec{r}'}{a}|} \rho(\vec{r}')d^3r d^3r' +$$

$$+ V_0^{COUL} \iint \rho(\vec{r}) \frac{a^{COUL}}{|\vec{r}-\vec{r}'|} \rho(\vec{r}')d^3r\, d^3r'. \qquad (5)$$

In the case of a spherical nucleus the nuclear macroscopic potential energy must have the expression

$$(V_{LDM}^{SURF})_{(0)} = -\frac{2}{3}\frac{r_0}{a} c_s A + c_s A^{2/3} - (\frac{a}{r_0})^2 c_s +$$

$$+ (A^{1/2} + \frac{a}{r_0})^2 c_s \exp(-\frac{2r_0}{a} A^{1/3}), \qquad (6)$$

where A is the number of the nucleons in the mother nucleus,

$$c_s = a_s[1 - \kappa(\frac{N-Z}{A})^2]$$

in which a_s is the surface energy constant and κ, the surface asymmetry constant ($a_s = 24.7$ *MeV*, $\kappa = 4.0$), $r_0 = 1.16$ *fm*, $a = 1.4$ *fm*.

The integration in eq. (5) is over the volume of the shape. The densities $\rho(\vec{r})$ are obtained by an averaging procedure:

$$\rho(\vec{r}) = \int dr\, \Psi^*(x_1 \ldots x_A)\hat{\rho}\,\Psi(x_1 \ldots x_A) \qquad (7)$$

with

$$\hat{\rho} = \sum_{i=1}^{A} \delta(\vec{r}_i - \vec{r}). \qquad (8)$$

Here

$$\Psi(x_1 \ldots x_A) = \langle x_1 \ldots x_A | \prod_\nu a_\nu^+ | 0 \rangle \qquad (9)$$

are the total intrinsic many body wave functions, normalized to unity, given by the asymmetric two center shell model (ATCSM) [39]. As is pointed out in the papers [9-12] these functions depend not only on the distance between the two centres of the fragments (R), but also on the other collective coordinates: the mass-fragmentation [9] (η), the charge-fragmentation [10] (η_Z), the surface vibrational and rotational coordinates ($\alpha_1^{[\lambda]}$, $\alpha_2^{[\lambda]}$) of the fission fragments, and so on.

Thus, the V_{LDM}-term given by eq. (5) is a function of all these collective coordinates:

$$V_{LDM} = V_{LDM}(R, \eta, \eta_Z, \alpha_1^{[\lambda]}, \alpha_2^{[\lambda]}). \qquad (9')$$

The second term in eq. (4) is usually called the Strutinsky correction [43] term [38]:

$$\Delta V_{sc} = \sum_\nu \epsilon_\nu - \int \tilde{\epsilon}(n)\, dn, \qquad (10)$$

where ϵ_ν are the single particle ATCSM-energies and $\tilde{\epsilon}(n)$ - the smooth energy curve [38]. ΔV_{sc} also depends on the collective coordinates mentioned in eq. (9), because of the ϵ_ν dependence on those coordinates.

Finally, the third term

$$\Delta V_{pc} = E_{pc} - \tilde{E}_{pc} \qquad (11)$$

is the pairing correction term, where

$$E_{pc} = 2(\sum_{k=1}^{N_p} \epsilon_k v_k^2 - \sum_{k=1}^{\frac{1}{2}N_p} \epsilon_k) - \frac{\Delta^2}{G} - G(\sum_{k=1}^{N_p} v_k^4 - \sum_{k=1}^{\frac{1}{2}N_p} 1) \qquad (12)$$

and $\tilde{E}_{pc}$ is the pairing energy of the uniform distribution of the levels. The summations are over the pairs of particles considered in the pairing interaction untill

$$N_p = \sum_{k=1}^{N_p} [1 - \frac{\epsilon_k - \lambda}{[(\epsilon_k - \lambda)^2 + \Delta^2]^{1/2}}]. \qquad (13)$$

Here Δ is the pairing gap, λ is the BCS Fermi energy and v_k^2 is the probability that the level k is occupied by a pair.

It is easy to see that ΔV_{pe} -term depends also on the same collective coordinates as V_{LDM} term in eq. (9). Thus

$$V_{COLL} = V_{COLL}(R, \eta, \eta_z, \alpha_1^{[\lambda]}, \alpha_2^{[\lambda]}). \qquad (14)$$

We construct the Hamiltonians describing asymptotically the motion of the fission fragments assuming that all the motions corresponding to the R, $\eta, \eta_z, \alpha_1^{[\lambda]}, \alpha_2^{[\lambda]}$ - collective degress of freedom are performed adiabatically, i.e.,

$$H_0^{(s)} = H_0^{(s)}(R) + H_0^{(s)}(\eta) + H_0^{(s)}(\eta_z) + H_0(\alpha_1^{[\lambda]}) + H_0(\alpha_2^{[\lambda]}). \qquad (15)$$

The first term

$$H_0^{(s)}(R) = -\frac{\hbar^2}{2\mu_s} \nabla_R^2 + V_s(R) \qquad (16)$$

describes the relative motion of the fission pairs (s). The potential is obtained as follows:

$$V_s(R) = \lim_{\substack{\eta \to \eta_s \\ \eta_z \to \eta_{z_s}}} \min_{\alpha_1^{[\lambda]}, \alpha_2^{[\lambda]}} V_{COLL}(R, \eta, \eta_z, \alpha_1^{[\lambda]}, \alpha_2^{[\lambda]}), \qquad (17)$$

where

$$\eta_s = A^{-1}(A_1^{(s)} - A_2^{(s)}); \qquad A = A_1^{(s)} + A_2^{(s)} \qquad (18)$$

$$\eta_{z_s} = Z^{-1}(Z_1^{(s)} - Z_2^{(s)}); \qquad Z = Z_1^{(s)} + Z_2^{(s)}. \qquad (19)$$

The second term

$$H_0^{(s)}(\eta) = -\frac{\hbar^2}{2} B_{\eta\eta}^{-1/4} \frac{\partial}{\partial \eta} B_{\eta\eta}^{-1/2} \frac{\partial}{\partial \eta} B_{\eta\eta}^{-1/4} + V_s(\eta), \qquad (20)$$

where $B_{\eta\eta}$ mass parameter is obtained in the framework of the cranking model [12,44,45]

$$B_{\eta\eta} = 2\hbar^2 \sum_{\mu\nu} (\epsilon_\mu - \epsilon_\nu)^{-1} |\langle \mu | \frac{\partial}{\partial \eta} | \nu \rangle|^2 (u_\mu v_\nu + u_\nu v_\mu)^2 \qquad (21)$$

and the potential [9]

$$V_s(\eta) = \lim_{\substack{R \to R_0 \\ \eta_z \to \eta_{z_s}}} \min_{\alpha_1^{[\lambda]}, \alpha_2^{[\lambda]}} V_{COLL}(R, \eta, \eta_z, \alpha_1^{[\lambda]}, \alpha_2^{[\lambda]}). \qquad (22)$$

The third term

$$H_0^{(s)}(\eta_z) = -\frac{\hbar^2}{2} B_{\eta_z\eta_z}^{-1/4} \frac{\partial}{\partial \eta_z} B_{\eta_z\eta_z}^{-1/2} \frac{\partial}{\partial \eta_z} B_{\eta_z\eta_z}^{-1/4} + V_s(\eta_z), \qquad (23)$$

where $B_{\eta_z\eta_z}$ and $V_s(\eta_z)$ have similar expressions with those given in eqs. (21) and (22), respectively, with interchanged η and η_z collective coordinates [10].

The last two terms have the classical expression given in the text-books [46,47].

The spectra of $H_0(\alpha_i^{[\lambda]})$ are discrete spectra. Using the recent information [11-13] the other two Hamiltonians $H_0^{(s)}(\eta)$ and $H_0^{(s)}(\eta_z)$ present discrete spectra also. The only Hamiltonians producing both discrete and continuum spectra are $H_0^{(s)}(R)$, because the potentials (17) are of the Gamov-type [53] with strong attractive part at the medium distances, with strong repulsive part at very short distances and a Coulomb-type behaviour at large distances. Combining all the partial [4-8] spectra we obtain in the total spectrum of the $H_0^{(s)}$-asymptotic Hamiltonian bound states, BSEC-states and continuum states distributed in different channels of the pair-fragmentation (s).

Following the procedure of refs. [4-8,54] we introduce in the channel fragmentation (s) four projectors onto subspaces of $H_0^{(s)}$-states

$$P_s = |b, s\rangle\langle b, s|; \qquad Q_s = \sum_c \int d\epsilon\, |\phi_{c\epsilon}^{(s)}\rangle\langle\phi_{c\epsilon}^{(s)}|$$

$$q_s = \sum_{c' \neq c} q_{c'}^{(s)} \qquad P_s + Q_s + q_s + A_s = 1. \qquad (24)$$

The states $|b,s>$ are BSEC-states [6,8,35,54] having the eigenenergy $E_0^{(s)}$ in the nearest vicinity of the decay energy (ϵ_f), Q_s projects onto the "active" open channels (the matrix elements of $H_s' = H - H_0^{(s)}$ between these channel states $|\phi_c^{(s)}>$ and $|b,s>$ we consider to be of the first order, they describe the most intensive transitions from the mother nucleus into the fission channel fragmentation (s)), q_s projects onto the "passive" open channels (the corresponding H_s' matrix elements are of the second order of magnitude as compared with the above mentioned ones) and A_s projects onto the rest of (closed) channels.

4. *DYNAMICS OF THE FISSION*

Inserting eqs. (1) and (2) into the expression trace ρW we obtain

$$\text{trace}\ \rho W(t) = \sum_k \omega(k) \int^R dx\, |<x|e^{-iHt}|\lambda_k>|^2 . \qquad (25)$$

As is pointed out in § 2 the prepared state $|\lambda_k>$ is a superposition *

$$|\lambda_k> = \sum_s <s|k>|b,s> \qquad (26)$$

of BSEC-states of $H_0^{(s)}$ asymptotic Hamiltonians corresponding to the fragmentation (s), those BSEC-states mentioned in eq. (24).

Using the "sharp resonance" condition we can neglect [4–8,32–36] the contribution to the integral over x in eq. (25) of the vectors $<x|(Q_s+q_s+A_s)e^{-iHt}|\lambda_k>$, hence

$$<x|e^{-iHt}|\lambda_k> \cong \sum_s <s|k><x|b,s>\, a_s(t), \qquad (27)$$

where

$$a_s(t) = <b,s|e^{-iHt}|b,s>. \qquad (28)$$

The vector $|b,s>$ is sufficiently nice in the sense that $d<b,s|E_\lambda|b,s>$ falls off exponentially, where dE_λ is the spectral measure associated to H, then [6,8,34,36]

$$a_s(t) = (2\pi i)^{-1} \int_C e^{-izt} <b,s|G(z)|b,s> \qquad (29)$$

when C is a contour running from $ip+\infty$ to $ip-\infty$ and the resolvent

$$G(z) = (z-H)^{-1} \qquad (30)$$

defined in the complex z plane so that has no singularities lying above C. To compute the residuum we write G(z) in our four subspaces [4–8,54]:

$$G(z) = PG + QG + qG + AG = \frac{A}{z-H_0-AH'A} + \Omega_{A_r}(z)\frac{q}{z-H_0-qVq}\Omega_{A_\ell}(z) + \Omega_{A_r}(z)\Omega_{q_r}(z) \times \frac{Q}{z-H_0-QWQ}\Omega_{q_\ell}(z)\Omega_{A_\ell}(z) + \Omega_{A_r}(z)\Omega_{q_r}(z) \times \Omega_{Q_r}(z)\frac{P}{z-H_0-PRP}\Omega_{Q_\ell}(z)\Omega_{q_\ell}(z)\Omega_{A_\ell}(z), \qquad (31)$$

where for the sake of simplicity we have omitted the index s, denoting the pair of fragments. The "right" and "left" Ω-operators are defined by

$$\Omega_{A_r}(z) = 1 + \frac{A}{z-H_0-AH'A}H'; \quad \Omega_{A_\ell}(z) = H'\frac{A}{z-H_0-AH'A} + 1, \qquad (32)$$

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* The contribution of other BSEC-states is natural, but for the large times their contribution to the expression (25) is negligibly small.

$$\Omega_{q_r}(z) = 1 + \frac{q}{z-H_0-qVq}V; \quad \Omega_{q_\ell}(z) = V\frac{q}{z-H_0-qVq}+1, \tag{33}$$

$$\Omega_{Q_r}(z) = 1 + \frac{Q}{z-H_0-QWQ}W; \quad \Omega_{Q_\ell}(z) = W\frac{Q}{z-H_0-QWQ}+1 \tag{34}$$

with

$$V = H' + H'\frac{A}{z-H_0-AH'A}H', \tag{35}$$

$$W = V + V\frac{q}{z-H_0-qVq}V, \tag{36}$$

$$R = W + W\frac{Q}{z-H_0-QWQ}W, \tag{37}$$

Here the resolvents corresponding to the projected subspaces must be understood as the solutions for example of the eq.

$$(z-H_0-AH'A)\frac{A}{z-H_0-AH'A} = A. \tag{38}$$

Inserting eqs. (31), (29) in eq. (27) we obtain

$$\langle x|e^{-iHt}|\lambda_k\rangle \cong (2\pi i)^{-1}\int_C dz\, e^{-izt}\sum_s \frac{g_{ks}^{(b)}(x)}{z-E_0^{(s)}-R_{bb}^{(s)}(z)}, \tag{39}$$

where

$$g_{ks}^{(b)}(x) = \langle s|k\rangle\langle x|b,s\rangle, \tag{40}$$

$$R_{bb}^{(s)}(z) = \langle b,s|R(z)|b,s\rangle. \tag{41}$$

The complex operator $R(z)$ given by eq. (37), may be broken up as follows

$$R(z) = L(z) - \frac{i}{2}\Gamma(z) \tag{42}$$

with

$$\Gamma(z) = \Gamma_R + \Gamma_W + \Gamma', \tag{43}$$

$$\Gamma_R = 2\pi R^+ Q\,\delta(z-H_0)QR, \tag{44}$$

$$\Gamma_W = 2\pi W^+ q\,\delta(z-H_0)qW, \tag{45}$$

$\Gamma' \triangleq$ third and higher order in W terms. (46)

Taking into account the smallness of the widths $\Gamma_{bb}^{(s)}$ eq. (39) can be replaced by

$$\langle x|e^{-iHt}|\lambda_k\rangle \cong (2\pi i)^{-1}\int_C dz\, e^{-izt}\frac{\sum_s g_{ks}^{(b)}(x)}{z-\epsilon_f+\frac{i}{2}\Gamma_f}, \tag{47}$$

where

$$\Gamma_f = \sum_s \Gamma_{bb}^{(s)} \tag{48}$$

with

$$\Gamma_{bb}^{(s)} \cong 2\pi\sum_c |\langle\phi_{c\epsilon_f}^{(s)}|R^{(s)}(\epsilon_f)|b,s\rangle|^2 \tag{49}$$

and

$$\epsilon_f = E_0^{(s)} + L_{bb}^{(s)}(\epsilon_f). \tag{50}$$

Here we have neglected $(\Gamma_W)_{bb}$ and $(\Gamma')_{bb}$-terms (see eq. (45), (46)) as small terms. We have assumed also that in every channel fragmentation there is fulfilled eq. (50). This fact is based on the experimental data. Neglecting the small contributions to the other poles and the cuts eq. (47) becomes

$$\langle x|e^{-iHt}|\lambda_k\rangle \cong \{\sum_s g_{ks}^{(b)}(x)\}\exp\{-i(\epsilon_f - \frac{i}{2}\Gamma_f)t\}. \tag{51}$$

Introducing eq. (51) in (25) we obtain

$$\mathrm{trace}\,\rho\, W(t) \cong \sum_k \omega(k) \int^R dx\, |\sum_s g^{(b)}_{ks}(x)\, \exp\{-i(\epsilon_f - \frac{i}{2}\Gamma_f)t\}|^2 \tag{52}$$

and finally the ratio (3) becomes

$$P(t) \cong \exp\{-\Gamma_f t\}. \tag{53}$$

5. *FISSION WIDTH*

Following the treatment of ref. [4,8] the R operator (37) can be factorized as follows:

$$R(\epsilon_f) = \Omega_{Q_\ell}(\epsilon_f) H' \Omega_{A_r}(\epsilon_f) + \Omega_{Q_\ell}(\epsilon_f) H' \Omega_{A_r}(\epsilon_f) \frac{q}{\epsilon_f - H_0} V \Omega_{q_r}(\epsilon_f). \tag{54}$$

We can neglect the second term from the r.h.s. in the braket by virtue of the made approximations. Thus the fission width (48) becomes

$$\Gamma_f = 2\pi \sum_{s,c} |\langle \Omega^+_{Q_\ell}(\epsilon_f)\phi^{(s)}_{c\epsilon_f} | H'_s \Omega_{A_r}(\epsilon_f) | b,s\rangle|^2 . \tag{55}$$

We define the new initial and final states as follows

$$|\Phi^{(s)}_i\rangle = \Omega_{A_r}(\epsilon_f)|b,s\rangle \tag{56}$$

and

$$|\Psi^{(s)}_{c\epsilon_f}\rangle = \Omega^+_{Q_\ell}(\epsilon_f)|\phi^{(s)}_{c\epsilon_f}\rangle \tag{57}$$

or

$$\{\epsilon_f - Q(H+H'')Q\}|\Psi^{(s)}_{c\epsilon_f}\rangle = 0, \tag{58}$$

where

$$H'' = H'A \frac{1}{\epsilon_f - AHA} AH' + Vq \frac{P.V.}{\epsilon_f - H_0 - qVq} qV + i\pi Vq\,\delta(\epsilon_f - H_0 - qVq)qV. \tag{59}$$

The Hamiltonian $Q(H+H'')Q$ that generates the new channel function (57) may be interpreted as a generalized optical model Hamiltonian containing the local terms, the nonlocal terms and the imaginary terms [48]. The channel function (57) can be obtained in terms of the old basis by using the coupled-channel techniques [49–51]

$$|\Psi^{(s)}_{c\epsilon_f}\rangle = \sum_{c'} \Omega^{(s)}_{c'c} |\phi^{(s)}_{c\epsilon_f}\rangle . \tag{60}$$

The new initial wave function is a solution of the following eq.

$$\{\epsilon_f - (A+P)H(A+P)\}|\Phi^{(s)}_i\rangle = \{\epsilon_f - (H + H' \frac{A}{\epsilon_f - AHA} H')_{bb}\}|b,s\rangle \cong 0. \tag{61}$$

The terms from right hand side of this eq. can be neglected on the strength of the made untill now approximations [4,8]. Thus

$$|\Phi^{(s)}_i\rangle = \sum_n \Omega^{(s)}_{nb}|n,s\rangle , \tag{62}$$

where the $|n,s\rangle$ states are, besides the $|b,s\rangle$ -BSEC state, other BSEC-states of the $H^{(s)}_0$ -asymptotic Hamiltonian. We may neglect also the contribution of the closed channel states.

In the following we denote by

$$\Phi^{(s)}_{NLM}(\vec{R}) = \mathcal{R}_{NL}(R) Y_{LM}(\hat{R}) \tag{63}$$

the normalized to unity bound state eigenfunction of the Hamiltonian (16), by

$$g^{(s)}_{LM}(k_s, \vec{R}) = \frac{1}{R}\sqrt{\frac{2\mu_s}{\pi \hbar^2 k_s}}\, F_{L,S}(k_s,R)\, Y_{LM}(\hat{R}) \tag{64}$$

the scattering (continuum) eigenfunction, normalized to the Dirac delta function in energy, generated by (16), where $F_{L,S}(k_s,R)$ is the regular radial solution normalized to a sinfunction at large distances, by $\Psi^{(s)}_\nu(\eta)$ – the normalized to unity eigenfunction of the Hamiltonian (20), by $\Psi^{(s)}_{\nu_z}(\eta_z)$ - the normalized to unity eigenfunctions of the Hamiltonians (23) and by $\Phi^{(s)}_{\sigma}(a_1^{[\lambda]}, a_2^{[\lambda]})$ - the normalized to unity eigenfunctions of the last two Hamiltonian in the r.h.s. of eq. (15). The ensemble of quantum numbers n and c is defined by

$$n \equiv \{ N_n L_n M_n \nu_n \nu_{z_n} \sigma_n \} \qquad (65)$$

and

$$c \equiv \{ L M \nu \nu_z \sigma \}. \qquad (66)$$

Assuming a surface delta-interaction at the scision point $R = R_0$ for the H'_s -interaction in the braket [8]

$$H'_{c'c} = \frac{\hbar^2 k_s}{2\mu_s F_{L,S}(k_s, R_0) G_{L,S}(k_s, R_0)} \delta(R-R_0)\delta(\eta-\eta_s)\delta(\eta_z-\eta_{z_s}) \qquad (67)$$

the fission width becomes [22]

$$\Gamma_f = \sum_{s,c} \mathcal{P}_{L,S}(\epsilon_f, R_0)\gamma^2_{c,s}(R_0, \eta_s, \eta_{z_s}), \qquad (68)$$

where

$$\mathcal{P}_{L,S}(\epsilon_f, R_0) = \frac{2k_s R_0}{G^2_{L,S}(\epsilon_f, R_0)} \qquad (69)$$

is the channel penetrability factor corresponding to the fragmentation s, $G_{L,S}(\epsilon_f, R_0)$ - is the irregular radial solution of the relative motion of the fission fragments, normalized to a cos-function at large distance and

$$\gamma_{c,s}(R_0, \eta_s, \eta_{z_s}) = \left[\frac{h^2}{2\mu_s R_0^2}\right]^{1/2} \sum_{c'n} \Omega^{(s)}_{c'c}\Omega^{(s)}_{nb} \times$$

$$\times G_{LS}(\epsilon_f, R_0) G^{-1}_{L'S}(\epsilon_f, R_0) \Psi^{*(s)}_{\nu'}(\eta_s)\Psi^{(s)}_{\nu_n}(\eta_s) \times \Psi^{*(s)}_{\nu'_z}(\eta_{z_s}) \times$$

$$\times \Psi^{(s)}_{\nu_{zn}}(\eta_{z_s}) \mathcal{R}^{(s)}_{N_n L_n}(R_0)\delta_{L'L_n}\delta_{M'M_n}\delta_{\sigma'\sigma_n} \qquad (70)$$

is the amplitude of the reduced width (the probability of the formation of the fission fragments).

For the spontaneous fission, only the lowest $\Psi^{(s)}_{\nu}(\eta)$ and $\Psi^{(s)}_{\nu_z}(\eta_z)$ should be taken into account. In ref. [11] it is shown that

$$|\Psi^{(s)}_{\nu}(\eta_s)|^2 \sim Y^{(s)}_A(\eta_s), \qquad (71)$$

where $Y^{(s)}_A(\eta_s)$ is the fission mass-yield and

$$|\Psi^{(s)}_{\nu_z}(\eta_{z_s})|^2 \sim Y^{(s)}_Z(\eta_{z_s}), \qquad (72)$$

where $Y^{(s)}_Z(\eta_{z_s})$ is the fission charge-yield. Taking eqs. (71) and (72) into account we rewrite the eq. (68) as follows

$$\Gamma_f = \sum_{s,c} \mathcal{P}_{c,s}(\epsilon_f, R_0) Y^{(s)}_A(\eta_s) Y^{(s)}_Z(\eta_{z_s}) |\beta_{c,s}(R_0, \eta_s, \eta_{z_s})|^2 \qquad (73)$$

where the quantity

$$\beta_{c,s}(R_0, \eta_s, \eta_{z_s}) = \left(\frac{h^2}{2\mu_s R_0^2}\right)^{1/2} \sum_{c',n} \Omega^{(s)}_{c'n}\Omega^{(s)}_{nb} G_{LS} G^{-1}_{L'S} \times$$

$$\times \Psi^{*(s)}_{\nu'}(\eta_s)|Y^{(s)}_A(\eta_s)|^{-1/2} \Psi^{*}_{\nu'_z}(\eta_{z_s})|Y^{(s)}_Z(\eta_{z_s})|^{-1/2} \Psi^{(s)}_{\nu_n}(\eta_s) \times$$

$$\times \Psi^{(s)}_{\nu_{z_n}}(\eta_{z_s}) \mathcal{R}^{(s)}_{N_n L_n}(R_0)\delta_{L'L_n}\delta_{M'M_n}\delta_{\sigma'\sigma_n} \qquad (74)$$

must be a smooth function on η_s and η_{z_s}. From the experimental data [52], we see that the formation probability γ^2 of a certain pair of fission fragments, varies within 4-5 orders of magnitude. Thus different channels contribute differently.

Finally we conclude that the usual calculations of the fission half-lives [14-21] are very rough estimations of these quantities, because the Γ_f is determined practically only by one channel (fission mode) through the penetrability factor, while no variations are permitted on the preformation factor, this factor being estimated statistically only.

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REFERENCES

1. *W.D.Myers, W.I.Swiatecki. Nucl.Phys., 81, 1 /1966/.*
2. *W.Franz. Z. Physik, 184, 181 /1965/; "Quanten Theorie", Springer Verlag /1970/.*
3. *H.Kummel. Z.Naturforschung, 21A H.1/2, 79 /1966/.*
4. *O.Dumitrescu. In "Interaction Studies in Nuclei", Eds. H.Jochim, B.Ziegler/1975/, North-Holland Publ. Co., The Netherlands, p. 125.*
5. *O.Dumitrescu, H.Kummel. Annals of Physics, (N.Y.), 71, 556 /1972/.*
6. *O.Dumitrescu. Revue Roumaine de Physique, 18, 277 /1973/; 20, 641 /1975/.*
7. *F.Carstoiu, O.Dumitrescu, D.G.Popescu. Revue Roumaine de Physique, 20, 415 /1975/.*
8. *O.Dumitrescu, D.G.Popescu. Soobshchenie JINR, E4-9097, Dubna, 1975.*
9. *H.J.Fink, I.Maruhn, W.Scheid, W.Greiner. Z.Physik, 268, 321 /1974/.*
10. *Raj K.Gupta, W.Scheid, W.Greiner. Phys.Rev.Lett., 35, 353 /1975/.*
11. *W.Greiner. Proceedings of the International Conference on Reactions between Complex Nuclei., Nashville /1974/, p. 21.*
12. *W.Greiner. Proc. of the International School on Nuclear Reactions, Predeal - Romania /1974/.*
13. *W.Greiner. Proc. of the International School-Seminar on Reactions of Heavy- Ions with Nuclei and Synthesis of New Elements, Dubna, 1975.*
14. *T.Ledergerber. Proc. of the International School-Seminar on Reactions of Heavy-Ions with Nuclei and Synthesis of New Elements, Dubna, 1975.*
15. *A.Sobiczewski, ibid.*
16. *S.G.Nilson, ibid.*
17. *J.R.Nix, ibid.*
18. *E.O.Fiset, J.R.Nix. Nucl.Phys., A193, 647 /1972/.*
19. *S.G.Nilsson, J.R.Nix, A.Sobiczewski, Z.Szymanski, S.Wycech, C.Gustafson, P.Moller. Nucl.Phys., A115, 545 /1968/.*
20. *Chin Fu Tzang, S.G.Nilsson. Nucl.Phys., A140, 275 /1970/.*
21. *H.C.Pauli, R.Ledergerber. Nucl.Phys., A207, 1 /1973/.*
22. *O.Dumitrescu, Comment to the T.Ledergerber lecture at the Dubna International School-Seminar /1975/, (see ref. /14/).*
23. *A.F.Siegert. Phys.Rev., 56, 750 /1939/.*
24. *I.Humblet, L.Rosenfeld. Nucl.Phys., 26, 529 /1961/.*
25. *L.Rosenfeld. Nucl.Phys., 70, 1 /1965/.*
26. *G,Breit. Encyclopedia of Physics, v. 41, Springer Verlag, 1959.*
27. *A.M.Lane, R.G.Thomas. Rev.Mod.Phys., 30, 257 /1958/.*
28. *T.Berggren. Nucl.Phys., A109, 265 /1968/.*
29. *W.Romo. Nucl.Phys., A116, 618 /1968/.*
30. *H.Casimir. Physica (Haag), 1, 193 /1934/.*
31. *M.Born. Z.Physik, 58, 306 /1929/.*
32. *H.I.Mang. Sitzungsberichte der Heidelberger Akad. Wiss. /1959/, p. 299. Phys.Rev., 119, 1069 /1960/; Ann.Rev.Nucl.Sci., 14, 1 /1964/.*
33. *K.Ingolfsson. "Zur Formulierung der mathematischen Theorie der Naturlichen Linienbreite", Basel, Buchdruckerei, Birkhauser AG /1967/.*
34. *B.Simon. Annals of Mathematics (Japan), 97, No. 2, 247 /1973/.*
35. *C.Mahaux, H.Weidenmuller. "Shell Model Approach to Nuclear Reactions", North-Holland, London /1969/.*
36. *M.L.Goldberger, K.Watson. "Collision Theory", J.Wiley, N.Y., 1974.*
37. *J.R.Nix, A.I.Sierk. Physica Scripta, 10a, 94 /1974/.*
38. *M.Bolsterli, E.O.Fiset, J.R.Nix, I.L.Norton. Phys. Rev., C5, 1050 /1972/.*
39. *I.Maruhn, W.Greiner. Z.Physik., 251, 431 /1972/.*
40. *W.D.Myers, W.I.Swiatecki. Ann.Phys., (N.Y.), 55, 395 /1969/.*
41. *W.D.Myers, W.J.Swiatecki. Ann.Phys., (N.Y.), 84, 186 /1974/.*
42. *H.J.Krappe, J.R.Nix. Proc. Third IAEA Symp. on Physics and Chemistry of Fission, Rochester, 1973, b. 1, p. 159, IAEA, Vienna.*
43. *V.M.Strutinsky. Yad.Fiz., 3, 614 /1966/.*
44. *D.Inglis. Phys.Rev., 96, 1059 /1950/.*

45. P.Lichtner, D.Drechsel, J.Maruhn, W.Greiner. Phys.Rev.Lett., 28, 829 /1972/.
46. A.Bohr, B.Mottelson. "Nuclear Structure", Benjamin, v. 2 /1974/.
47. A.S.Davydov. "Theory of the Atomic Nucleus", Moscow, 1958.
48. O.Dumitrescu. "One and Two-Nucleon Transfer Reactions on Deformed Nuclei", Preprint CSEN-IFA-FT-108 /1974/; Rev.Roum. Phys., 18, 225 /1973/.
49. T.Tamura. Annual Rev.Nucl.Sci., 19, 99 /1969/; Rev.Mod.Phys., 37, 679 /1965/.
50. S.I.Drozdov. Yad.Fiz., 1, 407 /1965/.
51. O.Dumitrescu. V.K.Lukyanov, I.Z.Petkov, H.Shulz, H.J.Wiebecke. Nucl.Phys., A149, 253 /1970/.
52. D.C.Hoffman, M.M.Hoffman. Ann. Rev. Nucl. Sci., 24, 151 /1974/.
53. It can be shown that for potentials describing the nucleus-nucleus collision, their nonlocality generates the discrete spectrum. This may be the case when the local potential does not generate the discrete spectrum.
54. O.Dumitrescu, D.G.Popescu. "Alpha Decay in the Framework of Four Projectors Theory", to be published.

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