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ON APPLICABILITY OF THE GLAUBER MODEL FOR PION ELASTIC SCATTERING BY HELIUM IN THE Δ_{33} RESONANCE REGION

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ON APPLICABILITY OF THE GLAUBER MODEL FOR PION ELASTIC SCATTERING BY HELIUM IN THE Δ_{33} RESONANCE REGION

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О применимости модели Глаубера для описания упругого рассеяния пионов на гелии в области Δ_{33} резонанса

В рамках теории Глаубера вычислено упругое рассеяние пионов на ^3He, ^4He в области первого пион-нуклонного резонанса. Полностью учитывались эффекты спин-флипа и перезарядки. Выяснена природа ограничений для применения теории Глаубера при очень низких энергиях.

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On Applicability of the Glauber Model for Pion Elastic Scattering by Helium in the Δ_{33} Resonance Region

Elastic scattering of pions by ^3He and ^4He in the region of the first pion-nucleon resonance is calculated within the Glauber theory. The spin-flip and charge-exchange effects are fully taken into account. Calculated results are compared with experiment and with predictions obtained using the optical model. Some conclusions are drawn concerning the validity of approximations used in deriving the two models. The calculations make clearer some of the limitations of using the Glauber model for very low energies.

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1. *INTRODUCTION*

In the past few years, much attention has been paid to the examination of pion elastic scattering by light nuclei in the Λ_{33} resonance energy region. Recent experiments with carbon and helium were subjected to analyses on the basis of the optical model and Glauber theory (for review see, e.g. [1,2]). The optical model appeared to be useful tool for description of pion-nucleus scattering. The influence of nuclear structure details [3,4], of various off-shell continuations chosen for πN amplitude [5], of nuclear Fermi motion and kinematical effects [6,7], etc., on elastic scattering by the lightest nuclei was at least qualitatively understood in this frame.

The Glauber model, the essence of which is the eikonal approximation, is intended for description of high energy collisions ($E \geq 500$ *MeV*) in the range of small momentum transfers. Nevertheless, it was shown [8] that qualitative description of pion-nucleus elastic scattering is possible also in the region of the first pion-nucleon resonance. The relative success of the Glauber model is probably caused by the fact that large cancellation occurs [9] between corrections arising from three effects: deviations from eikonal propagation between scatterings, Fermi motion of struck nucleons and the kinematical transformation which relates the many-body scattering operators of the Watson theory to the physical two-body scattering amplitude. Thus the deviations of Glauber model predictions from the exact result are smaller than even significant corrections to the eikonal approximation could indicate. No systematical investigation of validity of the eikonal approximation was, however, performed in the Δ_{33} resonance region.

Recently, several elaborate Glauber model calculations of π-He cross sections have been done in the energy interval 80-300 *MeV*. Franco [10] succeeded to take into account all the spin-flip and charge-exchange effects produced by the spin and isospin dependent part of the πN amplitude in the case of $\pi - {}^4He$ scattering. These effects, being of order $1/A$ (A is the number of nucleons) were shown to influence strongly $\pi - {}^4He$ total cross section in the resonance region. Most of the spin- and isospin-dependent terms were included also in [11] in calculating ${}^3He(\pi^-, \pi^0){}^3He$ cross section. Despite the progress reached in [10,11], it remains unclear whether the Glauber model is in a position to produce more accurate results including more realistic features of both the target nucleus and the elementary πN amplitude or it is possible to obtain - especially in the case of the lightest nuclei - only some qualitative predictions in the Δ_{33} resonance region.

In order to obtain at least a partial answer to this problem, we proceed as follows. Firstly, the Glauber model results were compared with new experimental data on elastic differential cross sections for $\pi^{\pm} - {}^3He$ [12] and $\pi^{\pm} - {}^4He$ [13] reactions. Such a comparison is especially interesting for $\pi^{\pm} - {}^3He$ reaction, since no ${}^3He$ data were earlier available. Using the optical model, it was established in the previous paper [14] that owing to the nonzero value of nuclear spin and isospin ($J = T = 1/2$), there are considerable differences between $\pi^+ - {}^3He$ and $\pi^- - {}^3He$ elastic scatterings. It will be shown that such differences can be accounted for in the Glauber frame, too. In deriving the Glauber model, the full spin and isospin structure of πN amplitude was retained also in the more complicated case of ${}^3He$ nucleus. Further, we preferred to express the πN amplitude in terms of the experimentally known pion-nucleon phase shifts. Although the off-energy-shell continuation of such an amplitude has some drawbacks [8], the procedure seems to be less ambiguous than various high-energy parametrizations commonly used in the previous Glauber-type calculations. Besides the spin-flip and charge-exchange effects, the nuclear correlations connected with the nuclear recoil were studied also in detail. It will be shown that $\pi - {}^4He$ cross sections are influenced by these correlations to the surprisingly large extent in the energy region considered.

Secondly, the Glauber model results were compared with those obtained utilizing the first order optical potential. The approximations used in deriving the two models are discussed. If the underlying approximations are chosen to coincide as much as possible, some conclusions can be drawn from the comparison of the two models concerning the validity of the eikonal approximation.

In section 2 a brief derivation of the Glauber and the optical model is given. Different features of the two models are discussed in section 3. The calculated results are compared with experimental data in section 4, where some conclusions are also drawn concerning the validity of the approximations used.

2. FORMALISM

Earlier, a detailed derivation was given (see, e.g., [1,2] and [15]) of both the Glauber and the optical model starting from the famous Watson's multiple scattering series

$$\mathcal{T} = \sum_{i=1}^{A} \tau_i + \sum_{i \neq j} \tau_i G \tau_j + \dots , \qquad (1)$$

where τ_i denotes the collision operator for pion scattering by a bound nucleon. The pion-nucleus collision operator is denoted as $\mathcal{T}$ and the Green function G describes the propagation of the pion through a nucleus. Rather than to repeat the derivation here, we point out the different physical content of the two models. The first, common approximation used in eq. (1) consists of $\tau \simeq t$ where t denotes the collision operator for scattering by a free nucleon (impulse approximation). In the case of ground state elastic scattering, eq. (1) can be rewritten as follows

$$<0|\mathcal{T}(E)|0> = A<0|t(E)|0> + A(A-1)\sum_n <0|t(E)|n> G_{nn}(E) \times \qquad (2)$$
$$\times <n|t(E)|0> + \ldots,$$

where

$$G_{nn}(E) = <n|G(E)|n>.$$

2.1. *Glauber Model*

The pion-nucleus amplitude is obtained in the following steps.

(i) Closure approximation. If the nuclear excitation energy e_n corresponding to the nuclear state $|n>$ is neglected, $G_{nn}(E) \approx G_{00}(E)$, then the summation over all intermediate nuclear states in eq. (2) can be easily performed and we have

$$<0|\mathcal{T}(E)|0> = A<0|t(E)|0> + A(A-1)<0|t(E)G_{00}(E)t(E)|0> + \ldots \qquad (3)$$

(ii) Eikonal approximation. The term linear in transferred momentum is retained in $G_{00}(E)$ only. As a result, the series (3) becomes finite being terminated by the term involving A scatterings.

(iii) Parametrization of the pion-nucleon amplitude. The amplitude $f_{\pi N}$ is supposed to be a function of the transferred momentum $\vec q$ and of the pion energy E_{2c} in the pion-nucleon centre-of-mass system (will be denoted as 2CM). If the amplitude $f_{\pi N}$ is transformed properly from 2CM to the pion-nucleus centre-of-mass system * (referred to as ACM), the pion-nucleus amplitude will be as follows

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* Performing the transformation, "sudden passage" relativistic kinematics was used as described in /9/.

$$<\alpha'|F_C(\vec q,\vec t,\vec T,\vec J)|\alpha> = \frac{p_{Ac}}{2\pi i}\int d^2b\, e^{i\vec q\vec b} <0,JT\alpha'| \times$$
$$\times \prod_{j=1}^{A}[1-\Gamma(\vec b-\vec s_j)]-1|0,JT\alpha> \qquad (4)$$

in ACM. Here,

$$\Gamma(\vec b) = \frac{1}{2\pi i\, p_{2c}}\int d^2q\, e^{-i\vec q\cdot\vec b} f_{\pi N}(\vec q,\vec\sigma,\vec\tau,\vec t,E_{2c}),$$

p_{Ac} and p_{2c} denote the pion momentum in ACM and 2CM, respectively. As α', α $(\alpha = m_J, m_T, m_t)$ there are denoted projections of nuclear spin, isospin and of projectile isospin. One has

$$p_{2c} f_{\pi N}(\vec q,\vec\sigma,\vec\tau,\vec t,E_{2c}) = a_0 + a_1\vec t\cdot\vec\tau + i\vec\sigma\cdot\vec\nu(b_0 + b_1\vec t\cdot\vec\tau), \qquad (5)$$

where

$$a_i = a_i(q,E_{2c}) = \sum_{\ell=0}^{\infty} A_\ell^{(i)}(E_{2c})P_\ell\left(1-\frac{q^2}{2p_{2c}^2}\right) \qquad (6a)$$

$$b_i = b_i(q,E_{2c}) = \sum_{\ell=1} B_\ell^{(i)}(E_{2c})P'_\ell\left(1-\frac{q^2}{2p_{2c}^2}\right) \qquad (6b)$$

for $i=0,1$. The dimensionless coefficients $A_\ell^{(i)}$ and $B_\ell^{(i)}$ are simply connected with pion-nucleon phase shifts. In actual calculations the phase shifts /16/ were used. The unit vector $\vec\nu$ normal to the scattering plane is generally different for each individual scattering act. In our calculations, the vector $\vec\nu$ was approximated by the expression $[\vec q\times\vec K]/p_{Ac}^2$. If the momenta of incoming and outgoing pion are denoted as $\vec p_i$ and $\vec p_f$ $(|\vec p_i| = |\vec p_f| = p_{Ac})$; respectively, then $\vec K = (\vec p_i + \vec p_f)/2$.

(iv) Nuclear model. Nuclear wave functions were constructed using the single particle harmonic oscillator basis which yields

$$\psi(^3He) = \psi_0(r_1)\psi_0(r_2)\psi_0(r_3)\frac{1}{\sqrt{2}}(\chi_{m_J}\bar\eta_{m_T} - \bar\chi_{m_J}\eta_{m_T}), \qquad (7a)$$

$$\psi(^4\mathrm{He}) = \psi_0(r_1)\,\psi_0(r_2)\,\psi_0(r_3)\,\psi_0(r_4)\,\frac{1}{\sqrt{2}}(\chi\bar{\eta} - \bar{\chi}\eta), \qquad (7b)$$

where $\chi, \bar{\chi}\ (\chi_{m_J}, \bar{\chi}_{m_J})$ are mixed symmetry spin wave functions corresponding to the spin $S = 0\,(1/2)$. The isospin wave functions $\eta, \bar{\eta}\ (\eta_{m_T}, \bar{\eta}_{m_T})$ have a similar meaning. Further,

$$\psi_0(r) = \frac{1}{(a_0^2\pi)^{3/2}}\, e^{-r^2/2a_0^2}\,, \qquad (8)$$

where the parameter a_0 can be determined from electron scattering experiments. Whereas by the symbol $|0, J\,T m_J m_T\rangle$ the "inner" nuclear wave function is denoted in eq. (4), the expressions given by eqs. (7) being translationally not invariant contain some portion of the nuclear motion as a whole. In /17/ it has been shown that the nuclear recoil can be treated correctly even if not invariant wave functions are used. To this end, it is necessary to substitute in eq. (4)

$$\vec{s}_i \to \vec{s}_i - \frac{1}{A}\sum_{j=1}^{A}\vec{s}_j\ . \qquad (9)$$

In this way, we obtain for $A = 3$ and 4 after angular integrations

$$F_G(\vec{q},\vec{t},\vec{T},\vec{J}) = p_{Ac}\, e^{\frac{q^2a_0^2}{4A}} \{F_0(q) + 2\vec{t}\cdot\vec{T}\,F_T(q) + 2i\vec{J}\,\frac{[\vec{q}\times\vec{K}]}{p_{Ac}^2}(F_S(q) + 2\vec{t}\cdot\vec{T}\,F_{ST}(q))\}. \qquad (10)$$

Here

$$F_0(q) = \int_0^\infty J_0(qb)\,b\,db\,\{\tbinom{A}{1}\alpha_0 + i\tbinom{A}{2}[\alpha_0^2 - \tfrac{2}{3}\alpha_1^2 - \tfrac{1}{3}\cos^2\tfrac{\theta}{2}\times(\beta_2^2 + 2\beta_3^2)] - \tbinom{A}{3}[\alpha_0^3 - 2\alpha_1^2\alpha_0 - \cos^2\tfrac{\theta}{2}(\alpha_0\beta_2^2 + 2\alpha_0\beta_3^2 - 4\alpha_1\beta_2\beta_3)] - i\tbinom{A}{4}[\alpha_0^4 + \tfrac{10}{3}\alpha_1^4 - 4\alpha_0^2\alpha_1^2 - 2\cos^2\tfrac{\theta}{2}(\alpha_0^2\beta_2^2 + 2\alpha_0^2\beta_3^2 + 2\alpha_1^2\beta_2^2 + 2\alpha_1^2\beta_3^2 - 8\alpha_0\alpha_1\beta_2\beta_3) + \cos^4\tfrac{\theta}{2}(\beta_2^4 + \tfrac{10}{3}\beta_3^4 - 4\beta_2^2\beta_3^2)]\}, \qquad (11a)$$

where the coefficients $\binom{A}{i} = A!/i!(A-i)!$ are supposed to equal zero for $i > A$. We have for $A = 3$

$$F_T(q) = \int_0^\infty J_0(qb)\,b\,db\,\{\alpha_1 + 2i(\alpha_0\alpha_1 - \cos^2\tfrac{\theta}{2}\beta_2\beta_3) - \alpha_0^2\alpha_1 + \tfrac{5}{3}\alpha_1^3 - \cos^2\tfrac{\theta}{2}(\alpha_1\beta_2^2 - 2\alpha_0\beta_2\beta_3 + \tfrac{5}{3}\alpha_1\beta_3^2)\} \qquad (11b)$$

$$F_S(q) = \int_0^\infty J_0(qb)\,b\,db\,\{\beta_2 + 2i(\alpha_0\beta_2 - 2\alpha_1\beta_3) + 4\alpha_0\alpha_1\beta_3 - \alpha_0^2\beta_2 - 2\alpha_1^2\beta_2 + \cos^2\tfrac{\theta}{2}(\beta_2^3 - 2\beta_2\beta_3^2)\} \qquad (11c)$$

$$F_{ST}(q) = \int_0^\infty J_0(qb)\,b\,db\,\{-\beta_3 + 2i(\alpha_1\beta_2 - \alpha_0\beta_3) + \alpha_0^2\beta_3 + \tfrac{5}{3}\alpha_1^2\beta_3 - 2\alpha_0\alpha_1\beta_2 + \cos^2\tfrac{\theta}{2}(2\beta_2^2\beta_3 - \tfrac{5}{3}\beta_3^3)\}, \qquad (11d)$$

whereas $F_T(q) \equiv F_S(q) \equiv F_{ST}(q) \equiv 0$ for $A = 4$. We used the notation

$$\alpha_i \equiv \alpha_i(b, E_{2c}) = \frac{1}{p_{2c}^2}\int_0^\infty e^{-\frac{q^2a_0^2}{4}}\,a_i(q, E_{2c})\,J_0(bq)\,q\,dq \qquad (12a)$$

$$\beta_i \equiv \beta_i(b, E_{2c}) = \frac{1}{p_{2c}^2\,p_{Ac}}\int_0^\infty e^{-\frac{q^2a_0^2}{4}}\,b_i(q, E_{2c})\,J_1(qb)\,q^2\,dq. \qquad (12b)$$

In order to determine the parameter a_0, nuclear charge form factor

$$F_{ch}(q) = \exp\{-\frac{q^2}{6}(r_p^2 + \frac{3}{2}\,\frac{A-1}{A}\,a_0^2)\} \qquad (13)$$

was constructed from the wave functions (7). We put

$$r_p^2 + \frac{3}{2}\frac{A-1}{A}a_0^2 = R_{ch}^2 , \qquad (14)$$

where R_{ch} is the experimental charge radius ($R_{ch}(^3He)$= = 1.88 *fm* /18/ and $R_{ch}(^4He)$= 1.71 *fm* /19/ and r_p = 0.8 *fm* is the proton radius.

In our calculations, the Coulomb term $F_C(q) \sim F_{ch}(q)/q^2$ was also introduced. Finally, $\pi - {}^3He$ and $\pi - {}^4He$ cross sections were obtained.

2.2. *Optical Model*

Since the $\pi - {}^3He$ and $\pi - {}^4He$ optical model was derived in detail in /4/, we only briefly recapitulate the approximations utilized.

(i) Coherent scattering approximation. The terms containing nondiagonal matrix elements $<0|t(E)|n>$, $|n> \neq |0>$, are fully neglected in eq. (2). Thus,

$$<0|\frac{A-1}{A}\mathcal{T}(E)|0> = (A-1)<0|t(E)|0> + (A-1)^2<0|t(E)|0> \times$$

$$\times G_{00}(E)<0|t(E)|0> + \ldots = (A-1)<0|t(E)|0>(1+G_{00}(E)\times \qquad (15)$$

$$\times <0|\frac{A-1}{A}\mathcal{T}(E)|0>).$$

Here, the pion-nucleus elastic scattering is treated as a two-body problem, and the mutual interaction is described by the optical potential

$$<\alpha'\vec{p}_f|U(\vec{t},\vec{T},\vec{J},E)|\vec{p}_i\,\alpha> = (A-1)<0,\alpha'\vec{p}_f|t(E)|\vec{p}_i\,\alpha,0>. \qquad (16)$$

(ii) Parametrization of the pion-nucleon amplitude, the kinematical transformation of it from 2CM to ACM as well as the nuclear wave functions were chosen to be exactly the same as is shown in section 2.1. Then the optical potential becomes

$$<\vec{p}_f|U(\vec{t},\vec{T},\vec{J},E)|\vec{p}_i> = F_c(q) + \{a(0) + a(1)(1-\frac{q^2}{2p_{2c}^2}) + \frac{2i}{Ap_{2c}^2}\vec{J}\cdot[\vec{p}_i\times\vec{p}_f]\,b(1)\}F(q), \qquad (17)$$

where $\vec{q} = \vec{p}_f - \vec{p}_i$ and

$$a(\ell) = -4\pi\frac{p_{Ac}}{p_{2c}^2}\{A_\ell^{(0)}(E_{2c}) + \frac{2}{A}\vec{t}\cdot\vec{T}A_\ell^{(1)}(E_{2c})\}, \qquad (18a)$$

$$b(\ell) = -4\pi\frac{p_{Ac}}{p_{2c}^2}\{B_\ell^{(0)}(E_{2c}) - 2\vec{t}\cdot\vec{T}B_\ell^{(1)}(E_{2c})\}. \qquad (18b)$$

The coefficients $A_\ell^{(i)}(E_{2c})$ and $B_\ell^{(i)}(E_{2c})$ were introduced in eq. (6). Pion-nucleon s- and p-waves were considered only in deriving eq. (17). Nuclear body form factor F(q) is given by

$$F(q) = e^{-\frac{A-1}{4A}q^2 a_0^2}. \qquad (19)$$

In the case of $\pi - {}^4He$ elastic scattering, solely the scalar-isoscalar part of the potential (17) is, of course, different from zero.

3. *COMPARISON OF THE GLAUBER AND THE OPTICAL MODEL*

The main difference between the two models (apart from the eikonal approximation) consists in the fact that in the Glauber frame all intermediate nuclear excited states are approximately taken into account, while the optical model allows for intermediate elastic scatterings only. As a consequence, the Glauber model could tell us, if taken seriously in the energy region considered, something about the effects which do not enter the first order optical potential.

3.1. *Spin-Flip and Charge-Exchange Effects*

The spin- and isospin-dependent portion of the πN amplitude does not give any contribution to the first order optical potential for $J = T = 0$ nuclei. Only those transitions can be described by the potential for $J \neq 0$, $T \neq 0$ nuclei, which differ by value m_T and/or m_J from the ground state (e.g., $\pi^- + {}^3He \to \pi^o + {}^3H \to \pi^- + {}^3He$). On the other hand, the part of the Glauber amplitude, eqs. (10, 11), which is induced by vector-isovector component of πN amplitude, approximately describes virtual processes of the type $\pi^- + {}^4He \to \pi^o + n + {}^3He \to \pi^- + {}^4He$ etc.

3.2. *Nuclear Correlations*

As can be seen from eq. (17), nuclear structure enters the first order optical potential via nuclear form factor. It is well known [20], that the Glauber amplitude contains in addition to the formfactor also various nuclear correlation functions. For example, the double-scattering term in eq. (4) can be rewritten as follows

$$F_G^{(2)}(\vec{q}) \sim \int d^2q_1 d^2q_2 \delta^{(2)}(\vec{q}-\vec{q}_1-\vec{q}_2) f_{\pi N}(\vec{q}_1, E_{2c}) f_{\pi N}(\vec{q}_2, E_{2c}) \times C(\vec{q}_1, \vec{q}_2), \quad (20)$$

where

$$C(\vec{q}_1, \vec{q}_2) = \langle 0 | e^{i\vec{q}_1 \cdot \vec{s}_1} e^{i\vec{q}_2 \cdot \vec{s}_2} | 0 \rangle \quad (21)$$

and $\vec{q} = \vec{p}_f - \vec{p}_i$. Spin and isospin indices were suppressed in eqs. (20) and (21). Using the wave functions given by eq. (7) and the Gartenaus and Schwarz receipt [17] contained in eq. (9), we obtain

$$C(q_1, q_2) = e^{-\frac{a_o^2}{4}\frac{A-1}{A}(q_1^2+q_2^2)} e^{\frac{a_o^2}{2A}\vec{q}_1 \cdot \vec{q}_2} = e^{\frac{q^2 a_0^2}{4A}} e^{-\frac{a_0^2}{4}(q_1^2+q_2^2)} \quad (22)$$

in accordance with [21], where the correlations caused by nuclear recoil were studied in detail. In order to test the sensitivity of the Glauber model results to this effect, elastic and total cross sections were also calculated, when the recoil correlations were "turned-off". To this end, we substituted in eqs. (10) and (12) $\exp(q^2 a_0^2/4A)$ by 1 and a_0^2 by $(A-1)a_0^2/A$, respectively. It can be easily verified, that using this prescription, the recoil correlations are suppressed in all multiple-scattering terms and the recoil effects remain unaltered in the expression (19) for nuclear form factors.

3.3. *Optical Model and Eikonal Approximation*

Even if the eikonal approximation is applied to the optical model Green function $G_{00}(E)$ in eq. (15), the resulting optical and Glauber model pion-nucleus amplitudes will be different. The optical model amplitude for $J = T = 0$ nuclei becomes

$$F_{OM}(q) = \frac{A}{A-1} p_{Ac} i \int_0^\infty b J_0(qb) db (1 - e^{i(A-1)\alpha_0(b, E_{2c})}), \quad (23)$$

whereas

$$F_G(q) = p_{Ac} i \int_0^\infty b J_0(qb) db [1 - (1 + i\alpha_0(b, E_{2c}))^A] \quad (24)$$

is obtained in the Glauber model neglecting all spin-flip and charge-exchange effects as well as correlations connected with the nuclear recoil. Here, $\alpha_0(b, E_{2c})$ is given by eq. (12) provided that a_0^2 is substituted by $(A-1)a_0^2/A$. If we express the right-hand side of eq. (23) as a power series in terms of $\alpha_0(b, E_{2c})$, it can be easily verified that single- and double-scattering terms in eqs. (23) and (24) coincide. The difference between the two amplitudes is caused only by scattering terms of higher multiplicity. As was impressively shown by Eisenberg [15], such terms cancel out to the large extent in the Glauber model. Using the coherent scattering approximation in deriving the optical model, the structure of multiple-scattering terms is crudely simplified, therefore the cancellation cannot fully occur.

In this paper, pion-nucleus cross sections calculated utilizing eq. (23) are compared with the exact optical model results. Such a comparison can give us an idea about validity of the eikonal approximation in the energy region considered. Provided that the eikonal approximation is adequate, the comparison of results obtained using eqs. (23) and (24) could tell us something about the validity of the coherent scattering approximation.

4. *DISCUSSION AND CONCLUSIONS*

We have calculated elastic $\pi^{\pm} - {}^{3}He$ and $\pi^{\pm} - {}^{4}He$ differential cross sections, corresponding total cross sections and the total cross section for the reaction ${}^{3}H(\pi^{+},\pi^{0}){}^{3}He$ in the region of the first pion-nucleon resonance using the Glauber model. Several results were compared with those obtained utilizing the optical model. The main aim of this study was to investigate the validity of various approximations used in deriving the two models, therefore no attempt was made to adjust the input parameters in order to obtain better agreement with experiment.

(i) Differential cross sections. The Glauber model results are compared with the experimental $\pi^{\pm} - {}^{3}He$ differential cross sections /12/ in *Fig. 1* at several energies. A reasonable agreement is observed up to the first minimum. The calculated results overestimate somewhat the experimental data in the region of the secondary maximum. The difference between π^{+} and π^{-} cross sections is accounted for quite well, as displayed at energy 98 *MeV*. A similar situation occurs also for other energies. Differential cross sections are influenced rather weakly by the nuclear recoil correlations. An analogous comparison performed for $\pi^{\pm} - {}^{4}He$ reaction is given in *Fig. 2*. An agreement between theory and experiment is satisfactory up to the minimum, however the large angle discrepancies become more serious. The Glauber model is also in a position to

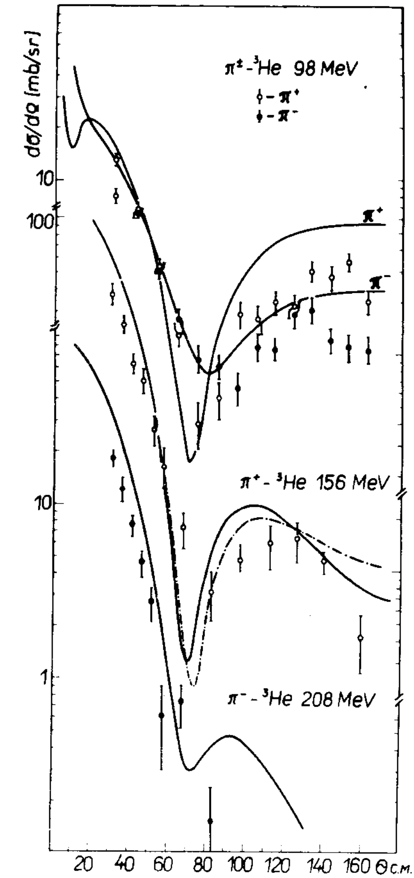

Fig. 1. Differential $\pi^{\pm} - {}^{3}He$ *cross sections. Full Glauber model results are compared with the experimental data* /12/ *(solid lines). If the recoil correlations are "turned off", the dot-dashed curve is obtained at 156MeV. Pion-nucleon* s– , p– , d– , f–, g– *and* h*-waves were considered.*

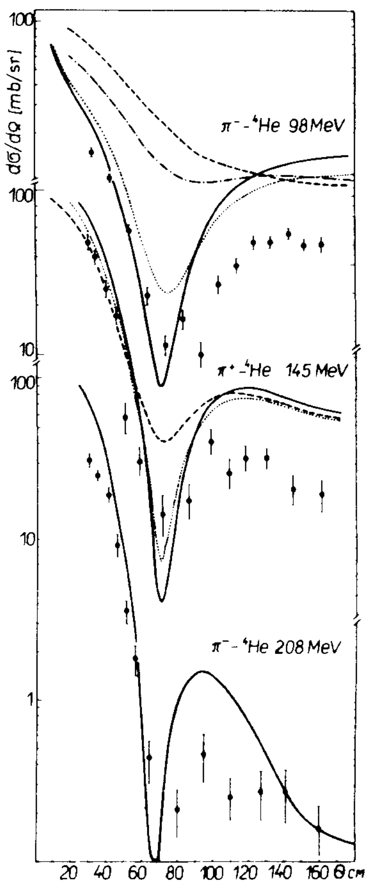

Fig. 2. Differential $\pi^{\pm}-{}^{4}$He *cross sections. Full Glauber model results are compared with the experimental data* /13/ *(solid lines). By dotted lines the results are shown obtained neglecting the spin-flip and charge-exchange effects. If the nuclear correlations are also neglected, the dashed curves are obtained. Six* πN *waves were considered. The difference between the dashed and dot-dashed lines at 98 MeV is caused by* d-, f-, g- *and* h *-waves, which are neglected in the latter case.*

describe correctly one of the most interesting features observed in the π - ^{3}He and π - ^{4}He differential cross sections - the fact that the minimum occurs at approximately the same angle over the whole resonance region. (Rather than in the angular variable, the dip position is a constant in transferred momentum in the case of heavier nuclei). The role of spin-flip and charge-exchange effects is also shown in *Fig. 2*. If the nuclear recoil correlations are "turned-off", changes in the calculated results are surprisingly large.

(ii) Charge exchange reaction ^{3}H(π^{+},π^{0}) ^{3}He. The calculated total cross section is shown in *Fig. 3*. If the correlations are neglected, a quite different result is obtained for $E_{\pi} < 100$ *MeV*. Unfortunately, this interesting reaction was not yet studied experimentally. Our

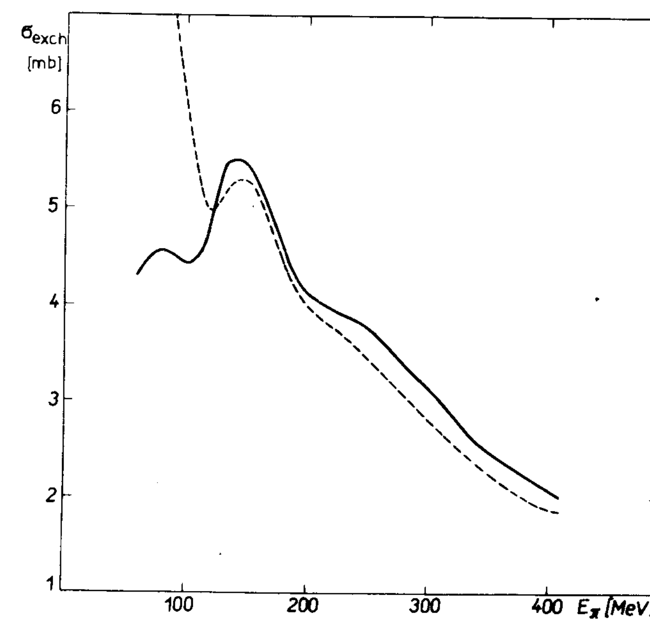

Fig. 3. Total cross section of ^{3}H(π^{+},π^{0}) ^{3}He *reaction. Solid line denotes the full Glauber model result. The dashed one represents a result of neglecting the recoil correlations. Six* πN *waves were considered.*

calculations predict somewhat bigger value of the cross section in the peak region and more rapid decrease for $E_\pi > 200$ *MeV* compared to the results obtained in /11,22/. The difference is caused by different parametrization of the pion-nucleon amplitude and by the fact that some spin-flip terms were neglected in /11,22/. There is a quite good agreement between our prediction and the optical model result as obtained utilizing eq. (17) (see /4/).

(iii) T o t a l c r o s s s e c t i o n s. The calculated $\pi^+ - {}^3He$ and $\pi^- - {}^3He$ total cross sections are shown in *Fig. 4*. It can be seen from *Fig. 4* that the recoil correlations produce a noticeable effect for energies $E_\pi < 200$ *MeV* only. For the sake of completeness, the Glauber model result for $\pi - {}^4He$ total cross section is drawn in

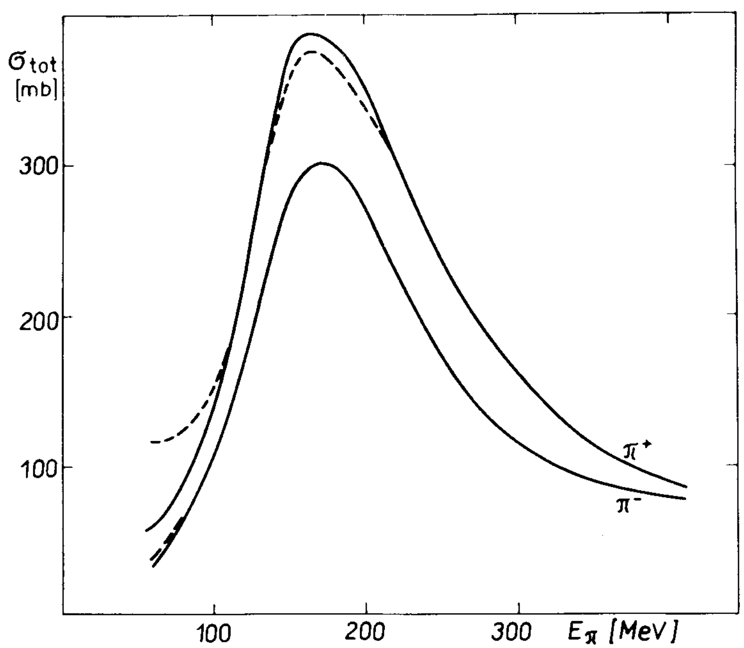

Fig. 4. Total $\pi^+ - {}^3He$ *and* $\pi^- - {}^3He$ *cross sections. The meaning of the curves is the same as in Fig. 3.*

Fig. 5. The spin and isospin dependent terms influence the cross section for $E_\pi < 200$ *MeV* considerably, as was pointed out earlier in /10/. The optical model curve, also shown in *Fig. 5,* is of similar shape as the Glauber

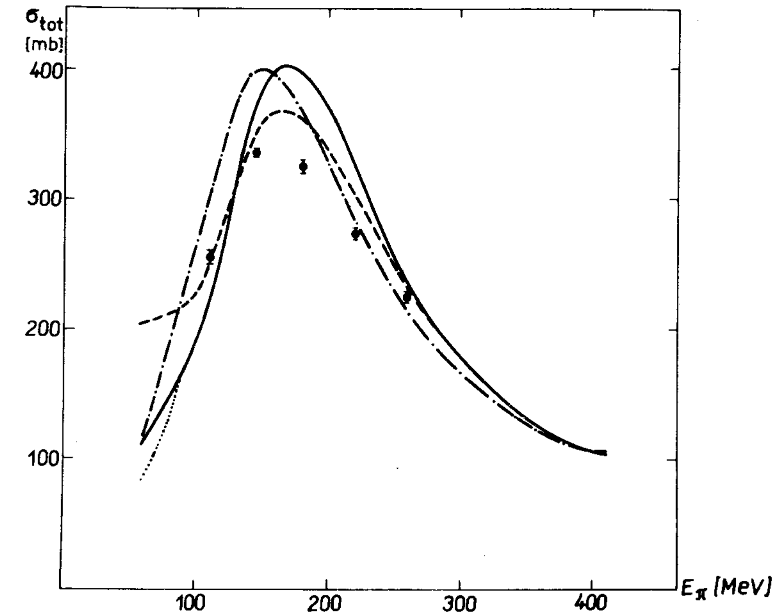

Fig. 5. Total $\pi - {}^4He$ *cross section. The results of the optical model, of the full Glauber model and of the Glauber model without spin-isospin effects are shown by dot-dashed, solid and dashed lines, respectively. Six* πN *waves were considered in the Glauber model. The full Glauber model result obtained using* s– *and* p*-waves only is given by dotted line. Experimental data were taken from* /24/.

model result. The measured maximum is smaller than the calculated ones. If the correlations were neglected, we obtained a total cross section curve (see *Fig. 6*), which differs substantially from those displayed in *Fig. 5*.

Another interesting observation can be made in *Figs. 2* and *5*. Pion-nucleon s– , p– , d–, f–, g– and h-partial waves were taken into account in constructing the Glauber model amplitude according to eqs. (10-12). If the predominating s– and p– waves are retained only, different results are obtained in the *small* energy region $E_\pi \lesssim 100$ *MeV* *. The effect of higher partial waves was described at first in connection with the reaction $^{18}O(\pi^+,\pi^-)^{18}Ne$ by Liu and Franko /23/. Unlike the authors of /23/, who concluded that higher partial waves represent an important constituent of the scattering amplitude in the region $E_\pi \sim 100$ *MeV*, we assume the effect to be spurious. This effect occurs due to the fact that higher powers of the transferred momentum q are not damped effectively enough by the Gaussian in eq. (12) for small energies. In fact, the integrals in eq. (12) are of the type

$$I = \int_0^\infty J_i(bq)\, e^{-\frac{q^2 a_0^2}{4}} P_\ell(x)\, q^{i+1} dq\,, \quad i = 0, 1, \qquad (25)$$

where $x = 1 - q^2/2p_{2c}^2$. It can be easily shown that if $(p_{2c}a_0) \lesssim 1$, the Glauber model results depend strongly on the parametrization chosen for the πN amplitude, since the contribution of even those q will be significant in eq. (25) for which $x < -1$ occurs. For higher partial waves the spurious effects will be, of course, more serious. Therefore the Glauber model is hardly adequate in the energy regime, where $p_{2c} \lesssim 1/a_0$ holds (at least in the case if the πN amplitude is expected to be a function of transferred momentum only). The inequality $(p_{2c}a_0) \gg 1$ represents some minimum condition, since the Glauber model can be applied with safety only when $(p_{lab}\, r) \gg 1$ /20/. Here, r characterizes the range of pion-nucleon interaction.

It is instructive to evaluate $(p_{2c} a_0)$ at $E_\pi = 100$ *MeV*. We have 1.34, 1.13 and 1.53 for ^3He, ^4He and ^{12}C, respectively. Since the condition $(p_{2c}\, a_0) \gg 1$ is fulfilled less well in the case of ^4He nucleus, somewhat better

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* The effect is much stronger if the recoil correlations are neglected.

results obtained for $\pi^\pm - {}^3He$ scattering are not surprising. Moreover, model dependent multiple-scattering terms give a more significant contribution to the $\pi - {}^4He$ cross sections compared to the $\pi - {}^3He$ ones.

Now, we can explain qualitatively also the remarkable sensitivity of the Glauber model results to the nuclear recoil correlations. It should be remainded that if the recoil correlations are neglected, the parameter a_0^2 is substituted by $(A-1)a_0^2/A$ in eqs. (12). Therefore, validity of the condition $(p_{2c}\, a_0)\sqrt{(A-1)/A} \gg 1$ is to be examined. This condition is fulfilled still worse than the previous one. Thus we may conclude that at least some part of the correlation effects reported at the beginning of this section is spurious. This statement is expected to be valid especially for $\pi - {}^4He$ scattering and for energies $E_\pi \lesssim 100$ *MeV*.

One of the most important results of this paper is obtained, if a more detailed comparison of the Glauber and the optical model is carried out. Such a comparison is more instructive and easier to perform for $\pi - {}^4He$ scattering. Total cross section calculated using the optical model is compared in *Fig. 6* with the "eikonalized" optical model result and with the Glauber model prediction obtained neglecting the correlation, spin-flip and charge-exchange effects. The three curves differ widely for $E_\pi < 100$ *MeV*, the "eikonalized" optical model result is very similar compared to the Glauber model prediction in the energy interval 100-200 *MeV* and all the curves practically coincide for $E_\pi > 200$ *MeV*. Having in mind the results of our discussion contained in section 3, the following conclusions can be drawn. Validity of the eikonal approximation for $E_\pi < 200$ *MeV* is very doubtful. Provided that the eikonal approximation is used, the intermediate nuclear excitations seem to be a rather unimportant constituent of the theory for $E_\pi > 100$ *MeV*. Finally, our version of the Glauber model is probably quite inadequate for energies $E_\pi < 100$ *MeV*. A large instability of the Glauber model concerning to the various corrections studied in this paper provides an additional argument for the validity of the last statement.

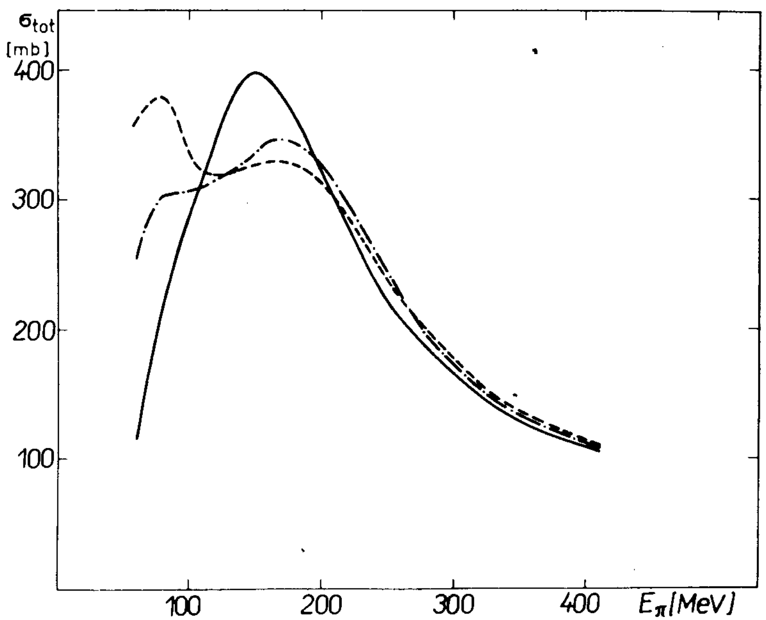

Fig. 6. Total $\pi - {}^4He$ cross section. The results of the full optical model, of the "eikonalized" optical model and of the Glauber model without correlations and spin-isospin effects are shown by solid, dot-dashed and dotted lines, respectively. Pion-nucleon s- and p-waves were considered only.

It is important to note that the marginal validity of the eikonal approximation for $E_\pi < 200$ *MeV* does not necessarily mean that the Glauber model breaks down completely. As was mentioned, the delicate cancellation which occurs in the model may cause that the full Glauber model result describes experiment quite well. Such an eventuality probably takes place for $\pi - {}^3He$ and $\pi - {}^4He$ elastic scattering in the energy interval 100-200 *MeV*. The mechanism of cancellation is not very well understood, especially if the spin-flip and charge-exchange effects are taken into account and we postpone this problem to further studies.

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