ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E4-95-411

A.B.Arbuzov, O.O.Voskresenskaja, E.A.Kuraev

ON A MODEL FOR HADRONIC CURRENT IN $K_{\mu 3}$-DECAY

Submitted to «Письма в ЖЭТФ»

1995

Semileptonic decays of K and η mesons provide an ideal laboratory to study the strangeness violating $\Delta Q = \Delta S = -1$ hadronic currents:

$$\begin{aligned} \langle \pi^0(q)|V_\mu|K^+(k)\rangle &\sim (k+q)_\mu f_+(t) + (k-q)_\mu f_-(t), \quad t=(k-q)^2, \\ \langle K^-(q)|V_\mu|\eta(k)\rangle &\sim (k+q)_\mu f_+^1(t) + (k-q)_\mu f_-^1(t). \end{aligned} \tag{1}$$

Form factors $f_\pm$ and their linear combination

$$f_0(t) = f_+(t) + \frac{t}{M^2-m^2} f_-(t), \tag{2}$$

where M and m are the masses of kaon and pion respectively, are the subject of experimental [1–5] and theoretical [6–9,16,17] treatment during last decades [1].

In this paper we suggest a mechanism which permits to take into account the strong interaction effects in all orders of Chiral perturbation theory. Namely, omitting the excited vector meson states we may restrict ourselves with the $VV\varphi$ and φ^4 contributions from the effective Chiral lagrangian [13–16]:

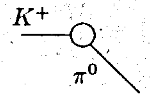
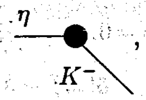
$$\begin{aligned} \mathcal{L} = &\frac{1}{2}\mathrm{Sp}\,(\partial_\mu\varphi)^2 - \frac{ig_{\rho\pi\pi}}{\sqrt{2}}\mathrm{Sp}\; V_\mu[\varphi,\partial_\mu\varphi] \\ &- \frac{1}{12f_\pi^2}\mathrm{Sp}\left((\varphi\partial_\mu\varphi)^2 - \varphi^2(\partial_\mu\varphi)^2\right) + \dots, \end{aligned} \tag{3}$$

where $f_\pi \approx 94$ MeV, $g_{\rho\pi\pi}$ is the coupling constant which determine the ρ meson width ($g^2_{\rho\pi\pi} \approx 32$). The octets of pseudoscalar and vector mesons are defined as

$$\varphi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}, \quad V_\mu = \begin{pmatrix} \frac{\rho+\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{-\rho+\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -\phi \end{pmatrix}_\mu . \tag{4}$$

Denoting graphically the amplitudes of ($K^+\pi^0$) and (ηK^-) transitions as empty and filled circles:

[1] The width of the semileptonic decay $\eta \to K e\nu$ is unfortunately too small ($\sim G^2(M_\eta - M)^5 \approx 3\cdot 10^{-11}$ eV) to be measured [17].

$M(K^+,\pi^0) \leftrightarrow$  and $M(\eta,K^-) \leftrightarrow$ ,

we put the system of graphical equations for them in fig. 1.

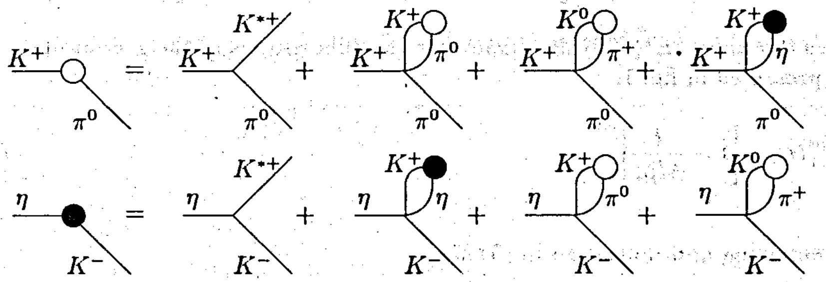


Figure 1: Graphical equation for transition form factors.

The corresponding system of integral equations for form factors $f_\pm$, $f_\pm^1$ turns out to be a pure algebraic one:

$$\begin{aligned} f_+ &= f_+^0(t), \quad f_+^1 = f_+^0(t)\sqrt{\frac{3}{2}}, \\ f_- &= (a_{K^+K^-\pi^0\pi^0} + \sqrt{2}a_{K^0K^-\pi^+\pi^0})A_1 + a_{K^+K^-\eta\pi^0}A_2, \\ f_-^1 &= a_{K^+K^-\eta\eta}A_2 + (\sqrt{2}a_{K^0K^-\pi^+\pi^0} + a_{K^+K^-\eta\pi^0})A_1, \\ A_1 &= \frac{M-m}{M+m}I_\pi f_+(t) + (C + \frac{t}{(M+m)^2}I_\pi)f_-(t), \\ A_2 &= \frac{M-M_\eta}{M+M_\eta}I_\eta f_+^1(t) + (C - C_0 + \frac{t}{(M+M_\eta)^2}I_\eta)f_-^1(t). \end{aligned} \tag{5}$$

The integrals over loop momenta are divergent. In fact this divergence is absent due to form factors denoted in (5) by a_i. The uncertainties induced by them can be taken into account by the introduction of the subtraction constant C in the dispersion relation for them. Moreover we may consider the vertex functions in the scattering–lengths approximation:

$$a_{K^-K^0\pi^+\pi^0}=0,\quad a_{K^+K^-\pi^0\pi^0}=\frac{M^2}{4\pi^2 f_\pi^2}[-\frac{1}{12}(1-\frac{M}{M_{K^*}})^{-1}], \tag{6}$$
$$a_{K^+K^-\eta\eta}=\frac{3(M_\eta^2-M^2)}{4\pi^2 f_\pi^2}[-\frac{1}{12}+\frac{1}{2}\alpha_K],$$
$$a_{K^+K^-\eta\pi^0}=-\frac{\sqrt{3}M(M_\eta+M)}{4\pi^2 f_\pi^2}[-\frac{1}{12}+\frac{1}{2}\alpha_K],$$
$$a_{K^-K^0\eta\pi^+}=-\frac{\sqrt{6}M(M_\eta+M)}{4\pi^2 f_\pi^2}[-\frac{1}{12}+\frac{1}{2}\alpha_K],\qquad \alpha_K=\frac{g_{\rho\pi\pi}^2 f_\pi^2}{M_{K^*}^2}=0.36.$$

We use the *naive* $K^*(890)$ dominance for tree-like graphs in the graphical equations presented in fig. 1:

$$f_+^0(t)=\left[1-\frac{t}{M_{K^*}^2}\right]^{-1}. \tag{7}$$

The remaining notations used in (5) are:

$$I_\pi=I_\pi\Big(\frac{t}{(M+m)^2}\Big),\qquad I_\eta=I_\eta\Big(\frac{t}{(M+M_\eta)^2}\Big), \tag{8}$$
$$I_i(z)=\int_1^\infty\frac{dx}{x^2(x-z)}\lambda_i^{\frac{1}{2}}(x),\qquad \lambda_\pi(x)=\lambda(x,\frac{m^2}{(m+M)^2},\frac{M^2}{(m+M)^2}),$$
$$\lambda_\eta(x)=\lambda(x,\frac{M_\eta^2}{(M_\eta+M)^2},\frac{M^2}{(M_\eta+M)^2}),$$
$$C_0=\int_1^{(\frac{M_\eta+M}{m+M})^2}\frac{dx}{x^2}\lambda_\pi^{\frac{1}{2}}(x)+\int_{(\frac{M_\eta+M}{m+M})^2}^\infty\frac{dx}{x^2}(\lambda_\pi^{\frac{1}{2}}(x)-\lambda_\eta^{\frac{1}{2}}(x))\approx 0.47,$$
$$\lambda(a,b,c)=a^2+b^2+c^2-2ab-2ac-2bc.$$

and M_η, M_{K^*} are the masses of η and K^* mesons respectively. To determine the subtraction constant C we use the Callan–Treiman relation [5]:

$$f_+(t_0)+f_-(t_0)=\frac{f_K}{f_\pi}=1.22,\quad t_0=M^2-m^2. \tag{9}$$

There are two values for the subtraction constant C from the numerical solution of (5,9):

$$C_1=-1.8,\qquad C_2=4.35. \tag{10}$$

Using these values we obtain for the experimentally observable quantities

$$\xi(0)=\frac{f_-(t)}{f_+(t)}\Big|_{t=0},\qquad \lambda_{+,0}=m^2\frac{d}{dt}f_{+,0}(t)\Big|_{t=0} \tag{11}$$

the results, which are given in the Table 1. We put there also the world average values [4]. On can see that at the value $C=-1.8$ there is reasonable agreement with the experimental data.

Table 1. Parameters $\xi(0)$, λ_+ and λ_0 for different choices of C in comparison with experimental data.

	$C=-1.8$	$C=4.35$	PDG [4]
$\xi(0)$	-0.14	-0.11	-0.35 ± 0.15
λ_+	0.024	0.024	0.033 ± 0.008
λ_0	0.017	0.020	0.004 ± 0.007

Note also that the experimental data from the $K_{\mu3}^0$ decay [4] differs considerably from the charged kaon decay data. Existing theoretical predictions [6–9] are also far from agreement. Further experimental investigations are desired.

Acknowledgement

We are grateful to A. Bel'kov, S. Gerasimov, V. Kurbatov, Z. Silagadze and to the participants of the JINR LTPh seminar for fruitful discussions. One of us (A.B.A.) is thankful to the Royal Swedish Academy of Sciences for the financial support via an ICFPM grant.

References

[1] L. Chounet, J. Guillard and M. Guillard, Phys. Rep. **C4** (1972) 199.

[2] R. Whiteman et al., Phys. Rev. **D21** (1980) 652.

[3] G. Donaldson et al., Phys. Rev. **D9** (1974) 2960.

[4] Particle Data Group, Phys. Rev. **D50** (1994) 1530.

[5] V.N. Bolotov, V. Kurbatov et al., (in preparation).

[6] C. Callan and S. Treiman, Phys. Rev. Lett. **16** (1965) 153.

[7] S.B. Gerasimov, Yad. Fiz. **29** (1979) 513.

[8] M.K. Volkov, V.N. Pervushin, D.I. Kazakov, Phys. Lett. **B64** (1976) 201.

[9] J. Gasser and H. Leutwiller, Nucl. Phys. **B250** (1985) 465.

[10] J.F. Donoqhue and B.R. Holtein, Phys. Rev. **D40** (1989) 2378.

[11] J. Gasser, in Proc. of Workshop on physics and detectors for DAFNE, 1991, p.291.

[12] J. Bijnens, G. Colangelo, G. Ecker and J. Gasser, preprint hep-ph/94–11–311 (1994).

[13] A.P. Bukhvostov, S.I. Kruglov and E.A. Kuraev, in Proc. of XXVI LIYaPh winter school, 1991, p.52.

[14] E.A. Kuraev and Z.K. Silagadze, Phys. Lett. **B292** (1992) 377.

[15] O. Kaymackalan, S. Radjeev and J. Schlechter, Phys. Rev. **D30** (1984) 594.

[16] S. Gaziorovich and D.A. Geffen, Rev. Mod. Phys.**41** (1969) 531.

[17] P. Singer, Phys. Rev. **B139** (1965) 483.