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NEW APPROACH TO THE OLD PROBLEM OF MUON STICKING IN μCF

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1 Introduction

Although a considerable effort has been made so far on the problem of muon sticking to helium produced in muon catalyzed fusion reactions, the continuing disagreement of the theoretical evaluations with experimental data demands further investigation (see [1-3] and references therein).

The aim of the present work is to avoid one of the questionable aspects of the theoretical treatments of the problem to date, namely use of the framework of the "sudden approximation" (SA)[4,5], which was an essential element of all previous calculations (see, for example, [4-12,3]). Because the fusion time ($\tau = 1/\Gamma_{in} < 10^{-20}$ s) is much shorter then the times characteristic for muonic processes ($\tau_\mu = 2.4\times10^{-17}$ s$/m_\mu = 1.2\times10^{-19}$s), it has been assumed that the sticking events of the type

$$dt\mu \to \mu\,{}^4\mathrm{He} + n \tag{1}$$

are sudden ($\tau/\tau_\mu \to 0$) and therefore that in evaluating ω_s one can apply the Migdal formula[13]

$$\omega_s = \mid \int \psi^*_{nl}(\vec{R})\, e^{-i\vec{q}\cdot\vec{R}}\, \psi_{in}(\vec{R})\, d\vec{R}\mid^2 . \tag{2}$$

This is the simple overlap integral between the initial ψ_{in} and the final wave function $\psi_f = \psi_{nl}\, e^{i\vec{q}\cdot\vec{R}}$ of the $(\mu\,{}^4\mathrm{He})_{nl}$ atom moving with the velocity $\vec{V}$ ($\vec{q} = m_\mu \vec{V}$), which is defined by the energy output of the reaction (1).

Here we analyze the application of the formula (2) to the case of the reaction (1). The key point is a computation of the time evolution for the muon wave packet

$$\psi_{in} = \psi(\vec{R}, t=0) \to \psi(\vec{R}, t \to \infty) = \psi_f$$

due to emission of the fusion neutron, by direct solution of the time-dependent three-dimensional Schrödinger equation.

2 Time-dependent equation

First we develop the time-dependent Schrödinger equation describing the process (1). Simple semi-classical considerations give a rough estimate for the time-dependent part of the interaction potential $\Delta U(\vec{R}, t)$ acting on the muon. Due to receiving the momentum $\Delta\vec{p} = \vec{q} = m_\mu \vec{V}$, the muon is subject to the mean force $\vec{F} \sim \Delta\vec{p}/\Delta t = m_\mu \vec{V}/\tau$ during the time $\Delta t = \tau$ of the fusion event (1). On the other hand, the force is equal to $\vec{F} = -\frac{\partial}{\partial z}\Delta U(\vec{R},t)\vec{n_z}$, which gives the following estimate for the interaction potential:

$$\Delta U \sim -\vec{F}\cdot\vec{R} \sim -\frac{m_\mu \vec{V}\cdot\vec{R}}{\tau}. \quad (3)$$

Here the z-axis direction $\vec{n_z}$ coincides with $\vec{V}$, the direction of the emitted neutron.

More precisely, the time-dependent potential $\Delta U(R,t)$ may be determined as

$$\Delta U(\vec{R},t) = -\frac{d\vec{p}}{dt}\vec{R} = -\frac{m_\mu V R\cos\theta}{\tau}\exp\{-\frac{t}{\tau}\}. \quad (4)$$

This is a consequence of the evolution of the muonic momentum during the fusion event (1):

$$\vec{p}(t) = m_\mu \vec{V}(1-\exp\{-\frac{t}{\tau}\}),$$

reproducing the limiting cases $\vec{p}(t=0)=0$ and $\vec{p}(t\to\infty)=\vec{q}$.

The fusion time $\tau = 1/\Gamma_{in}$ is determined by the nuclear width Γ_{in} of the compound nucleus $^5He^*$ for the initial state $(\mu\,^5He^*)$ in the transition (1). For fixing this value we use the parameter Γ_{in}=340 keV ($\tau = 1/\Gamma_{in} = 0.017$ m.a.u.) of the three-coupled-channels model $(dt, {}^5He^*, n\,{}^4He)$[14] giving the best fit to the experimental data for the fusion reaction $d + t \to {}^4He + n$.

3 Method of calculation

The method of global approximation on a subspace grid [15], recently applied to a number of stationary problems [16], is extended here to the time-dependent Schrödinger equation. Following the key idea of the papers [15,16] we seek a solution as an expansion

$$\psi^N(R,x,t) = \frac{1}{R}\sum_{lj}^{N} P_l(x)\,P_{lj}^{-1}\,\psi_j(R,t), \quad (5)$$

where $P_l(x)$ are the Legendre polynomials and P_{lj}^{-1} is the $N\times N$ matrix inverse to $\{P_l(x_j)\}$ defined on the grid $x_j = \{\cos\theta_j\}_1^N$, coinciding with the set of the nodes of the Gauss quadrature on $[-1,1]$. In this way the initial problem is reduced to the system of Schrödinger-type equations with respect to the vector $\vec{\psi}(R,t) = \{\psi_j(R,t)\}_1^N = \{\psi(R,x_j,t)\}_1^N$ of unknown coefficients in the expansion (5)

$$i\frac{\partial}{\partial t}\vec{\psi}(R,t) = \hat{H}(R,t)\,\vec{\psi}(R,t), \quad (6)$$

where

$$\hat{H}(R,t) = H_{kj}(R,t) = \{-\frac{1}{2}\frac{\partial^2}{\partial R^2} - \frac{2}{R} + \frac{m_\mu V R x_k}{\tau}\exp\{-\frac{t}{\tau}\}\}\delta_{kj}$$

$$+\frac{1}{2R^2}\sum_l l(l+1)\,P_l(x_k)\,P_{lj}^{-1}.$$

We seek a solution of the equation (6) on a discrete set of the points $R_{n'}$ and t_n, in the spatial $R_{n'} \in [0, R_m]$ and temporal $t_n \in [0, t_m]$ dimensions. For propagation in time $t_n \to t_n + \Delta t$ the simple Crank-Nickolson scheme has been used

$$(1+\frac{i}{2}\Delta t\,\hat{H}(R,t_n))\,\vec{\psi}(R,t_n+\Delta t) = (1-\frac{i}{2}\Delta t\,\hat{H}(R,t_n))\,\vec{\psi}(R,t_n), \quad (7)$$

which is stable, preserves unitarity and may be integrated over R by applying the implicit inverse technique.

Note that we analyze Eq (6) in a deep nonperturbative region because the diagonal part of the effective potential $\hat{H}(R,t)$ satisfies the following relation:

$$-\frac{2}{R} + \frac{m_\mu V R x_k}{\tau}\exp\{-\frac{t}{\tau}\} \simeq 10^3\,R x_k\exp\{-10^2\,t\} \gg 1$$

for $t_n \le t_m = 10\,\tau < 0.2$ m.a.u. Solving (7) imposes rather tough demands on the computational method. (Henceforth we use the muonic atomic units (m.a.u.) : $m_\mu = \hbar = e = 1$).

4 Results and discussion

Fig. 1 illustrates the evolution in time of the muon wave packet $\psi(\vec{R},t)$ due to the emission of the neutron in the transition (1). The real part $\Re\{\psi(R,x,t)\}$ of the wave packet, calculated by solving the system (6) with the initial condition $\psi(R,x,t=0) = \psi_{1s}(R)$, is presented here for a few points of t. By projecting the wave packet on the bound states $k=(nl)$ of the $(\mu\,^4He)_{nl}$ atom one can evaluate the sticking coefficient $\omega_s(k,t) = |<\psi_k\,|\,\psi(t)>|^2$ as a function of time. The calculated quantities $\omega_s(k,t)$ are given in Fig. 2, which demonstrates that the last point of integration over t, $t_m = 10\,\tau = 0.17$ m.a.u., is already in the asymptotic region of the reaction, and may be chosen as a final state of the transition (1) for a few initial states k giving the main contribution to the coefficient $\omega_s = \sum_k \omega_s(k)$.

The convergence of the calculated values $\omega_s(\Gamma_{in}, N)$ with respect to $N \to \infty$, as well as the dependence on the input parameter Γ_{in} of the developing model, is illustrated in Fig. 3. This figure demonstrates that in the limiting case $\tau = 1/\Gamma_{in} \rightarrow 0$ our approach gives the coefficient, $\omega_{1s}(\Gamma_{in} \geq 500$ keV, $N \geq 18)$, close to the SA result ω_{1s}^{SA}, calculated by the Eq. (2). The value $\omega_{1s}(\Gamma_{in}{=}340$ keV,$N \geq 18)$, calculated with the parameter $\tau{=}1/\Gamma_{in}$ of the three-coupled-channels model[14], exceeds the SA result only by a few percent.

The way proposed above for treating the initial sticking problem has advantages, compared with the standard SA procedure. Since the suggested computational scheme gives the muon wave packet $\psi(\vec{R},t)$ as a function of time, it makes possible evaluation of the energy radiated from the muon emitted during the fusion events, according to the formulas:

$$P = \int P(\Omega)\, d\Omega, \qquad (8)$$

$$P(\Omega) = \frac{\alpha^3}{3\pi} \int e^{i\Omega t} \,|< \psi(t) \mid \ddot{z} \mid \psi(t) >|^2\, dt. \qquad (9)$$

According to the Ehrenfest theorem, $< \psi \mid \ddot{z} \mid \psi >= -< \psi \mid \frac{\partial}{\partial z} U \mid \psi >$, one can estimate the quantity P as

$$P = \frac{2\alpha^3}{3} \int |< \psi \mid \frac{2z}{R^3} + \frac{m_\mu V}{\tau} e^{-t/\tau} \mid \psi >|^2 \, dt \simeq \frac{1}{3} \alpha^3\, m_\mu^2\, V^2\, \Gamma_{in}. \qquad (10)$$

The numerical evaluation confirms the above estimation with sufficient accuracy. The proportionality (10) of the radiated energy P to the decay width Γ_{in} permits us to exclude this parameter, and instead use the dependence $\omega_s(\Gamma_{in})$ of the sticking coefficient on the width Γ_{in} (see Fig. 3) to get the dependence $\omega_s(P)$ on the energy P. The calculated curve $\omega_{1s}(P)$ (Fig. 4) gives a new possibility for experimental analyses of the "initial sticking". By measuring the energy P, radiated during the fusion event (1), one can evaluate the corresponding value $\omega_{1s}(P)$ by using the theoretical curve presented in Fig. 4 . This approach gives also the possibility of evaluating the coefficients $\omega_k(P)$ for sticking in excited states $k \neq 1s$, as well as the spectral density of the radiation $P(\Omega) = \frac{\alpha^3}{3\pi} \int e^{i\Omega t} \,|< \psi \mid \frac{\partial}{\partial z} U \mid \psi >|^2\, dt$, which may permit, in principle, measurement of the time dependence of the perturbative interaction $\Delta U(R, x, t)$.

This approach gives also the energy and angular distributions for the muon emitted during the fusion event (1):

$$\frac{dW(E_k)}{dE_k} = \sum_l \mid \int \psi^*(\vec{R}, t \rightarrow \infty)\, \psi_{kl}(R)\, Y_{l0}(\hat{R})\, e^{-i\vec{q}\cdot\vec{R}}\, d\vec{R} \mid^2 , \qquad (11)$$

$$\frac{d^2W(E_k,\theta)}{dE_k d\theta} = 2\pi \mid \sum_l Y_{l0}^*(\hat{k}) \int \psi^*(\vec{R}, t \rightarrow \infty)\, \psi_{kl}(R) Y_{l0}(\hat{R})\, e^{-i\vec{q}\cdot\vec{R}}\, d\vec{R} \mid^2 \qquad (12)$$

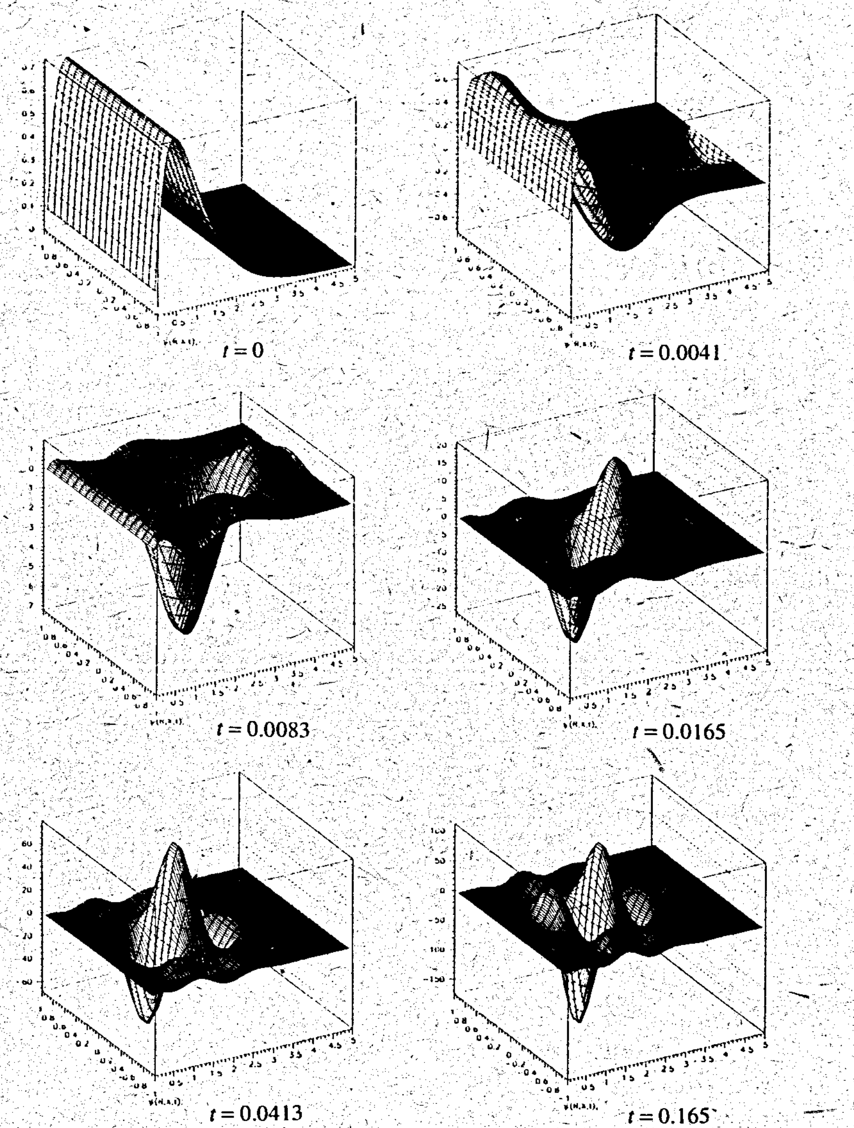

Fig. 1 Real part $\Re\{\psi(R, x, t)\}$ of the muon wave packet propagating from th bound state $(\mu\,^5\mathrm{He})_{1s}$ due to the neutron emission in the transition (1) The computation has been done in the spatial $[0, R_m] = [0, 5]$ and tem poral $[0, t_m] = [0, 10\,\tau]$ ($\tau = 1/340$ keV $= 0.017$ m.a.u.) dimensions witl the constant $\Delta t = \tau/200$, and $\Delta R = 0.025$ steps of integration.

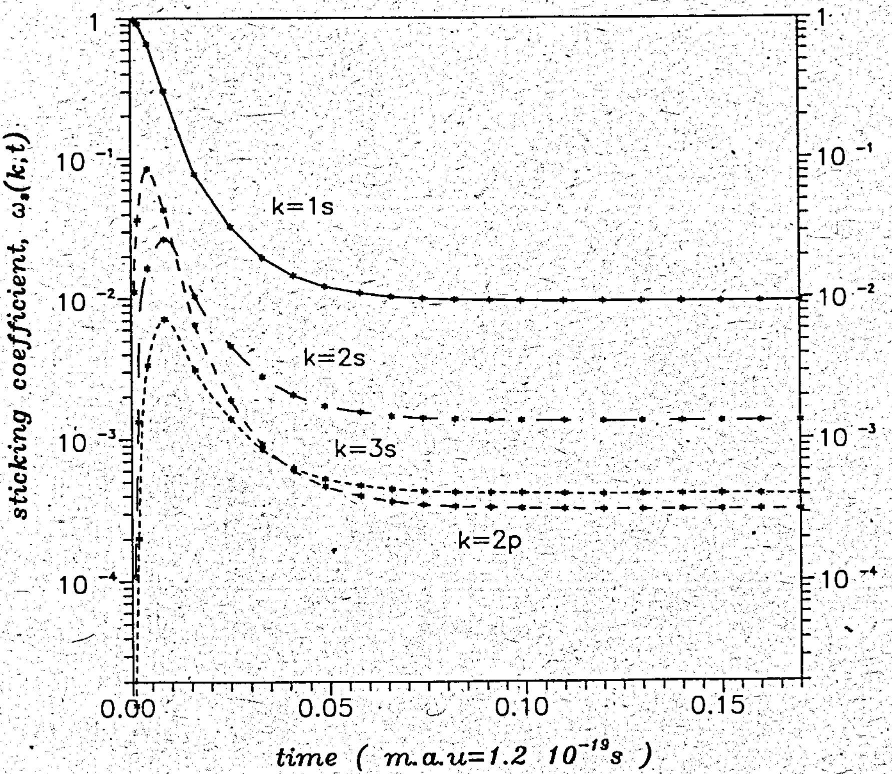

Fig. 2 The time evolution of the main partial sticking coefficients $\omega_s(k,t) = |< \psi_k | \psi(t) >|^2$ in the sum $\omega_s = \sum_k \omega_s(k)$.

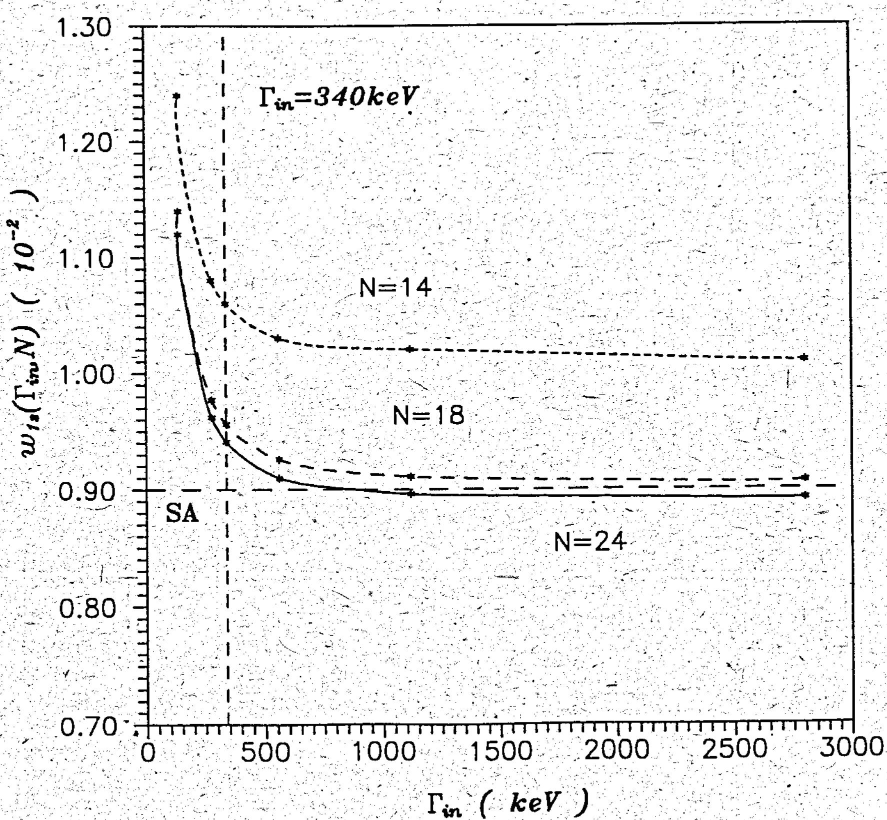

Fig. 3 The convergence with respect to $N \to \infty$ of the sticking coefficient $\omega_{1s}(\Gamma_{in}, N) = |< \psi_{1s} \mid \psi^N(t) >|^2$, calculated for a few values of the input parameter Γ_{in}. The points Γ_{in} =340 keV correspond to the value Γ_{in} taken from the three-coupled-channels model[14]. The points $\Gamma_{in} \geq$ 500 keV, $N \geq 18$ correspond to the limiting case $\tau = 1/\Gamma_{in} \to 0$ of SA (horizontal line "SA").

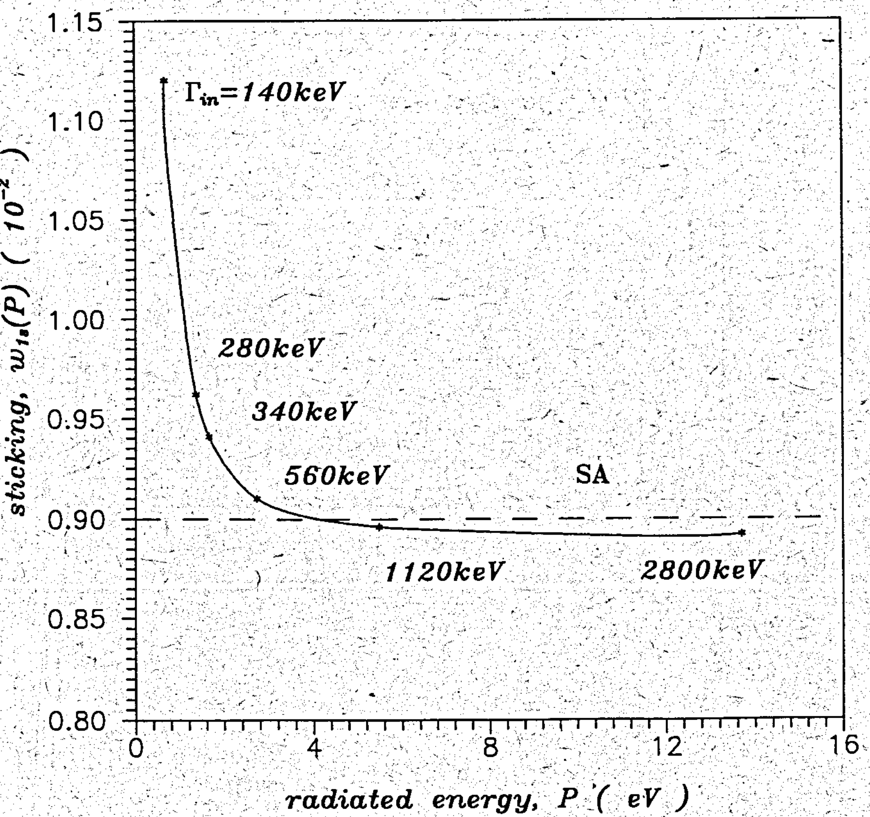

Fig. 4 The calculated dependence of the sticking coefficient $\omega_{1s}(P)$ on the energy P, radiated during the fusion event (1) due to the muon emission.

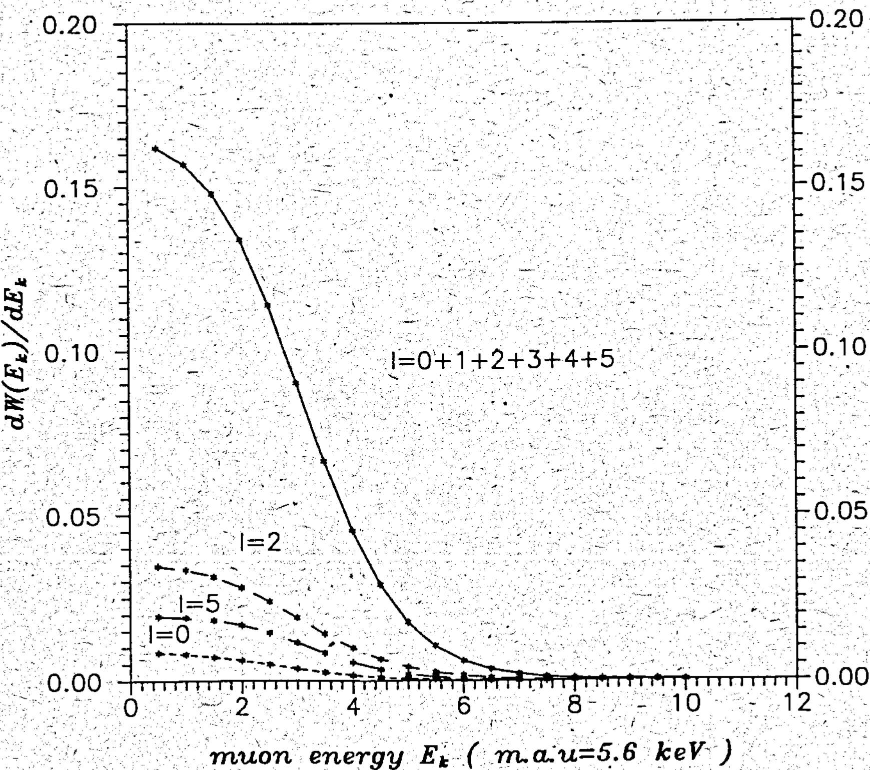

Fig. 5 The energy distribution for the muon emitted during the fusion event (1). Here only a few values of angular momentum l have been included in the summation formula (11).

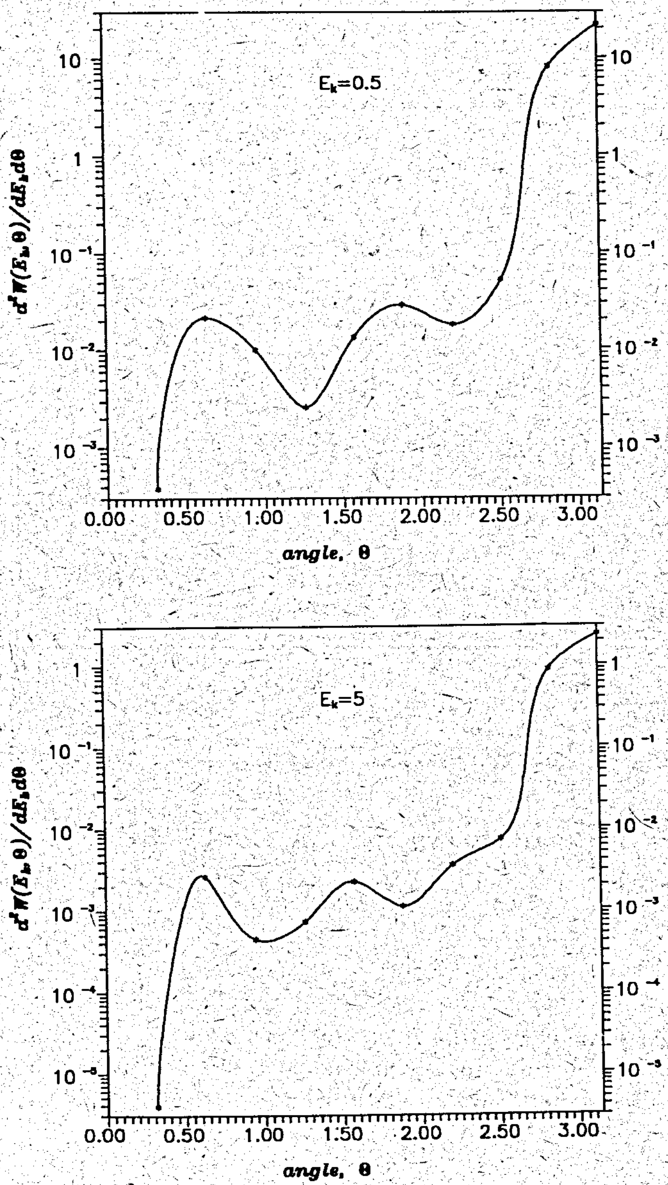

Fig. 6 The illustration of the angular distributions for the muon emitted during the fusion event (1) for the case of different muon energies E_k.

The curves calculated by virtue of Eq. (11) and (12) are presented in Figs. 5 and 6, respectively. These characteristics of the sticking process (1) are observable values, and for special conditions may be experimentally detectable.

The above calculations have been done with the initial condition $\psi(\vec{R}, t = 0) = \psi_{1s}(\vec{R})$, where $\psi_{1s}(\vec{R})$ was the wave function of $(\mu^5\mathrm{He})_{1s}$. But, it is known, that using more accurate wave function $\psi(\vec{R}) = \sum_n a_n \psi_{n0}(R) + \int a_k \psi_{k0}(R) dk$ of the muonic molecule $dt\mu$ instead of $\psi_{1s}(\mu^5\mathrm{He})$ decreases the initial sticking coefficients to $\sim 20 - 25\%$[7-11]. The developed approach permits such improvement in describing of the sticking processes by changing of the initial condition.

5 Conclusion

A new approach avoiding the framework of SA has been developed for the treatment of muon sticking. It has been shown that SA is a limiting case, $\tau = 1/\Gamma_{in} \to 0$, for the suggested approach. As the input parameter of the computational scheme is $\Gamma_{in} \geq 500$ keV, the SA result ω_s^{SA} has been reproduced with satisfactory accuracy for the reaction $dt\mu \to \mu^4\mathrm{He} + n$. For a realistic value of the parameter $\Gamma_{in} = 340$ keV of the three-coupled-channel model[14], the calculated ω_s exceeds SA result only by a few percent.

In addition, the new approach gives the energy and angular distributions for the emitted muon, as well as the dependence of the sticking coefficient on the energy, radiated during the muon catalyzed fusion event. These observable characteristics of the sticking processes have been calculated for the case of the reaction (1).

A special interest is in the dependence of the sticking coefficient $\omega_s(P)$ on the energy P, radiated during the fusion event, which may open a new possibility for experimental analysis of the sticking processes.

There are no limitations to extension of the results, obtained here for the reaction (1), to other muon catalyzed fusion processes.

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