ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

E4 - 9601

V.B.Belyaev, K. Möller

EIGENVALUES OF THE FADDEEV EQUATION KERNEL FOR A SYSTEM OF THREE SPINLESS PARTICLES

1976

E4 - 9601

V.B.Belyaev, K. Möller

EIGENVALUES OF THE FADDEEV EQUATION KERNEL FOR A SYSTEM OF THREE SPINLESS PARTICLES

Submitted to "Zeitschrift für Physik (A)"

1. INTRODUCTION

To calculate the eigenvalues and eigenfunctions of the kernel of the Faddeev equations [1] is important at least for two reasons, firstly for the search for resonances in three-body systems and secondly for the separable expansion of the three-particle amplitude.

The theoretical investigation of three-particle resonances represents a poor elaborated branch of the three-particle problem as compared, e.g., with the study of the bound states. This is caused, on the one hand, by mathematical difficulties in solving the corresponding Faddeev equations and, on the other hand, by the absence of unambiguous experimental indication of the existence of such resonances in nonrelativistic systems. In brief, the experimental situation in systems with $A = 3$ [2,9] may be characterized by the fact that if resonances exist, then they are produced with very small cross sections and, probably, have large widths. Up to now it is difficult to understand the experimental results from the theoretical point of view, since it is very little known on the conditions under which three-particle resonances appear and on the sensitivity of their physical characteristics to a change of the two-particle interaction parameters. Concerning the importance of the investigation of three-particle resonances one should remember that at present such systems are the only multiparticle systems with several decay channels for which, starting from a given two-particle interaction, accurate calculations of resonance states can be performed. Moreover the question of three-particle resonances is of interest in elementary particle physics to explain the existence of mesons like $A_{1,2}$, C, D, E, ω on the basis of our knowledge on the interaction between the elementary particles.

As to the second point mentioned above, concerning the separable expansion of the three-particle amplitude, it is of great importance in connection with the solution of the four-body problem, since the three-particle amplitude forms the kernel of the Faddeev-Yakubovsky equations for four particles [3]. The separable expansion of these kernels allows one [4,5] to essentially simplify the complicated four-particle equations. The separable expansion may also be used for the calculation of vertex coupling constants of three and four-particle systems which are important characteristics of bound states, together with the binding energy and the mean square radius.

The properties of resonances in a system of three elementary particles have been studied in papers [7] on the basis of a relativistic version of the Faddeev equations. The authors have found a considerable sensitivity of the eigenvalues to the shape of the two-particle interaction. Resonances in a model approach with one heavy particle ("nucleus") and two light ones (n,p) for negative total energy have been investigated in detail in papers [15]. Eigenvalues and eigenfunctions of a system of modified three-particle Lippmann-Schwinger equations with Gaussian-type two-body potential have been calculated in papers [8]. The existence of resonances in a system consisting of three neutrons has been investigated in paper [10]. Two opposite cases with respect to the two-particle interaction have been considered: a large-range square well potential and a δ -potential. Resonances were shown to be absent in both cases. A promising approach in the study of three-particle resonances permitting an extension to the relativistic case was proposed in [11]. Starting from the three-particle unitarity relations and using the N/D-method an integral equation has been derived for the scattering amplitude describing the scattering of a particle on a two-particle resonance. This integral equation turned out to be very suitable for the investigation of the conditions for the existence of three-particle resonances.

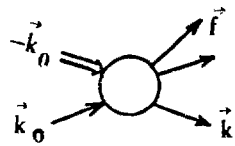
As is known, the mathematical difficulties in the investigation of three-particle resonances consist in the

fact that at positive total energy there appear moving singularities of logarithmic type in the kernel of the Faddeev equations. Recently methods have been proposed for solving this problem [12,13,14]. However, up to now these methods have been applied only to scattering problems and not to resonances.

In the present paper we calculate the eigenvalues and eigenfunctions of the Faddeev-kernel for a system of three spinless particles interacting via separable Yamaguchi potential using the mathematical methods [13,14]. In Section 2 we present the initial equations and give a definition of the three-particle resonances. In Section 3 we show for a sufficiently general class of separable potentials that it is possible to transform the kernel of the integral equation in such a way that after interpolation of the solution the remaining singular integrals can be calculated analytically. In Section 4 we present the numerical results of the calculation.

2. *THE HOMOGENEOUS FADDEEV EQUATION AND THREE-PARTICLE RESONANCES*

We start from the amplitude $T(\vec{f},\vec{k};\vec{k}_0,z_3)$ describing the following process

$T(\vec{f},\vec{k};\vec{k}_0,z_3) =$ 

For three spinless particles the Faddeev equation [1] may be written in the form

$$T(\vec{f},\vec{k};\vec{k}_0,z_3) = t_s(\vec{f},\vec{p}_{20},\bar{z}_2)\,\phi_d(\vec{p}_{10}) + \int \frac{d\vec{k}'}{(2\pi)^3} \cdot \frac{t_s(\vec{f},\vec{p}_2,\bar{z}_2)\,T(\vec{p}_1,\vec{k}';\vec{k}_0,z_3)}{[z_3 - \frac{\hbar^2}{m}(\vec{k}^2 + \vec{k}\vec{k}' + \vec{k}'^2)]}, \quad (1)$$

where z_3 is the total c.m.-energy of the three-particle system (in what follows z_3 takes any complex value), $z_2 = z_3 - (3/4)(\hbar^2/m)k^2$, $\vec{p}_{10}=(\vec{k}+\vec{k}_0/2)$, $\vec{p}_{20}=(\vec{k}'/2+\vec{k}_0)$, $\vec{p}_1=(\vec{k}+\vec{k}'/2)$, $\vec{p}_2=(\vec{k}/2+\vec{k}')$, ϕ_d is the wave function of the two-particle state ("deuteron") and m - the nucleon mass. The symmetrized two-particle t-matrix is defined as $t_s(\vec{k}',\vec{k},z_2)=t(\vec{k}',\vec{k},z_2)+t(-\vec{k}',\vec{k},z_2)$. The matrix elements $t(\vec{k}',\vec{k},z_2)$ obey the equation

$$t(\vec{k}',\vec{k},z_2)=v(\vec{k}',\vec{k})+\int \frac{d\vec{k}''}{(2\pi)^3}\cdot\frac{v(\vec{k}',\vec{k}'')t(\vec{k}'',\vec{k},z_2)}{z_2-\frac{\hbar^2}{m}k''^2}, \qquad (2)$$

where $v(\vec{k}',\vec{k})=\langle\vec{k}'|v|\vec{k}\rangle$ with $\langle\vec{k}'|\vec{k}\rangle=(2\pi)^3\delta(\vec{k}'-\vec{k})$. The quantity v is the two-particle potential. Separating the angular variables and allowing for interaction only in the S-state we can reduce eq. (1) to the following equation

$$T_L(f,k;k_0,z_3)=T^{(0)}+\int_0^\infty \frac{k'dk'}{k\pi^2}\int_{|k-k'/2|}^{k+k'/2} pdp \times$$

$$\times\frac{t_0(f,p_2(p),y)P_L(y)T_L(p,k';k_0,z_3)}{z_3-\frac{\hbar^2}{m}(p^2+\frac{3}{4}k'^2)}, \qquad (3a)$$

where

$$y=\frac{p^2-k^2-k'^2/4}{kk'}, \quad p_2(p)=\sqrt{p^2+\frac{3}{4}k'^2-\frac{3}{4}k^2} \qquad (3b)$$

Here L is the total angular momentum and $P_L(y)$ the corresponding Legendre polynomial. For the two-body t-matrix t_0 we have

$$t_0(k',k,z_2)=v_0(k',k)+\frac{1}{2\pi^2}\int_0^\infty \frac{k''^2dk''\,v_0(k',k'')t_0(k'',k,z_2)}{z_3-\frac{\hbar^2}{m}k''^2}. \qquad (4)$$

The inhomogeneous term $T^{(0)}$ will not be specified in detail, since in what follows we will deal with the homogeneous equation only.

We restrict ourselves to separable potentials of the form

$$v_0(k', k) = -\lambda g(k') g(k) . \tag{5}$$

Then we have for the t -matrix

$$t_0(k', k, z_2) = g(k') g(k) \tau(\sqrt{z_2}) , \tag{6a}$$

where

$$\tau(\sqrt{z_2}) = -\left[\lambda^{-1} + \int_0^\infty \frac{k^2 dk}{2\pi^2} \cdot \frac{g^2(k)}{z_2 - \frac{\hbar^2}{m} k^2}\right]^{-1} \quad \mathrm{Im}\sqrt{z_2} > 0 \tag{6b}$$

The condition $\mathrm{Im}(z_2)^{1/2} > 0$ defines the physical sheet of the two-particle t -matrix.

Inserting eq. (6) into (3a) we get

$$T_L(f, k; k_0, z_3) = g(f) \tau(\sqrt{z_2}) F_L(k, z_3) , \tag{7}$$

where $F_L(k, z_3)$ obeys the one-dimensional integral equation

$$F_L(k, z_3) = F^{(0)} + \frac{1}{2\pi^2} \int_0^\infty k'^2 dk' \times \tag{8}$$

$$\times \left[2 \int_{|k-k'/2|}^{k+k'/2} \frac{p\, dp}{kk'} \cdot \frac{g(p_2) P_L(y) g(p) \tau(\sqrt{z_2})}{z_3 - \frac{\hbar^2}{m}(p^2 + \frac{3}{4} k'^2)} \right] F_L(k', z_3) .$$

We define resonances as poles of the amplitude $T_L(f, k; k_0, z_3)$ in the complex z_3-plane. From eq. (7) it is clear that one should distinguish two types of poles:

$$\begin{aligned} &1) \quad \tau(\sqrt{z_2}) = \tau\left(\sqrt{z_3 - \frac{3}{4}\frac{\hbar^2}{m} k^2}\right) = \infty \qquad z_2 = \epsilon_1, \epsilon_2, \ldots \\ &2) \quad F_L(k, z_3) = \infty . \end{aligned} \tag{9}$$

The first type of them is due to poles in the two-particle amplitude. These poles correspond to bound states or resonances (on the unphysical sheet in the z_2-plane) of the two-body subsystem. The three-particle amplitude will contain these poles at any energy z_3 under the condition that k is given by $k = (2/\hbar)((m/3)(z_3 - \epsilon_n))^{1/2}$.

The second type of poles arises at those energies z_3 for which the homogeneous equation corresponding to (8) has a solution. Unlike the first type the positions of the poles of the function $F_L(k, z_3)$ depend only on the total energy z_3 and are independent of the kinematical variables. Due to the fact that eq. (8) is an integral equation of Fredholm-type the positions of the poles of F in the z_3-plane form a discrete set of energies $(z_3^{(1)}, z_3^{(2)}, \ldots)$. A part of these poles corresponds to bound states of the three-particle system. We call the remaining ones three-particle resonances. The statement that the three-particle amplitude contains all the poles of the two-particle amplitude, in addition to the true three-particle poles, is not restricted to separable potentials but holds in general. This fact can be proved as follows. As is known, in the vicinity of a pole the two-particle amplitude has the form

$$t(\vec{k}', \vec{k}, z_2) \simeq \frac{G_n(\vec{k}')G_n(\vec{k})}{z_2 - \epsilon_n}; \quad z_2 \sim \epsilon_n. \tag{10}$$

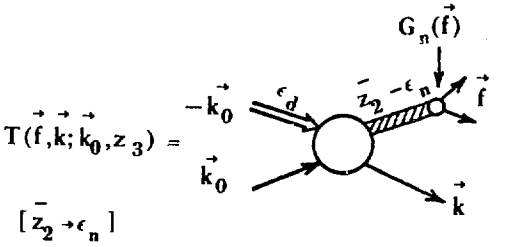
Inserting eq. (10) into eq. (1) we have

$$T(\vec{f}, \vec{k}; \vec{k}_0, z_3) = \frac{2G_n(\vec{f})}{z_3 - \frac{3}{4}\frac{\hbar^2}{m}k^2 - \epsilon_n}[G_n(\vec{p}_{20})\phi_d(\vec{p}_{10}) + \tag{11}$$

$$+ \int \frac{d\vec{k}' G_n(\vec{p}_2) T(\vec{p}_1, \vec{k}'; \vec{k}_0, z_3)}{(2\pi)^3 [z_3 - \frac{\hbar^2}{m}(k^2 + \vec{k}\vec{k}' + k'^2)]}]$$

$$\frac{3}{4}\hbar^2 k^2 \simeq z_3 - \epsilon_n.$$

The graphic representation of (11) is



(12)

From eq. (11) and the graph (12) it follows immediately that the three-particle amplitude contains all the poles of the two-particle one. If one regards the three-particle amplitude with three free particles in the initial state, then its amplitude will have additional poles corresponding to two-particle bound states and resonances in the entrance channel.

It is difficult in practice to find resonances because the poles corresponding to resonances are located on the unphysical sheet of the energy plane z_3. The integral equation, however, is formulated on the physical sheet. This means, to find three-particle resonances the calculated quantities must be analytically continued to the unphysical sheet. The analytical structure of the amplitude $F(k,z_3)$ is shown in *Fig. 1*.

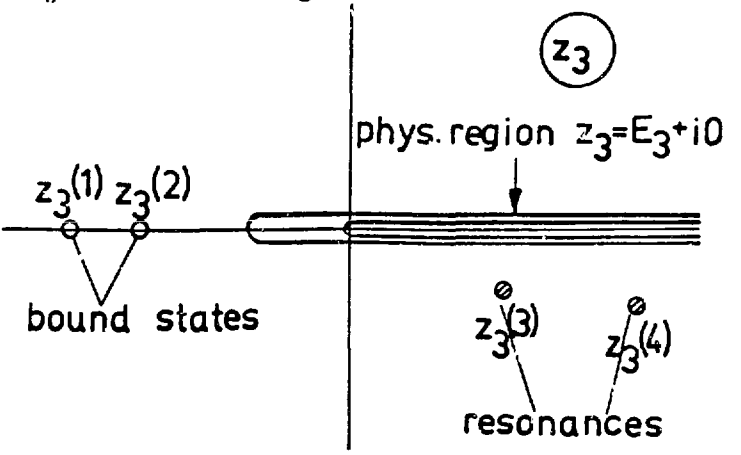


Fig. 1. The analytical structure of the amplitude $F(k,z_3)$.

The analytical continuation is performed in such a way that at first one calculates the eigenvalues $\lambda_n(z_3)$ on the physical sheet according to the following equation

$$\lambda_n(z_3)\,|\,F_n(z_3)\rangle = K\,|\,F_n(z_3)\rangle\,. \tag{13}$$

where K is the kernel of eq. (8). Then the complex functions $\lambda_n(z_3)$ can be continued numerically to the unphysical sheet where one looks for those energies $z_3^{(res)}$ for which the condition holds $\lambda_n(z_3^{(res)}) = 1$. We will regard only resonances which are not very far from the physical sheet. For them there exists a domain on the physical sheet where the condition holds

$$\mathrm{Re}\,\lambda_n \approx 1, \quad |\mathrm{Im}\,\lambda_n| \ll 1 \tag{14}$$

Our numerical search for resonances will rest upon this criterion.

3. *NUMERICAL SOLUTION*

Here we present the procedure for the numerical solution of eq. (13). We use the notations

$$c = \sqrt{\frac{4}{3}\frac{m}{\hbar^2}|z_3|}\,,\quad x = \frac{\kappa}{c},\quad x' = \frac{k'}{c},\quad \gamma_3 = \frac{z_3}{\frac{\hbar^2}{m}c^2};\quad |\gamma_3| = \frac{3}{4}. \tag{15}$$

Moreover the integration over p is substituted by the integration over y using relation (3b). Then the homogeneous equation reads

$$F_L(x,\gamma_3) = \int_0^\infty dx'\,K_L(x,x',\gamma_3)\,F_L(x',\gamma_3), \tag{16a}$$

where

$$K_L(x,x',\gamma_3) = W_L(x,x',\gamma_3)\,r\left(\sqrt{\gamma_3 - \frac{3}{4}x'^2}\right) \tag{16b}$$

$$W_L(x,x',\gamma_3) = -\frac{x'}{x}\int_{-1}^{1} dy\,\frac{f_L(x,x',y)}{y - q(x,x',\gamma_3)}$$

$$f_L(x,x',y) = g(p_0(x,x',y))\,P_L(y)\,g(p_0(x',x,y)),$$

$$p_0(x,x',y) = \sqrt{x'^2 + \frac{x^2}{4} + xx'y}, \qquad q(x,x',y_3) = \frac{y_3 - x^2 - x'^2}{xx'}.$$

Now we consider the singularities of the kernel $K(x,x',y_3)$ in the x'-plane. The function $W_L(x,x',y_3)$ has logarithmic singularities at the points

$$z_L^{(1,2,3,4)}(x) = -\frac{x}{2} \pm \sqrt{y_3 - \frac{3}{4}x^2}\,. \tag{17}$$

The function $\tau((y_3 - (3/4)x'^2)^{1/2})$ has two square-root branch points and two poles (if the two-particle system has a bound state). The branch points and the poles are located at

$$z_b = \pm\frac{2}{\sqrt{3}}\sqrt{y_3}\,, \quad |z_b|^2 = 1\,, \quad z_p = \pm\frac{2}{\sqrt{3}}\sqrt{y_3 + \frac{\epsilon_d \cdot m}{\pi^2 \cdot c^2}}. \tag{18}$$

For the case $\mathrm{Re}\,y_3 > 0$, $0 < \mathrm{Im}\,y_3 \ll 1$ and $\sqrt{3}/2 < x < 1$ the singularities of x' are shown in *Fig. 2.*

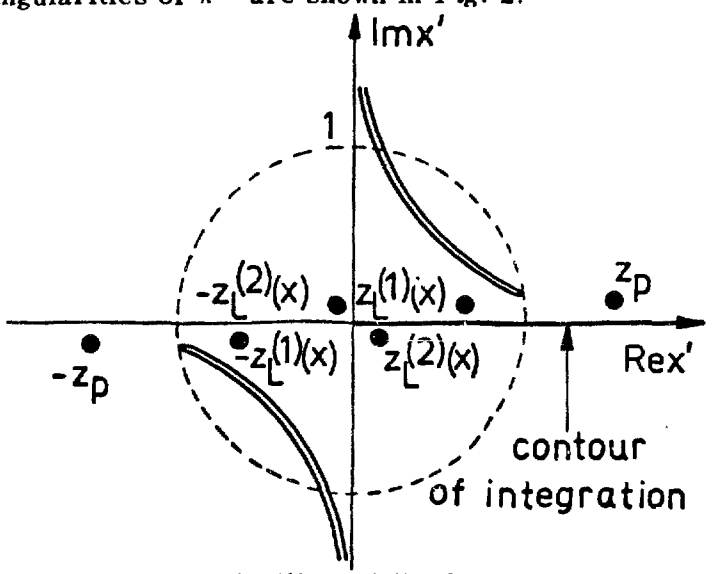


Fig. 2. The singularities of the kernel $K(x,x',y_3)$ *in the* x'*-plane.*

The singular integral equation (16a) has been solved by generalizing the methods proposed in refs. [13,14] to complex energies. At first the function $W(x,x')$ is divided into a smooth part and a singular one

$$W(x,x') = [-\frac{x'}{x}\int_{-1}^{1} dy\ \frac{f(x,x',y)-f_q(x,x')}{y-q(x,x')}] - \frac{x'}{x} f_q(x,x') \times \tag{19}$$

$$\times \int_{-1}^{1} \frac{dy}{y-q(x,x')} \equiv W_g(x,x') + W_q(x,x')\, Q_0(q(x,x')),$$

where $Q_0(q)$ is the Legendre function of the second kind which contains all the logarithmic singularities. As is shown in the *Appendix*, the function in the first integral is smooth, if $f_q(x,x')$ is taken as follows

$$f_q(x,x') = f(x,x',y_q) + (q-y_q)\frac{df(x,x',y)}{dy}\Big|_{y=y_q}, \tag{20}$$

where

$$y_q = \begin{cases} \mathrm{Re}\, q & -1 \le \mathrm{Re}\, q \le 1 \\ 1 \quad \text{for} & \mathrm{Re}\, q > 1 \\ -1 & \mathrm{Re}\, q < -1 \end{cases}$$

On account of the special choice (20) of the function $f_q(x,x')$ we need not continue function $f(x,x',y)$ to complex values of y.

The second function $\tau((y_3-(3/4)x'^2)^{1/2})$ in the kernel (16b) is splitted as follows

$$\tau(\sqrt{y_3-\frac{3}{4}x'^2}) = [\tau(\sqrt{y_3-\frac{3}{4}x'^2}) - \frac{\mathrm{Res}_{z_p}\tau}{x'-z_p}] + \frac{\mathrm{Res}_{z_p}\tau}{x'-z_p} =$$

$$= \tau_g(x') + \frac{\mathrm{Res}_{z_p}\tau}{x'-z_p}. \tag{21}$$

Here $\mathrm{Res}_{z_p}\tau$ is the residue of τ at the point z_p. After subtraction the function $\tau_g(x')$ still contains a pole at the point $-z_p$, which may influence the accuracy of the numerical integration under the condition that $|z_p| \ll 1$,

i.e., for energies z_3 in the vicinity of the two-particle threshold. For the energies considered in the present paper the presence of the pole at $-z_p$ has not noticeably affected the results of the calculations.

Inserting expressions (19) and (21) into the first of eqs. (16b) we have

$$K(x,x') = \sum_{m=1}^{4} K^{(m)}(x,x') S^{(m)}(x,x'). \tag{22}$$

where

$$\begin{aligned}
&K^{(1)}(x,x') = W_g(x,x') \tau_g(x'), \quad S^{(1)}(x,x') = 1 \\
&K^{(2)}(x,x') = W_g(x,x') \operatorname{Res}_{z_p} \tau, \quad S^{(2)}(x,x') = \frac{1}{x'-z_p} \\
&K^{(3)}(x,x') = W_\rho(x,x') \tau_g(x'), \quad S^{(3)}(x,x') = Q_0(q(x,x')) \\
&K^{(4)}(x,x') = W_\rho(x,x') \operatorname{Res}_{z_p} \tau, \quad S^{(4)}(x,x') = \frac{Q_0(q(x,x'))}{x'-z_p}
\end{aligned} \tag{23}$$

Moreover the integral (16a) can be rewritten in the form

$$\int_0^{\infty} dx' K(x,x') F(x') = \sum_{i=1}^{N} \int_{a_i}^{b_i} dx' K(x,x') F(x'). \tag{24}$$

The limits a_i and b_i of the integration are taken in such a way, that in the interval (a_i, b_i) the function $F^{(i)}(x')$ can be approximated with sufficient accuracy by the Lagrange interpolating polynomials

$$F^{(i)}(x') = \sum_{n=0}^{4} \Pi_n^{(i)} x'^n . \tag{25}$$

In an analogous way the smooth parts $K^{(m)}(x,x')$ of the kernel (22) have been represented in the form of polynomials

$$K^{(m)}(x,x') = \sum_{r=0}^{4} K_r^{(m)}(x) x'^r . \tag{26}$$

Having applied formulae (25) and (26) we are left with

the following integrals which contain all singularities of the initial integral equation

$$J_p^{(n)} = \int dx' \frac{x'^n}{x'-z_p},$$

$$J_\ell^{(n)} = \int dx' x'^n \cdot Q_0(q(x,x')), \tag{27}$$

$$J_{\ell_p}^{(n)} = \int dx' x'^n \frac{Q_0(q(x,x'))}{x'-z_p}.$$

The integrals $I_p^{(n)}$ and $I_\ell^{(n)}$ can be solved analytically. The integrals $I_{p\ell}^{(n)}$ have been calculated by means of an approximation procedure with high accuracy. For all the three types the integrals for different n have been calculated by recurrence formulae.

4. *RESULTS*

In the calculations the form factor (5) has been choosen in the Yamaguchi form

$$g(k) = \frac{(2\pi)^{3/2}}{k^2+\beta^2}. \tag{28}$$

Then we have for $\tau(p)$

$$\tau(p) = \frac{-\lambda}{\left[1-\left(\frac{\alpha+\beta}{\beta-ip}\right)^2\right]^2}; \quad \lambda = \frac{\beta(\alpha+\beta)^2}{\pi^2} \tag{29}$$

For α and β we have choosen the usual triplet parameters

$$\alpha = 0.2307 \text{ fm}^{-1}, \quad \beta = 1.45 \text{ fm}^{-1}. \tag{30}$$

The behaviour of the first two eigenvalues for $L=0$ and $L=1$ is shown in *Figs. 3* and *4*, respectively.

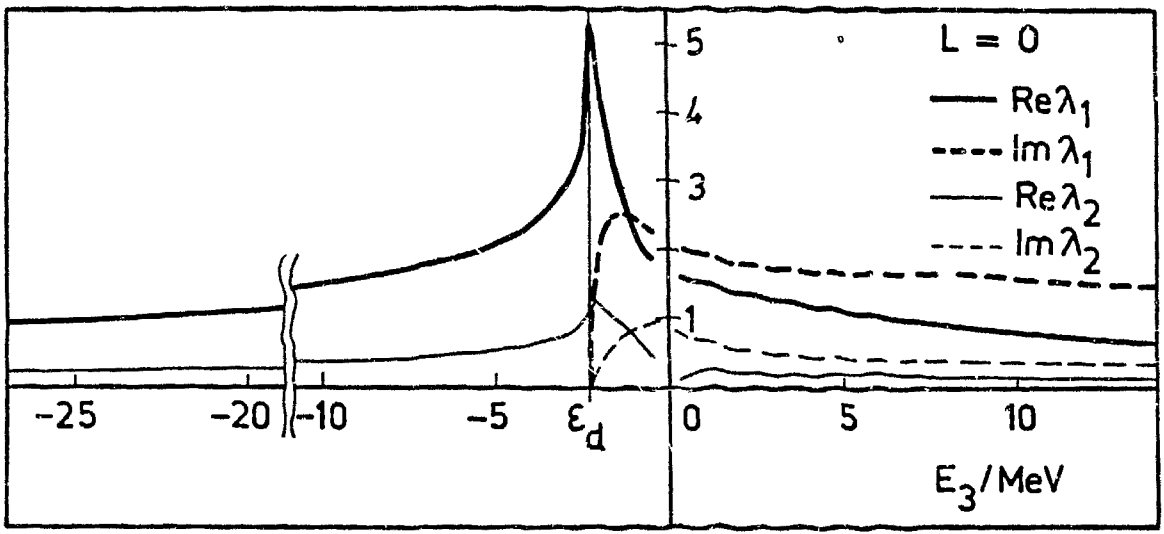


Fig. 3. The energy dependence of the first two eigenvalues of the Faddeev-kernel for L = 0.

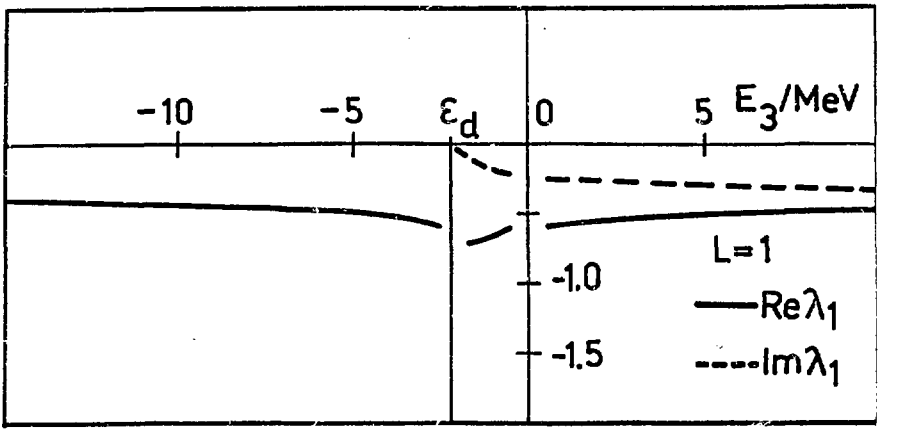


Fig. 4. The energy dependence of the first eigenvalue of the Faddeev-kernel for $L=1$.

The calculations were done for energies of the form $z_3 = E_3 + i10^{-4}$, i.e., we consider the region just above the cuts (cf. *Fig. 1*). The sensitivity of the eigenvalues as a function of the imaginary part of the energy is shown in *Fig. 5*. The typical behaviour of the first eigenfunction for $L=0$ and $L=1$ at different energies is represented in *Figs. 6* and *7*, respectively. The *Tables 1-3* give the eigenvalues in more detail.

5. *DISCUSSION AND CONCLUSIONS*

The results obtained can be summarized as follows. From *Figs. 3* and *4* it is clear that in the nonrelativistic system of three spinless particles interacting via the Yamaguchi potential with triplet parameters there are no resonances in states with the total orbital momentum $L=0$ and $L=1$. In the figures we show only those eigenvalues which have a modulus of the order of unity, since only these eigenvalues are of interest for the search for resonances (comp. (14)). As can be seen from *Fig. 3* the curves $\mathrm{Re}\,\lambda_1(E_3)$ and $\mathrm{Re}\,\lambda_2(E_3)$ intersect unity at negative

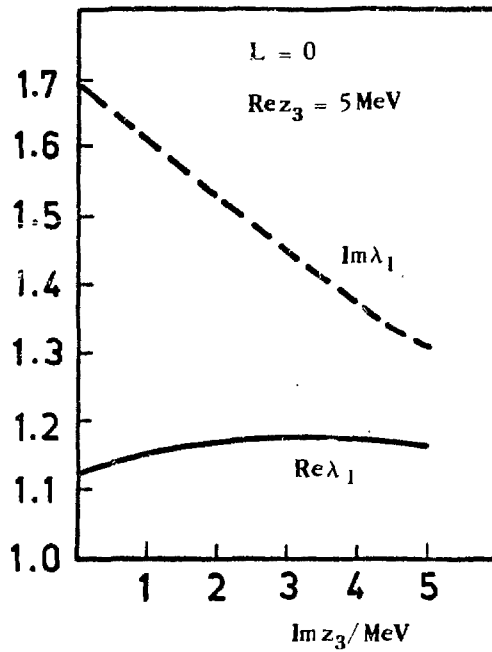


Fig. 5. The dependence of the first eigenvalue on the imaginary part of the energy.

energies with the imaginary part of the eigenvalues being zero in this region. This corresponds to a bound and an excited state of the system. The last result as well as the behaviour of the eigenvalues for energies below the two-particle threshold is consistent with the results from refs. [16,17]. If we exclude the region near the two-particle threshold, we can state that the dependence of the eigenvalues on the energy is rather smooth in the interval of several dozens of *MeV*. An analogous conclusion has been made by the authors of ref. [10]. It is interesting to note that in the relativistic three-body system [7] the behaviour of the greatest eigenvalues is not so monotonous (in its own energy scale, i.e., in an energy range of several pion masses).

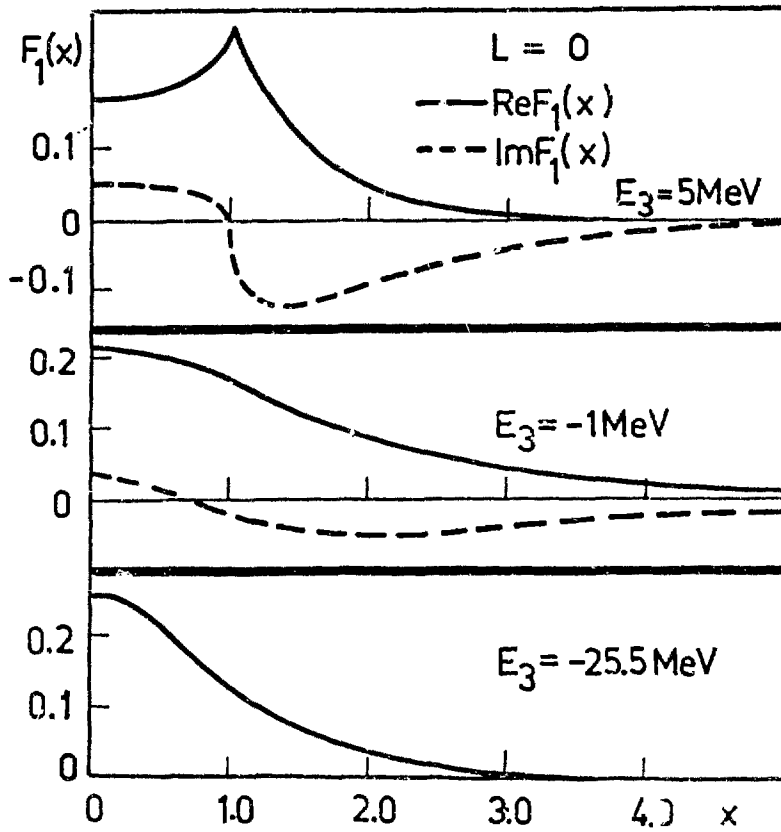


Fig. 6. Typical behaviour of the first eigenfunction in different energy regions ($L = 0$).

Finally we will discuss the reliability of the numerical calculations. Besides the methods described here, the eigenvalues have been calculated by two other independent procedures. The first is based on the Gaussian integration and works for not too small $\mathrm{Im} z_3$. The second one (the results will be published independently) is based on the subtraction of the moving logarithmic singularities in a similar way like usually the pole singularity is extracted

(this computer code works for $\mathrm{Im} z_3 = +0$). All the three procedures led to the same results within the range of their applicabilty. The accuracy of the calculated eigenvalues has been checked by comparing the results with 35 mesh points with those with 40 mesh points. *Tables 1* and *2* give the digits which coincide in both cases.

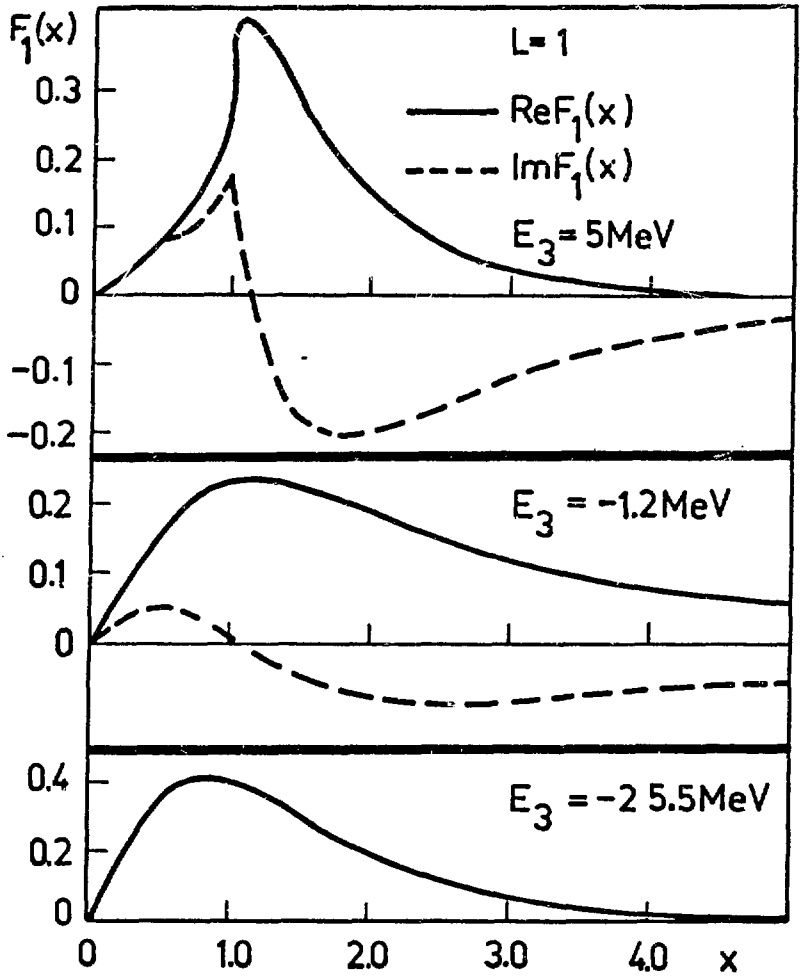


Fig. 7. Typical behaviour of the first eigenfunction in different energy regions (L = 1).

Table 1
The eigenvalues for L = 0

E_T/MeV	$Re\lambda_1$	$Im\lambda_1$	$Re\lambda_2$	$Im\lambda_2$	E_T/MeV	$Re\lambda_1$	$Im\lambda_1$	$Re\lambda_2$	$Im\lambda_2$
-30.0	0.927	0	0.24	0	-1.0	2.26	2.46	0.80	0.86
-25.5	0.998	0	0.25	0	-0.5	1.88	2.24	0.49	0.96
-20.0	1.11	0	0.28	0	-0.1	1.67	2.25	0.09	0.91
-15.0	1.26	0	0.32	0	0.3	1.64	2.09	0.12	0.79
-10.0	1.51	0	0.39	0	0.6	1.62	1.96	0.25	0.71
- 6.0	1.90	0	0.50	0	1.0	1.56	1.91	0.25	0.65
- 4.0	2.35	0	0.62	0	2.0	1.42	1.82	0.26	0.55
- 2.5	3.47	0	0.91	0	3.0	1.30	1.77	0.25	0.5
- 2.3	4.08	0	1.05	0	5.0	1.12	1.69	0.21	0.45
- 2.2	5.27	0.447	1.26	0.07	8.0	0.928	1.61	0.17	0.42
- 2.1	4.79	1.57	1.23	0.26	11.0	0.790	1.54	0.16	0.39
- 1.8	3.72	2.40	1.13	0.51	14.0	0.678	1.48	0.13	0.37

APPENDIX

The integrand in (16b) can be splitted into a smooth and a singular part (q -complex)

$$\frac{f(y)}{y-q} \equiv \frac{f(y)-f(q)}{y-q} + \frac{f(q)}{y-q} . \qquad (A.1)$$

This splitting is not suitable since it requires the extension of the domain of definition of the function $f(y)$ to complex y. This considerably complicates the problem since we must investigate the analytic structure of each form factor used. To avoid these difficulties we replace the function $f(q)$ by a function f_q defined as

$$f_q = f(y_q) + \left(\frac{df}{dy}\right)_{y_q} (q - y_q) ,$$

where

$$y_q = \begin{cases} \mathrm{Re}\, q & -1 \le \mathrm{Re}\, q \le 1 \\ 1 & \text{for} \quad \mathrm{Re}\, q > 1 \\ -1 & \mathrm{Re}\, q < -1 \end{cases} \qquad (A.2)$$

Table 2
The eigenvalues for $L = 1$

E_3/MeV	$\mathrm{Re}\,\lambda_1$	$\mathrm{Im}\,\lambda_1$	E_3/MeV	$\mathrm{Re}\,\lambda_1$	$\mathrm{Im}\,\lambda_1$
-25.5	-0.32	0	-1.2	-0.67	-0.16
-20.0	-0.35	0	-0.7	-0.64	-0.19
-15.0	-0.38	0	0.5	-0.60	-0.23
-10.0	-0.42	0	1.0	-0.58	-0.25
- 6.0	-0.48	0	2.0	-0.57	-0.26
- 4.0	-0.53	0	3.0	-0.55	-0.27
- 2.5	-0.60	0	4.0	-0.54	-0.29
- 2.2	-0.65	$-2\cdot10^{-4}$	5.0	-0.52	-0.30
- 1.7	-0.71	-0.09	10.0	-0.46	-0.33

In the vicinity of y_q the function $f(y)$ may be written as

$$f(y) = f(y_q) + \left(\frac{df}{dy}\right)_{y_q} (y - y_q) + \frac{1}{2}\left(\frac{d^2 f}{dy^2}\right)_{y_q} (y - y_q)^2 + \ldots \qquad (A.3)$$

Hence it follows

$$\frac{f(y)-f_q}{y-q} = \left(\frac{df}{dy}\right)_{y_q} + \frac{1}{2}\left(\frac{d^2f}{dy^2}\right)_{y_q} \frac{(y-y_q)^2}{y-q}. \qquad (A.4)$$

It is easy to see that the function in (A.4) is in fact a smooth function of y in the vicinity of the pole.

Table 3

The eigenvalues in dependence on the imaginary part of the energy ($Re\, z_3 = 5$ *MeV*, L=0)

$\frac{Im z_3}{MeV}$	$Re\lambda_1$	$Im\lambda_1$	$Re\lambda_2$	$Im\lambda_2$
0	1.123	1.69	0.21	0.45
1.0	1.154	1.61	0.23	0.44
2.0	1.170	1.53	0.25	0.42
3.0	1.176	1.45	0.25	0.40
4.0	1.172	1.38	0.26	0.38
5.0	1.160	1.31	0.26	0.35

REFERENCES

1. *L.D.Faddeev. JETP (USSR), 39, 1459 /1960/.*
2. *S.Fiarman, S.S.Hanna. Nucl.Phys., A251, 1 /1975/.*
3. *O.A.Yakubovsky. Yad. Fiz. (USSR), 5, 1312 /1967/.*
4. *I.M.Narodetsky. Nucl.Phys., A221, 191 /1974/.*
5. *J.A.Tjon. Phys.Lett., 56B, 217 /1975/.*
6. *A.G.Baryshnikov, I.M.Blokhintsev, I.M.Narodetsky. Phys.Lett., 51B, 432 /1974/.*

7. J.L.Basdevant, R.Kreps. Phys.Rev., 141, 1398 /1965/; 141, 1404 /1965/.
J.Y.Pasquier, R.Pasquier. Proc. 1st Int. Conf. on the Three Body Problem, Birmingham, 1969, p. 461.
8. W.Glöckle. Nucl.Phys., A158, 257 /1970/; A175, 337 /1971/.
9. Proc. Int. Conf. on Few Particle Problems in the Nuclear Interaction. Los Angeles, 1972, pp. 519-550.
10. A.M.Badalyan, E.S.Birger, N.B.Konyukhova. Yad. Fiz. (USSR), 20, 1147 /1974/.
11. A.M.Badalyan, Yu.A.Simonov. Yad. Fiz. (USSR), 21, 890 /1975/.
12. J.H.Hetherington, L.H.Schick. Phys.Rev., 137B, 935 /1965/.
13. F.Sohre, H.Ziegelmann. Phys.Lett., 34B, 579 /1971/.
14. V.B.Belyaev, V.N.Efimov, E.G.Tkachenko, H.Schulz. Yad. Fiz. (USSR), 18, 779 /1973/.
15. P.Beregi, I.Lovas. Z.Physik, 241, 410 /1971/; 257, 440 /1972/.
16. V.F.Kharchenko. Thesis, ITF Kiev, 1970.
17. I.M.Narodetsky, E.S.Galpern, V.N.Lyakhovitsky. Yad. Fiz. (USSR), 16, 707 /1972/.

Received by Publishing Department
on March 12, 1976.