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D.Janssen, R.V.Jolos

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I. Introduction

In paper /1/ spectra of collective excitations of isotopes $^{190,192,194}Pt$ were investigated through the (α, xn) reactions. The following results obtained in /1/ are of most interest:

1. At angular momenta $I = 12$ in $^{190,192}Pt$ and $I = 10$ in ^{194}Pt a smooth growth of the energy difference $(E(I+2) - E(I))$ with increasing I is breaking for levels of the ground state quasi-rotational bands.

2. E2-transitions $12^+ \to 10^+$ in $^{190,192}Pt$ and E2-transition $10^+ \to 8^+$ in ^{194}Pt are slowed down as compared with the pure collective transitions. This slowing down is significant in ^{194}Pt.

3. In $^{190,192}Pt$ near the state 12^+ several almost degenerated in energy levels are found with $I^\pi = 10^+$.

II. *Spherical solutions of the Schrodinger equation with the collective Hamiltonian for which the potential energy does not depend on γ and has a minimum at $\beta \neq 0$*

Consider the quadrupole collective Hamiltonian with terms up to the fourth order in powers of phonon operators. We require that the seniority v be a good quantum number. This means that the collective Hamiltonian should not contain anharmonic terms of the third order:

$$H_{coll.} = c_1 \sum_\mu b^+_{2\mu} b_{2\mu} + c_2 \left(\sum_\mu (-1)^\mu b^+_{2\mu} b^+_{2-\mu} + h.c.\right) +$$

$$+ c_{41}(\sum_\mu (-1)^\mu b^+_{2\mu} b^+_{2-\mu} \cdot \sum_\nu (-1)^\nu b^+_{2\nu} b^+_{2-\nu} + \text{h.c.}) +$$

$$+ c_{43}(\sum_\mu (-1)^\mu b^+_{2\mu} b^+_{2-\mu} \cdot \sum_\nu b^+_{2\nu} b_{2\nu} + \text{h.c.}) +$$

$$+ \sum_{L=0,2,4} c_{4L} [[b^+_2 b^+_2]_L [b_2 b_2]_L]_{00} . \qquad (1)$$

By applying the linear canonical transformation for the operators $b^+_{2\mu}$, $b_{2\mu}$ Hamiltonian (1) can be transformed so that in terms of new phonon operators it will not include the term

$$\sum_\mu (-)^\mu b^+_{2\mu} b^+_{2-\mu} \cdot \sum_{\mu'} (-)^{\mu'} b^+_{2\mu'} b^+_{2-\mu'} + \text{h.c.}$$

Therefore, without loss of generality one can put $c_{41} = 0$.

The eigenstates of Hamiltonian (1) are specified by the following quantum numbers:

I, M the angular momentum and its projection;

v the seniority, Ω an additional quantum number.

The spectrum here is degenerated with respect to Ω and the degeneracy multiplicity n_Ω equals /2/:

$$n_\Omega = [(2v - I - 3\frac{1-(-1)^I}{2})/6] - [(v - I + 2)/3] + 1 - \delta_{I,2v-I} ,$$

where $[A]$ means the integer part of A.

In the general case the eigenstates of Hamiltonian (1) are a superposition of states with different numbers of phonons N. However, if the coefficients c_2 and c_{43} are of different signs, among the eigenstates there can appear also special states with fixed number of phonons N_0:

$$\Psi = |N_0, v = N_0, \Omega, I, M> . \qquad (2)$$

The value N_0 is determined by values of the constants c_2 and c_{43}.

Let us show that the states (2) are eigenstates of Hamiltonian (1). In Hamiltonian (1) the number of phonons is changed by the following terms

$$c_2(\sum_\mu (-)^\mu b^+_{2\mu} b^+_{2-\mu} + \sum_\mu (-)^\mu b_{2\mu} b_{2-\mu}) +$$

$$+ c_{43}(\sum_{\mu\nu} (-)^\mu b^+_{2\mu} b^+_{2-\mu} b^+_{2\nu} b_{2\nu} + \sum_{\mu\nu} (-)^\mu b^+_\nu b_\nu b_{-\mu} b_\mu . \qquad (3)$$

Since N_0 is the minimal number of phonons for the state with $v = N_0$, then the result of action of (3) on vector (2) is as follows:

$$(c_2 + c_{43} N_0) \sum_\mu (-)^\mu b^+_{2\mu} b^+_{2-M} | N_0, v = N_0, \Omega, I, M> .$$

If $N_0 = -\frac{c_2}{c_{43}}$, the state (2) is the eigenvector of Hamiltonian (1) with the eigenvalue

$$E_{N,I} = (c_1 + \frac{2}{7\sqrt{5}} c_{42} - \frac{3}{7} c_{44}) N_0 + (\frac{4}{7\sqrt{5}} c_{42} + \frac{1}{7} c_{44}) N_0^2 +$$

$$+ (\frac{1}{21} c_{44} - \frac{1}{7\sqrt{5}} c_{42}) I(I+1) . \qquad (4)$$

The ratio $|c_2/c_{43}|$ can be noninteger, as well. Then state (2) will include admixture of components with other numbers of phonons. However, these admixtures are negligible due to the factor of smallness $|(N_0 + \frac{c_2}{c_{43}})| < 1$ which weakens the interaction for states with $N = N_0$.

For nuclei, for which the potential energy of quadrupole oscillations has maximum at $\beta = 0$ and minimum at $\beta \neq 0$, the coefficient c_2 is negative and c_{43} positive. If in these nuclei the potential energy is γ-independent, then there exist the states of type (2). These are the pure spherical states with fixed number of phonons. Here there appears a set of such states with different I. Thus, there occurs the whole band of the spherical states, in contrast to the most of other eigenstates of (1) which wave functions are concentrated at $\beta \neq 0$ (around minimum). As is clear from (4) the energies of states of the spherical band depend on I in the same way as the energies of states of the deformed rotational band. If the coefficient $(\frac{1}{21} c_{44} - \frac{1}{7\sqrt{5}} c_{42})$ is small enough, the spherical band crosses the ground state band and the effect of "back bending" will be observed in the dependence of the moment of inertia on the rotation frequency. If the value of seniority for the spherical band is considerably larger than that for the ground state band at the intersection, then there appears as isomeric state due to the forbiddenness in seniority for E2-transitions. Besides, the states of the spherical band at sufficiently large N_0 and I are degenerated with multiplicity n_Ω.

In the following section we make use of the results obtained to interpret the experimental data of ref. /1/.

III. Collective states in ^{190,192,194}Pt isotopes

To calculate energies of the collective quadrupole excitations in ^{190,192,194}Pt isotopes, we use the Hamiltonian proposed in ref. /3/:

$$H = w_{21} \sum_\mu b^+_{2\mu} b_{2\mu} +$$

$$+ w_{20} ([b^+_2 b^+_2]_{00} \sqrt{(1 - \frac{\sum_\mu b^+_{2\mu} b_{2\mu}}{\omega})(1 - \frac{\sum_\mu b^+_{2\mu} b_{2\mu} + 1}{\omega})} + h.c.) +$$

$$+ w_{31} ([b^+_2 b^+_2 b_2]_{00} \sqrt{1 - \frac{\sum_\mu b^+_{2\mu} b_{2\mu}}{\omega}} + h.c.) +$$

$$+ \sum_{I=0,2,4} w_{42I} [[b^+_2 b^+_2]_I [b_2 b_2]_I]_{00}. \tag{5}$$

Here ω is a positive integer. As was pointed out in ref. /3/, it can be put to equal approximately half of number of nucleons in the nonfilled shells of the nucleus.

For ^{190}Pt we have $\omega = 9$. In what follows we make use of this value also for calculations of ^{192,194}Pt. It was checked that sligth changes in ω do not influence, in principle, the calculation results.

The numerical values of the coefficients w_{21}, w_{20}, w_{31}, w_{42I} were fiexd to minimalize the deviations of theoretical energy values from the experimental ones for the first six low-lying states 2^+_1, 2^+_2, 4^+_1, 4^+_2, 3^+_1, 0^+_1. The results are presented in *Table* I. From *Table I*

Table 1

The values of the coefficients w_{21}, w_{20}, w_{31}, w_{42I} *(in KeV) and quantity* $\chi = \sqrt{\sum_{i=1}^{6} \frac{(E^i_{exp} - E^i_{th})^2}{(E^i_{exp})^2}}$ *in isotopes* ^{190,192,194}Pt

Nuclei	^{190}Pt	^{192}Pt	^{194}Pt
W_{21}	14,3	-73,3	123,0
W_{20}	-1505,9	-1828,4	-1682,7
W_{31}	-23,2	-4,7	-1,56
W_{420}	57,7	97,7	-146,9
W_{422}	-121,6	-105,2	-122,1
W_{424}	39,1	-10,3	-22,3
χ	0,11	0,26	0,32

it is seen that the coefficient $|w_{31}|$ is considerably smaller than $|w_{20}|$ and therefore its contribution may be neglected. If, in addition, in (5) we neglect by $\frac{1}{\omega} = \frac{1}{9}$ as compared to unity in (5) under the square-root sign then Hamiltonian (5) can be rewritten as follows

$$H = w_{21}\sum_{\mu} b^{+}_{2\mu} b_{2\mu} + w_{20}\left([b^{+}_{2} b^{+}_{2}]_{00}\left(1-\frac{1}{\omega}\sum_{\mu} b^{+}_{2\mu} b_{2\mu}\right) + \text{h.c.}\right) +$$

$$+\sum_{I=0,2,4} w_{42I}\,[[b^{+}_{2} b^{+}_{2}]_{I}\,[b_{2} b_{2}]_{I}]_{00}\,. \qquad (6)$$

This expression is equivalent to Hamiltonian (1), and the states of the spherical band with $N_0 = 9$ are the eigenstates of Hamiltonian (6) with the energy

$$E = 9\left(w_{21} + \frac{2}{7\sqrt{5}} w_{422} - \frac{3}{7} w_{424}\right) +$$

$$+\left(\frac{4}{7\sqrt{5}} w_{422} + \frac{1}{7} w_{424}\right) 81 + \left(\frac{1}{21} w_{424} - \frac{1}{7\sqrt{5}} w_{422}\right) I(I+1).$$

The degeneracy of the states of the spherical band at $N_0 = 9$ are given in *Table 2.*

To answer the question whether the spherical band intersects with the ground state one in ^{190,192,194}Pt isotopes we have found the eigenvalues of Hamiltonian (5) for states with large spins. The values of coefficients were taken from *Table 1.*

Table 2

The values of angular momenta and degeneracy multiplicities of states of the spherical band at $N = V = 9$

I^{π}	0^+	3^+	4^+	6^+	7^+	8^+	9^+	10^+	11^+	12^+	13^+	14^+	15^+	16^+	18^+
n_{Ω}	1	1	1	2	1	1	2	2	1	2	1	1	1	1	1

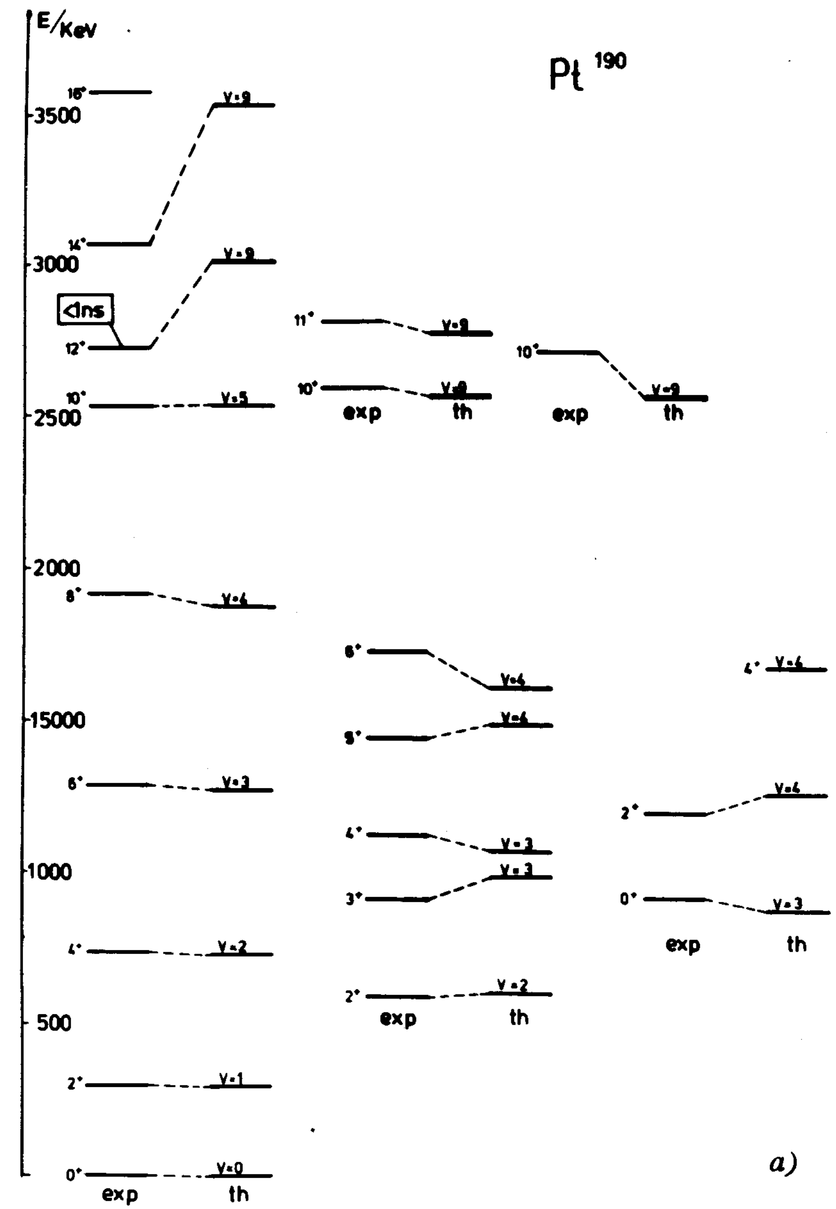

Fig. 1. Theoretical and experimental values of energies of collective states in ^{190,192,194}Pt.

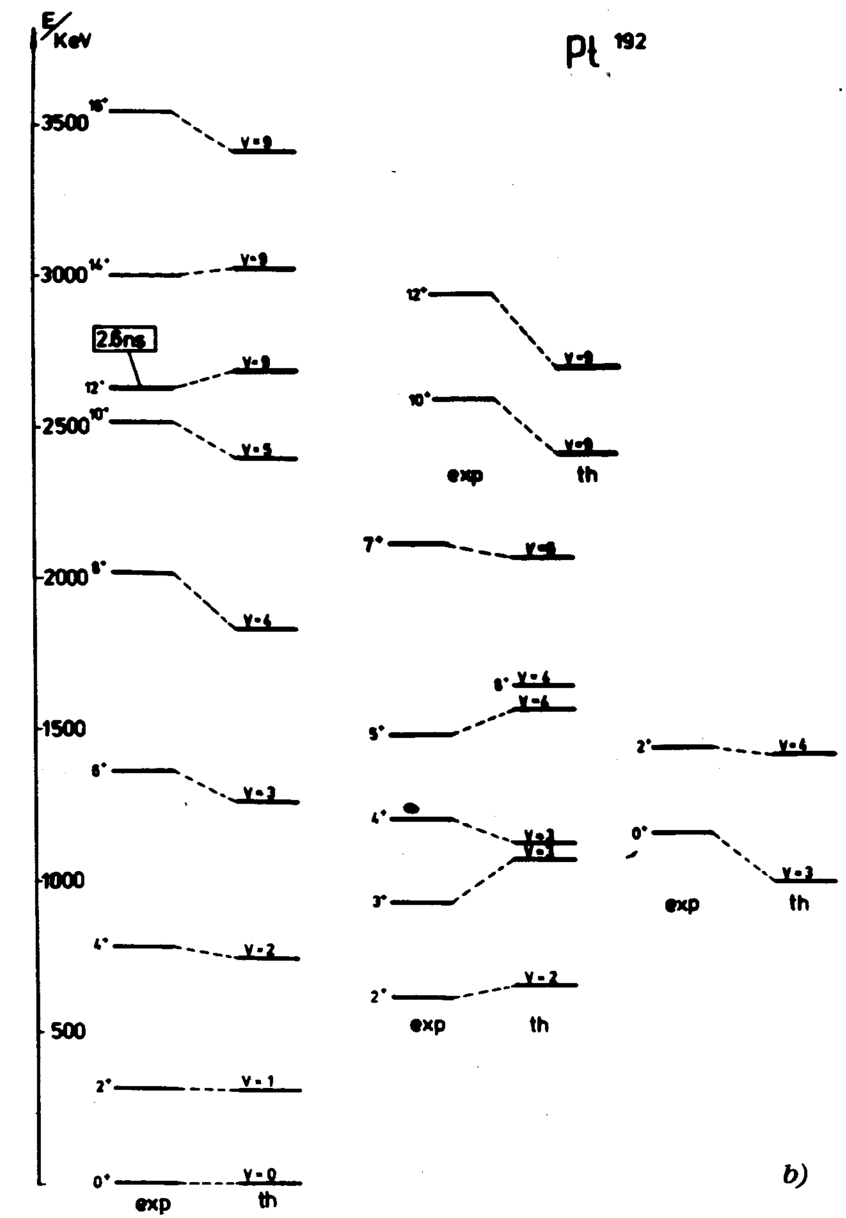

Fig. 1b.

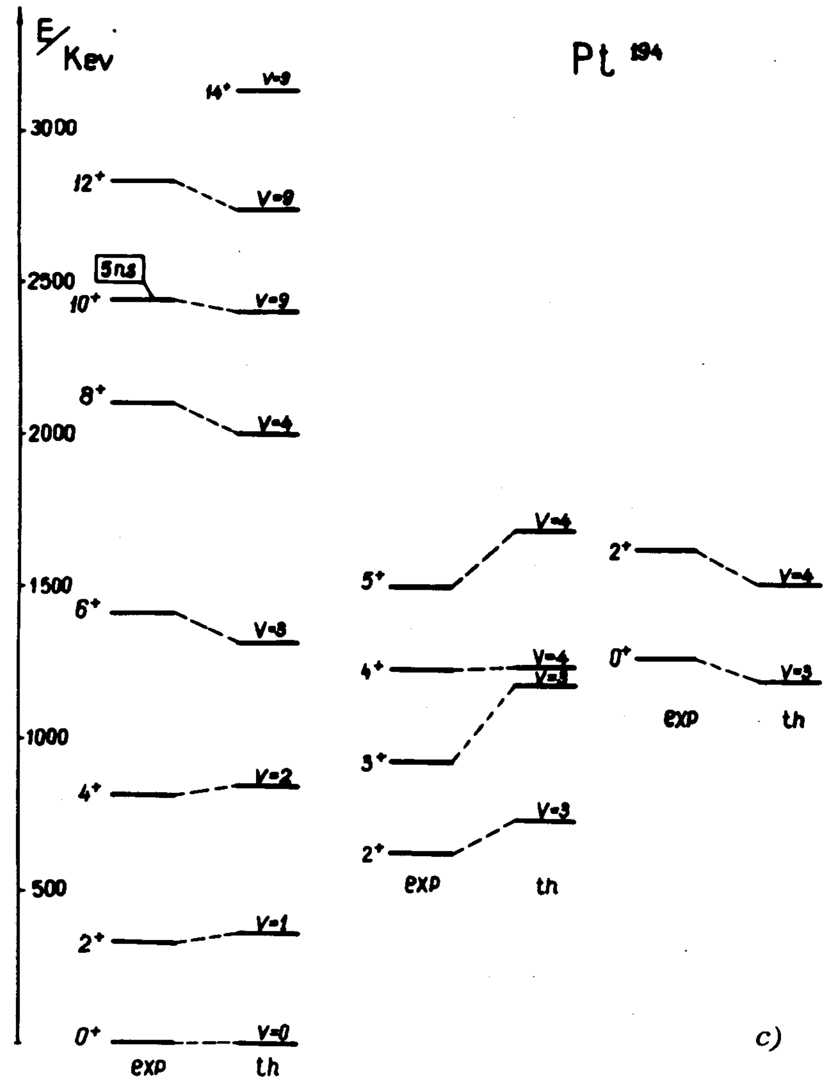

Fig. 1c.

The calculation results are shown in *Fig. 1* only with the states of the spherical bans belonging to the irrast-line. *Figure 2* shows the wave functions of the states belonging to the irrast-line in ^{190}Pt. It is seen that up

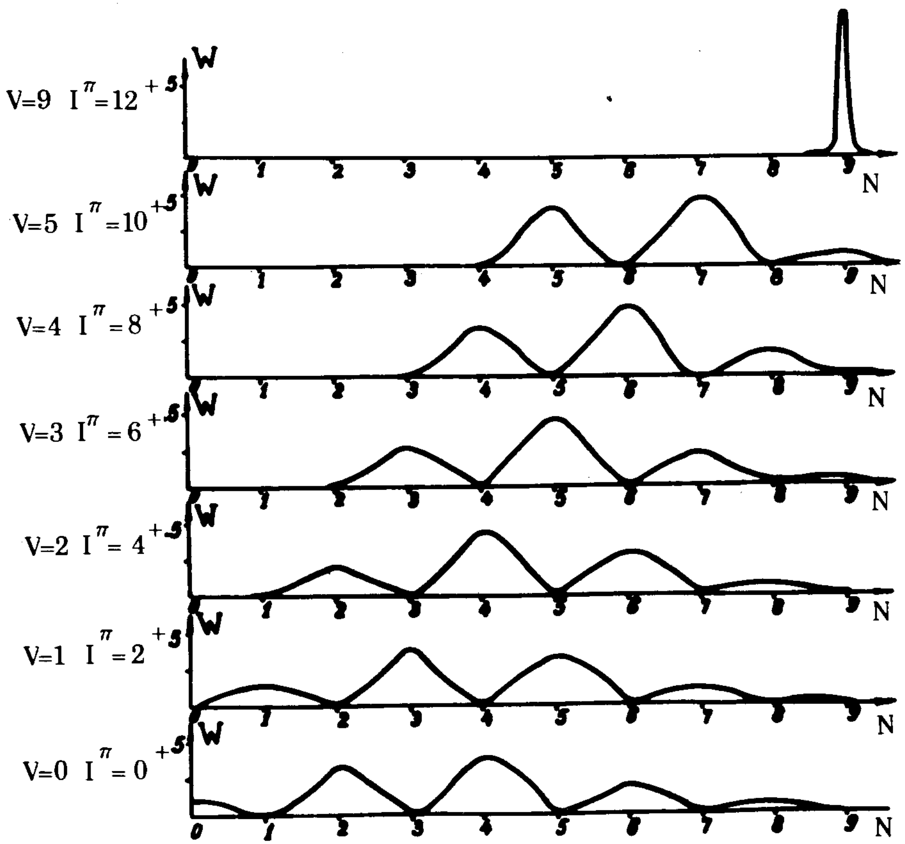

Fig. 2. The phonon structure of wave functions of collective states of the ground quasirotational band. N *is the number of phonons,* w *the relative contribution to the wave function norm.*

to spin $I = 10^+$ the irrast-line is composed of the states of the ground state quasirotational band, and from spin $I = 12^+$ there starts the spherical band. Due to large difference in the values of seniority ($\Delta v = 4$) between states 10^+ and 12^+ the E2-transition $12^+ \to 10^+$ is slowed down. It is interesting that in this nucleus there, in addition, are found two more almost degenerated in

energy 10^+ states. Their energies in practice coincide with the energies of the corresponding states of the spherical band. Experimentally there also is found the 11^+ state of the spherical band.

Analogous results are obtained for $^{192,194}Pt$ isotopes. The theory explains also the appearance of additional states with $I = 10^+, 12^+$ in ^{192}Pt and sharp jump in change of the energy difference $(E(I+2) - E(I))$ with increasing I in the ground quasirotational band.

From the results for states with small values of the angular momentum it follows that in the considered nuclei there is violated the rule of correspondence proposed in ref. /4/ for description of the quasirotational bands in transitional nuclei. The state lying in the beginning of the β-vibrational band has seniority $v = 3$ and not $v = 0$ as was supposed in /4/. The large value of seniority of states of the β-vibrational band results in sharp decreases of the probabilities of E2-transitions from this band to the ground state band one. This effect has been found experimentally /5/. The ratio $B(E2; 2_3^+ \to 0_1^+)/B(E2; 2_3^+ \to 0_2^+)$ in $^{190,192}Pt$ equals $2 \cdot 10^{-4}$ while in such transitional nuclei as ^{150}Sm, ^{152}Gd, , ^{98}Mo this ratio is about 0.5.

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