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ON ELECTROMAGNETIC TRANSITIONS BETWEEN HIGHLY EXCITED STATES OF DEFORMED ODD-A NUCLEI

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1 Introduction

The microscopic method of studing low–lying nonrotational states of deformed nuclei has been developed in papers by V.G.Soloviev [1]. It provided a good description of experimental data and a number of predictions that were later confirmed experimentally. Further, this model was applied to investigate the states of nuclei at intermediate and high excitation energies [2] and was called the quasiparticle–phonon nuclear model (QPNM) [3, 4]. Studies of nuclei in the energy region 2 MeV $< E <$ 8 MeV (here called intermediate and high energies) are highly promising since they provide a qualitatively new information on the structure of atomic nuclei. Some difficulties arisen are caused by a complicated structure of nuclear states and their high density, which requires further improvement of the theoretical methods.

A specific property of deformed nuclei is that one–phonon states with the same projection of momentum onto the nuclear symmetry axis K and parity π can be generated by different multipole (λ) and spin–multipole (λL) interactions with the projection $\mu = K$. Independent consideration of the phonons of different multipolarities ($\lambda\mu$ and $\lambda L\mu$) would result in the corresponding nonphysical increase of the number of states for each value of K. For low energies, this problem was not crucial since the influence of components with values of λ, different from the multipolarity of lowest collective quadrupole and octupole states, is insignificant and the states of electric and magnetic types can sometimes be considered independently.

A correct description of similar states with a higher excitation energy required introducing a phonon operator with a certain projection μ that contains the components of electric and magnetic types with different λ and λL [5]. In this report, a similar consideration is carried out within the QPNM in a slightly different way.

Another problem is connected with complication of the state structure and increasing their density with growing excitation energy. Therefore, traditional methods developed for the study of low–lying states are to be further improved. In this connection, it is rather promising to describe the cross section of excitation of states and transition probabilities by using the method of strength functions (SF) [2, 6]. So, for deformed nuclei, a microscopic description was developed for widths of multipole giant resonances, neutron strength functions, and the fragmentation of the simplest components of wave functions was studied [7, 8] through averaging over the states at intermediate and high excitation energies.

In this work, the SF method is applied to analyse γ-ray transitions between states both of which can be at intermediate or high excitation energies, with averaging over both intervals.

This consideration is useful in a number of physical problems, in particular, in studying $(n, 2\gamma)$ reactions [9], when compound states decay with a subsequent emission of two γ-quanta. Experimentally observed anomalies in the reaction cross section cannot be described statistically, and the microscopic interpretation of arising substructures requires considering γ-ray transitions between highly excited states.

In refs. [10–12], also certain substructures in the excitation cross sections of nuclear isomers were detected which correspond to γ-ray transitions between states at intermediate excitation energies. Attempts at theoretical description of similar structures in deformed nuclei by a direct calculation of transition probabilities between particular states seem hopeless in view of a large number of possible combinations of transitions and a limited accuracy of a theoretical description of energy spectra of these nuclei.

2 Model

The QPNM Hamiltonian for nonrotationl states of deformed nuclei is taken in the form [4]:

$$H = H_{s.p.} + H_p + H_M + H_S, \tag{2.1}$$

where $H_{s.p.}$ describes the mean field of neutron and proton systems as the Saxon–Woods potential, H_p is the monopole pairing, H_M and H_S are multipole and spin–multipole interactions between quasiparticles.

Introduce the operators of quasiparticle pairs in the generalized form [1, 13]:

$$A^+_{q_1q_2}(mn) = -\frac{1}{\sqrt{2}}n(\alpha^+_{q_1+}\alpha^+_{q_1n} + mn\alpha^+_{q_1-}\alpha^+_{q_2-n}), \tag{2.2}$$

$$B_{q_1q_2}(mn) = \alpha^+_{q_1+}\alpha_{q_2-n} - mn\alpha^+_{q_1-}\alpha_{q_2n}, \tag{2.2'}$$

$$B_q = \alpha^+_{q+}\alpha_{q+} + \alpha^+_{q-}\alpha_{q-},$$

where $\alpha^+_{q\sigma}$ is the creation operator of a quasiparticle with quantum numbers of a one-particle state q of the Saxon–Woods potential of a deformed nucleus, $\sigma = \pm 1; m = +1$ corresponds to operators of the electric type; $m = -1$, to operators of the magnetic type; $n = \pm 1$,to the transition operators with $\mu = | K_1 + nK_2 |$.

These operators in RPA obey the following commutation relations:

$$[A_{q_1'q_2'}(m'n'), A^+_{q_1q_2}(mn)] \cong \delta_{mm'}\delta_{nn'}(\delta_{q_1q_1'}\delta_{q_2q_2'} - nm^{(1-n)/2}\delta_{q_1q_2'}\delta_{q_2q_1'}), \tag{2.3}$$

$$[A_{q_1'q_2'}(m'n'), A_{q_1q_2}(mn)] = 0, \tag{2.3'}$$

$$[B_q, A^+_{q_1q_2}(mn)] = A^+_{q_1q_2}(mn)(\delta_{qq_1} + \delta_{qq_2}). \tag{2.3''}$$

The matrix elements of multipole moment

$$f^{\lambda\mu}(m) = \frac{1}{2}(Y_{\lambda\mu} + m(-1)^\mu Y_{\lambda-\mu})R_\lambda(r) \tag{2.4}$$

or spin–multipole moment

$$f^{\lambda L\mu}(m) = \frac{1}{2}((\mathbf{Y}_\lambda\boldsymbol{\sigma})_{L\mu} + m(-1)^\mu(\mathbf{Y}_\lambda\boldsymbol{\sigma})_{L-\mu})R_\lambda(r) \tag{2.4'}$$

over one-particle states q obey the following symmetry properties:

$$f^{\tau n\lambda\mu}_{q_1q_2}(m) = -nm^{(1-n)/2}f^{\tau n\lambda\mu}_{q_2q_1}(m), \tag{2.5}$$

$$f^{\tau n\lambda L\mu}_{q_1q_2}(m) = n^{L+\lambda+1}m^{(1-n)/2}f^{\tau n\lambda L\mu}_{q_2q_1}(m). \tag{2.5'}$$

The radial dependence of forces $R_\lambda(r)$ is usually taken to be either r^λ or $\partial V/\partial r$, where V is the central part of the Saxon–Woods potential, and $\tau = \{N, Z\}$ for neutron or proton systems. The introduced notation corresponds to [1, 4, 5]. In what follows, we will use the generalized expression $f^{\tau n\ell\mu}_{q_1q_2}(m)$ for matrix elements of the electric ($\ell = \lambda$) and magnetic ($\ell = \lambda L$) types.

Let us introduce the phonon creation operator:

$$Q^+_{\mu i}(m) = \frac{1}{2}\sum_{\tau nq_1q_2}\{\psi^{\tau n\mu i}_{q_1q_2}(m)A^+_{\tau q_1q_2}(mn) - \phi^{\tau n\mu i}_{q_1q_2}(m)A_{\tau q_1q_2}(mn)\} \tag{2.6}$$

and write the wave function of a one-phonon state in the form

$$Q^+_{\mu i}(m)\Psi_0, \tag{2.7}$$

where Ψ_0 is the phonon vacuum, being the ground state of an even–even nucleus. Here i is the number of a solution of the secular equation in RPA in the frame of which the phonon operators satisfy the commutation relations:

$$[Q_{\mu'i'}(m'), Q^+_{\mu i}(m)] = \delta_{\mu\mu'}\delta_{ii'}\delta_{mm'}, \tag{2.8}$$

$$[Q^+_{\mu'i'}(m'), Q^+_{\mu i}(m)] = [Q_{\mu'i'}(m'), Q_{\mu i}(m)] = 0. \tag{2.8'}$$

These relations provide the orthonormalization condition for the wave functions of ground and excited states and the functions $\psi^{\tau n\mu i}_{qq'}$ and $\phi^{\tau n\mu i}_{qq'}$ obey certain constraints.

If we take into account the above transformation, we can rewrite the operator of multipole moment in the form

$$M^{\tau\gamma\ell\mu}(m) = \frac{1}{\sqrt{2}}\sum_i D^{\tau\gamma\mu i}_\ell(m)\left(Q^+_{\mu i}(m) + \gamma_\ell mQ_{\mu i}(m)\right) + \sum_{nq_1q_2} f^{\tau n\lambda\mu}_{q_1q_2}(\gamma_\ell m)v^{(-\gamma)}_{q_1q_2}B_{q_1q_2}(mn), \tag{2.9}$$

where

$$D^{\tau\gamma\mu i}_\ell(m) = \sum_{nq_1q_2} f^{\tau n\lambda\mu}_{q_1q_2}(\gamma_\ell m)u^{\tau(\gamma)}_{q_1q_2}\left(\psi^{\tau n\mu i}_{q_1q_2}(m) + \gamma m\phi^{\tau n\mu i}_{q_1q_2}(m)\right); \tag{2.10}$$

$u^{(\pm)}_{qq'}, v^{(\pm)}_{qq'}$ are combinations of the Bogolubov uv-transformation coefficients, $\gamma = +1$ for electric transitions and $\gamma = -1$ for magnetic transitions.

3 RPA equations for even nuclei

Having performed transformations, the QPNM Hamiltonian (2.1) acquires the form:

$$H_{QPNM}=\sum_{q\tau}\varepsilon_q^{\tau}B_{\tau q}-\frac{1}{2}\sum_{\tau\rho\ell\mu m}(\kappa_0^{(\ell\mu)}+\rho\kappa_1^{(\ell\mu)})M^{\tau\gamma\ell\mu}(m)M^{\rho\tau\gamma\ell\mu}(m)\equiv H_q+H_Q+H_{qQ},\quad(3.1)$$

where H_q describes free quasiparticles; H_Q, their multipole and spin-multipole particle-hole interaction; H_{qQ}, the quasiparicle–phonon interaction; ε_q^{τ} is the energy of quasiparticles; $\rho=\pm1$. For deformed nuclei, isoscalar and isovector constants of multipole $(\kappa_0^{(\lambda\mu)},\kappa_1^{(\lambda\mu)})$ and spin–multipole forces $(\kappa_0^{(\lambda L\mu)},\kappa_1^{(\lambda L\mu)})$ depend on the projection μ of momenta λ and L onto the symmetry axis of a nucleus.

Calculating the average of H_q+H_Q over the wave function of a one–phonon state (2.7) and using the variational principle [4], we obtain the following system of equations for the energy $\omega_{\mu i}$ and coefficients of the wave function (2.7):

$$\left.\begin{array}{l}\varepsilon^{\tau}_{q_1q_2}g^{\tau n\mu i}_{q_1q_2}(m)-\omega_{\mu i}w^{\tau n\mu i}_{q_1q_2}(m)-2\sum_{\lambda}f^{\tau n\lambda\mu}_{q_1q_2}(m)u^{\tau(+)}_{q_1q_2}\sum_{\rho}(\kappa_0^{(\lambda\mu)}+\rho\kappa_1^{(\lambda\mu)})D^{\rho\tau\mu i}_{\lambda}(m)=0\\ \varepsilon^{\tau}_{q_1q_2}w^{\tau n\mu i}_{q_1q_2}(m)-\omega_{\mu i}g^{\tau n\mu i}_{q_1q_2}(m)-2\sum_{\ell=\bar\lambda L}f^{\tau n\ell\mu}_{q_1q_2}(\gamma_\ell m)u^{\tau(-)}_{q_1q_2}\sum_{\rho}(\kappa_0^{(\ell\mu)}+\rho\kappa_1^{(\ell\mu)})D^{\rho\tau\mu i}_{\ell}(m)=0\end{array}\right\}.\quad(3.2)$$

Here $\varepsilon^{\tau}_{q_1q_2}=\varepsilon^{\tau}_{q_1}+\varepsilon^{\tau}_{q_2},\gamma_\ell=(-)^{\bar\lambda+L}$,
$g^{\tau n\mu i}_{q_1q_2}(m)=\psi^{\tau n\mu i}_{q_1q_2}(m)+m\phi^{\tau n\mu i}_{q_1q_2}(m)$, $w^{\tau n\mu i}_{q_1q_2}(m)=\psi^{\tau n\mu i}_{q_1q_2}(m)-m\phi^{\tau n\mu i}_{q_1q_2}(m)$.
The normalization condition

$$\sum_{\tau q_1q_2}g^{\tau n\mu i}_{q_1q_2}w^{\tau n\mu i}_{q_1q_2}=2.\quad(3.3)$$

Using (2.12) we transform this system and derive the RPA equations for the energies $\omega_{\mu i}$ and quantities $D^{\tau\lambda\mu i}$:

$$\begin{pmatrix}(\kappa_0^{(\ell'\mu)}+\kappa_1^{(\ell'\mu)})X^{N\mu i}_{\ell\ell'}-\delta_{\ell\ell'} & (\kappa_0^{(\ell'\mu)}-\kappa_1^{(\ell'\mu)})X^{N\mu i}_{\ell\ell'}\\ (\kappa_0^{(\ell'\mu)}-\kappa_1^{(\ell'\mu)})X^{Z\mu i}_{\ell\ell'} & (\kappa_0^{(\ell'\mu)}+\kappa_1^{(\ell'\mu)})X^{Z\mu i}_{\ell\ell'}-\delta_{\ell\ell'}\end{pmatrix}\begin{pmatrix}D^{N\mu i}_{\ell'}\\ D^{Z\mu i}_{\ell'}\end{pmatrix}=0,\quad(3.4)$$

where

$$X^{\tau\mu i}_{\ell\ell'}=2\sum_{q_1q_2n}\frac{(\varepsilon^{\tau}_{q_1q_2}\delta_{\gamma_\ell\gamma_{\ell'},1}+\omega_{\mu i}\delta_{\gamma_\ell\gamma_{\ell'},-1})f^{\tau n\ell\mu}_{q_1q_2}(\gamma_\ell m)f^{\tau n\ell'\mu}_{q_1q_2}(\gamma_{\ell'}m)u^{\tau(\gamma')}_{q_1q_2}u^{\tau(\gamma)}_{q_1q_2}}{(\varepsilon^{\tau}_{q_1q_2})^2-\omega^2_{\mu i}}.\quad(3.5)$$

By solving this system we determine the quantities $g^{\tau n\mu i}_{q_1q_2}$ and $w^{\tau n\mu i}_{q_1q_2}$.

Now we shall calculate the reduced probability of $E\lambda$ and $M\lambda$ transitions:

$$B\big(E\lambda(M\lambda);0^+_{g.s.}\to(I^{\pi}K)_i\big)=(2-\delta_{\mu0})(00\lambda\mu\mid IK)e^2\left|M^{\lambda\mu i}\right|^2,\quad(3.6)$$

where the transition amplitude is given by

$$M^{\lambda\mu i}=\left\langle\Psi^*_i(K^{\pi})\middle|M(E\lambda(M\lambda)\mu)\middle|0\right\rangle=\frac{1}{\sqrt2}\sum_{\tau}e^{(\tau\lambda\mu)}_{eff}\sum_{\ell'}\tilde X^{\tau\mu i}_{\lambda\ell'}\tilde D^{\tau\mu i}_{\ell'},\quad(3.7)$$

$$\tilde D^{\tau\mu i}_{\ell}=\sum_{\rho}(\kappa_0^{(\ell\mu)}+\rho\kappa_1^{(\ell\mu)})D^{\rho\tau\mu i}_{\ell},\quad(3.8)$$

and $e^{(\tau\lambda\mu)}_{eff}$ is the effective charge for $E\lambda$ transitions.

The quantity $\tilde X^{\tau\mu i}_{\lambda\ell'}$ can be determined from (3.5) by substituting instead of $f^{\tau n\lambda\mu}_{q_1q_2}(m)$ the matrix element of (2.4) $p^{\tau n\lambda\mu}_{q_1q_2}$ with the radial dependence r^{λ} for $E\lambda$ transitions or the matrix element

$$p^{\tau n\lambda\mu}_{q_1q_2}=\mu_N\sqrt{\lambda(2\lambda+1)}\left\langle q_2\middle|r^{\lambda-1}\left[\frac{1}{2}g^{s\tau}(\sigma\mathbf{Y}_{\lambda-1})_{\lambda\mu}+\frac{2}{\lambda+1}g^{l\tau}(\mathbf{lY}_{\lambda-1})_{\lambda\mu}\right]\middle|q_1\right\rangle,\quad(3.9)$$

for $M\lambda$ transitions.
Here $g^{s\tau}$ and $g^{l\tau}$ are spin and orbital gyromagnetic ratios, and μ_N is a nuclear magneton.

The results presented here describe nonrotational states with arbitrary K^{π} except 0^+. The excited 0^+–states require special consideration as there the spurious state is to be eliminated [1, 4, 5], and we will not dwell upon it here.

4 RPA equations for odd–A nuclei

Consider nonrotational states of odd–A deformed nuclei when multipole and spin–multipole forces of the residual interaction with arbitrary μ for each λ and $\bar\lambda L$ are taken into account.

Making use of eqs. (3.4) and (3.8) we can write the model Hamiltonian (3.1) in the form:

$$\begin{aligned}H_{QPMN}&=\sum_q\varepsilon_qB_q-\tfrac14\sum_{\tau\ell\ell'\mu mii'}X^{\tau\mu i}_{\ell\ell'}\tilde D^{\tau\mu i}_{\ell}\tilde D^{\tau\mu i}_{\ell'}\left[Q_{\mu i}(m)Q^+_{\mu i'}(m)+Q^+_{\mu i}(m)Q_{\mu i'}(m)\right]-\\ &-\sum_{\tau nq_1q_2\mu im\gamma}\Gamma^{\tau n\gamma\mu i}_{q_1q_2}(m)\left\{\left[Q_{\mu i}(m)+\gamma mQ^+_{\mu i}(m)\right]B^{\tau}_{q_1q_2}(mn)+h.c.\right\},\end{aligned}\quad(4.1)$$

where

$$\Gamma^{\tau n\gamma\mu i}_{q_1q_2}(m)=\frac{1}{2\sqrt2}\sum_{\ell}f^{\tau n\ell\mu}_{q_1q_2}(\gamma_\ell m)v^{\tau(-\gamma)}_{q_1q_2}\tilde D^{t\mu i}_{\ell}.\quad(4.2)$$

Write the wave function of an odd–A nucleus in the form:

$$\Psi_{\nu}(K^{\pi})=\frac{1}{\sqrt2}\sum_{\sigma}\left\{\sum_{\rho}C_{\rho}\alpha^+_{\rho\sigma}+\sum_{\nu\mu i}D_{\nu\mu i}\alpha^+_{\nu\sigma}Q^+_{\mu i}(m)\right\}\Psi_0\quad(4.3)$$

with the normalization condition

$$\sum_{\rho}C^2_{\rho}+\sum_{\nu\mu i}D^2_{\nu\mu i}=1.\quad(4.3')$$

When studying highly excited states, we should take account of more complicated configurations as compared to (4.3), however, in this work we don't consider terms more complicated than a quasiparticle⊗phonon.

We determine the average of H_{QPNM} over (4.3) and using the variational principle, we obtain the system of equations for state energies η_r of an odd–A nucleus and coefficients of the wave function (4.3):

$$\sum_{\rho'} C^r_{\rho'} \left[(\varepsilon_\rho - \eta_r)\delta_{\rho\rho'} - V^r_{\rho\rho'}\right] = \|(\varepsilon_\rho - \eta)\delta_{\rho\rho'} - V_{\rho\rho'}\| \cdot C_{\rho'} \equiv \|\Delta_{\rho\rho'}\| \cdot C_{\rho'} = 0, \quad (4.4)$$

$$D^r_{\nu\mu i} = (\varepsilon_\nu + \omega_{\mu i} - \eta_r)^{-1} \sum_{\rho n \gamma m} C^r_\rho \Gamma^{\tau n \gamma \mu i}_{\rho\nu}(m), \quad (4.5)$$

where

$$V_{\rho\rho'} = \sum_{\nu n \gamma \mu i} \frac{\Gamma^{\tau n \gamma \mu i}_{\rho\nu}(m) \Gamma^{\tau n \gamma \mu i}_{\rho'\nu}(m)}{\varepsilon_\nu + \omega_{\mu i} - \eta}. \quad (4.6)$$

Solving this system we can calculate the reduced probability of γ- ray transitions in odd–A nuclei [4]. The amplitude of the reduced probability of the $E\lambda(M\lambda)$ transition between states described by the wave function (4.3) is of the form

$$M_{if}(\lambda_0\mu_0) \equiv \langle \Psi_f(K^\pi) | M(E\lambda_0(M\lambda_0)\mu_0) | \Psi_i(K^\pi) \rangle = \quad (4.7)$$

$$= e_{eff}^{(\tau\lambda_0\mu_0)} \left[\sum_{\rho_i\rho_f} C_{\rho_i} C_{\rho_f} v^{\tau(-\gamma)}_{\rho_f\rho_i} p^{\tau n \lambda_0 \mu_0}_{\rho_f \rho_i} + \sum_{\nu_i \nu_f \mu j} D_{\nu_i \mu j} D_{\nu_f \mu j} v^{\tau(-\gamma)}_{\nu_f \nu_i} p^{\tau n \lambda_0 \mu_0}_{\rho_f \rho_i} \right] +$$

$$+ \sum_j M^{\lambda_0\mu_0 j} \left[\sum_{\rho_f} C_{\rho_f} D_{\rho_f \mu_0 j} + \sum_{\rho_i} C_{\rho_i} D_{\rho_i \mu_0 j} \right].$$

The reduced probability of the $E\lambda(M\lambda)$ transition is given by

$$B\Big(E\lambda(M\lambda)\mu; (I^\pi K)_i \to (I^\pi K)_f\Big) \sim \Big|M_{if}(\lambda\mu)\Big|^2. \quad (4.8)$$

It is expressed [4] through the transition amplitudes (4.7) and the Clebsch–Gordan coefficients (omitted here and below).

The matrix element (4.7) contains terms that we will denote by CC, DD and CD. The first two of them describe transitions without changing the number of phonons between one-quasiparticle components and between quasiparticle⊗phonon components (4.3). The latter describe transitions between components C and D with changing the number of phonons by unity. When phonons $\lambda_0\mu_0 j$ are collective, those terms give a major contribution to the reduced transition probability.

5 Strength functions of the reduced probabilities of electromagnetic transitions

Calculation of the reduced probabilities of γ-ray transitions between excited states of odd-A deformed nuclei (4.7)-(4.8) directly provides information on the structure of

states, their collectivity and on the contribution of different multipolarities to the wave function (4.3). However, a similar detailed analysis and comparison of calculations with experimental data directly for every transition can be carried out only for the low-lying part of the spectrum. As the excitation energy increases, the state density grows, and the amount of transitions between them increases to still a greater extent. A possible way to overcome difficulties in analysing this part of the spectrum is to apply the strength–function method. A similar analysis was earlier performed in doubly even nuclei [7] when studying the multipole giant resonances and in odd-A nuclei when studying the fragmentation of one-quasiparticle components of (4.3) [8].

Making transformations similar to those of ref. [6, 8] for (4.8) with the averaging function

$$\rho(\eta - \eta_f) = \frac{1}{2\pi} \cdot \frac{\Delta}{(\eta - \eta_f)^2 + (\Delta/2)^2}, \quad (5.1)$$

where Δ is the avereging interval, and using the analytic properties of the secular equation (4.4), we obtain the following expression for the strength function of the reduced probability of electromagnetic transition:

$$b(E\lambda_0(M\lambda_0)\mu_0, \eta) = \sum_f B\Big(E\lambda_0(M\lambda_0)\mu_0; (K^\pi\eta)_i \to (K^\pi\eta)_f\Big)\rho(\eta - \eta_f) =$$
$$= \frac{1}{\pi} \cdot Im \left\{ \frac{1}{\Delta(\eta^*)} \cdot \begin{vmatrix} b_0 & -b_{\rho'_f} \\ b_{\rho_f} & \Delta_{\rho_f\rho'_f} \end{vmatrix} \right\}_{\eta^* = \eta + i\Delta/2}. \quad (5.2)$$

Here $\Delta_{\rho_f\rho'_f}$ are elements of the matrix (4.4),

$$\Delta(\eta^*) \equiv det\Delta_{\rho_f\rho'_f} \equiv | \Delta_{\rho_f\rho'_f} |. \quad (5.3)$$

In the bordered determinant from (5.2) :

$$b_0 = \sum_{\rho_i j} \frac{(C_{\rho_i} M^{\lambda_0\mu_0 j})^2}{\varepsilon_{\rho_i} + \omega_{\mu_0 j} - \eta^*} + \quad (5.4)$$

$$+ (e_{eff}^{(\lambda_0\mu_0)})^2 \sum_{\nu'\mu j} \frac{\left[\sum_\nu D_{\nu\mu j} v^{(-\gamma)}_{\nu'\nu} p^{\lambda_0\mu_0}_{\nu'\nu}\right]^2}{\varepsilon_{\nu'} + \omega_{\mu j} - \eta^*} + 2 \cdot e_{eff}^{(\lambda_0\mu_0)} \sum_{\rho_i j} \frac{C_{\rho_i} M^{\lambda_0\mu_0 j} \sum_\nu D_{\nu\mu_0 j} v^{(-\gamma)}_{\nu'\nu} p^{\lambda_0\mu_0}_{\rho_i\nu}}{\varepsilon_{\rho_i} + \omega_{\mu_0 j} - \eta^*},$$

$$b_{\rho_f} = \sum_j D_{\rho_f\mu_0 j} M^{\lambda_0\mu_0 j} \sum_{\rho_i j'} C_{\rho_i} \frac{\Gamma^{n\gamma\mu_0 j'}_{\rho_f\rho_i} M^{\lambda_0\mu_0 j'}}{\varepsilon_{\rho_i} + \omega_{\mu_0 j'} - \eta^*} + \quad (5.5)$$

$$+ e_{eff}^{(\lambda_0\mu_0)} \left[\sum_{\rho_i} C_{\rho_i} v^{(-\gamma)}_{\rho_f\rho_i} p^{\lambda_0\mu_0}_{\rho_f\rho_i} + \sum_{\nu\nu'\mu j} D_{\nu'\mu j} \frac{v^{(-\gamma)}_{\nu\nu'} p^{\lambda_0\mu_0}_{\nu\nu'} \Gamma^{n\gamma\mu j}_{\rho_f\nu}}{\varepsilon_\nu + \omega_{\mu j} - \eta^*} \right].$$

Using the strength function (5.2), we can calculate the reduced probability of transitions $(K^\pi)_i\eta_i \to (K^\pi)_f\eta$ from the initial state $K_i^{\pi_i}$ with energy η_i to highly excited states $K_f^{\pi_f}$ in the energy region η. Thus, unlike the calculations with formula (4.8), it is not necessary to compute the energies η_f and wave functions of all the states with $K_f^{\pi_f}$, which certainly simplifies their investigation.

In some cases, there arises a still more complicated problem of calculating γ-ray transition between states when both lie in the intermediate energy region with a large state density.

We have also applied the strength–function method [6, 7], expounded here, to solve that problem by introducing a complex function of two variables and using the analytic properties of more complex function (5.2). To this end, we average over initial states in the energy range ζ and over final states in the region η.

Since the total expression for the reduced probability SF of an electromagnetic transition $(K^\pi)_i\zeta \rightarrow (K^\pi)_f\eta$ of the multipolarity $\lambda_0\mu_0$ is rather complicated, we will limit ourselves to the most important transitions of type CC and CD, omitting transitions of type DD and interference terms from the total expression, which gives:

$$b(E\lambda_0(M\lambda_0)\mu_0,\zeta,\eta) =$$

$$= \frac{1}{\pi^2}\left\{(e_{eff}^{(\lambda_0\mu_0)})^2 \sum_{\rho_i\rho_i'\rho_f\rho_f'} v^{(-\gamma)}_{\rho_f\rho_i} p^{\lambda_0\mu_0}_{\rho_f\rho_i} v^{(-\gamma)}_{\rho_f'\rho_i'} p^{\lambda_0\mu_0}_{\rho_f'\rho_i'} \cdot Im R_{\rho_i\rho_i'} \cdot Im T_{\rho_f\rho_f'} + \right. \tag{5.6}$$

$$+ \sum_j (M^{\lambda_0\mu_0 j})^2 \left[\sum_{\rho_i} Im R_{\rho_i\rho_i} \cdot Im\frac{1}{\varepsilon_{\rho_i}+\omega_{\mu_0 j}-\eta^*} + \sum_{\rho_f} Im T_{\rho_f\rho_f} \cdot Im\frac{1}{\varepsilon_{\rho_f}+\omega_{\mu_0 j}-\zeta^*}\right] +$$

$$+ \sum_{\rho_i\rho_i'\rho_f\rho_f' jj'} M^{\lambda_0\mu_0 j} M^{\lambda_0\mu_0 j'} \Gamma^{n\gamma\mu_0 j}_{\rho_i\rho_f} \Gamma^{n\gamma\mu_0 j'}_{\rho_i'\rho_f'} \times$$

$$\times \left[Im R_{\rho_i\rho_i'} \cdot Im\frac{T_{\rho_f\rho_f'}}{(\varepsilon_{\rho_i}+\omega_{\mu_0 j}-\eta^*)(\varepsilon_{\rho_i}+\omega_{\mu_0 j'}-\eta^*)} + \right.$$

$$\left.\left. + Im\frac{R_{\rho_i\rho_i'}}{(\varepsilon_{\rho_f}+\omega_{\mu_0 j}-\zeta^*)(\varepsilon_{\rho_f}+\omega_{\mu_0 j'}-\zeta^*)} \cdot Im T_{\rho_f\rho_f'}\right] + \cdots\right\}.$$

Here $\zeta^* = \zeta + i\Delta_i/2, \eta^* = \eta + i\Delta_f/2$; Δ_i and Δ_f are intervals of averaging over initial and final states; $R_{\rho_i\rho_i'}$ and $T_{\rho_f\rho_f'}$ are elements of the corresponding inverse matrices $\| \Delta_{\rho_i\rho_i'}(\zeta^*) \|^{-1}$ and $\| \Delta_{\rho_f\rho_f'}(\eta^*) \|^{-1}$ of the system (4.6) for the initial and final states.

6 Results of calculations

The reduced probability of γ-ray transitions were calculated by formula (5.6) for several rare–earth deformed nuclei. In Fig. 1, we draw the results of calculation of the strength function of $E2$ and $M1$ transitions between states with $K_i^{\pi_i} = 3/2^-$ and $K_f^{\pi_f} = 1/2^-, 3/2^-$ from the energy interval $0 \div 2.2$ MeV in ^{185}W. The energy is reckoned from the nucleus ground state $(K^\pi)_{g.s.} = 3/2^- : E_i = \zeta - \zeta_{g.s.}; E_f = \eta - \zeta_{g.s.}$.

The strength function for M1 transitions is shown on the left part of the plane E_iEf; and for E2 transitions, on the right part. The points on the axis E_i denote the energies

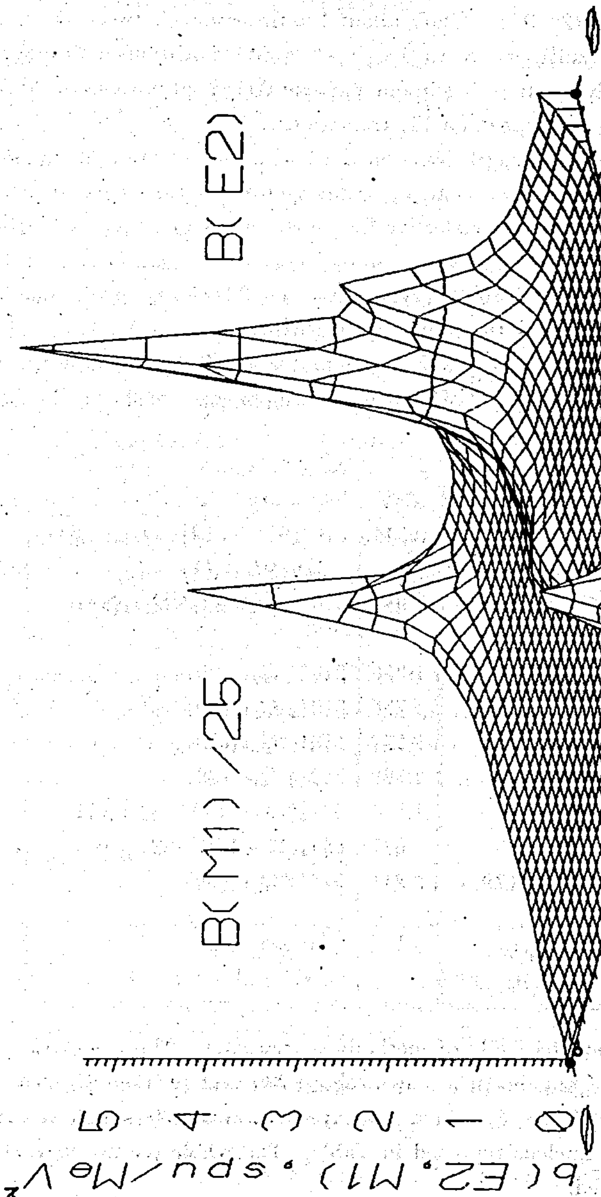

Fig. 1. Strength functions of $E2$– and $M1$–transitions for ^{183}W between the states $K_i^{\pi_i} = 3/2^-$(solid circles) and $K_f^{\pi_f} = 1/2^-$(open circles), $3/2^-$(solid circles). The rest of the notation is given in the text.

for lowest 11 roots of eq.(3.9) with $(K^\pi)_i = 3/2^-$; whereas the points on the axis E_f, for 21 states with $(K^\pi)_f = 1/2^-, 3/2^-$. Thus, about 160 transitions between the above states are possible per each multipolarity $\lambda\mu$ (i.e., in all, $\sim$ 800 transitions with different λ and μ involved). Strength functions are in one-particle Weisskopf units (spu/MeV^2) and the scale factor 1/25 is introduced for M1 transitions.

The Fig. 1 reproduces the strength function (5.6) with the intervals of averaging $\Delta_i = \Delta_f = 0.2MeV$, which allows us to distinguish only several strongest transitions from the total background: two collective E2 transitions $511\downarrow\otimes Q_{0+1} \rightarrow 511\downarrow$ and $512\downarrow\otimes Q_{2+1} \rightarrow 512\downarrow$, magnetic one-particle transition $511\downarrow \rightarrow 512\downarrow$ and a group of transitions between states of the complicated structure at $E_{i,f} \sim 2MeV$. Quantum numbers of one-particle states q are given in the conventional Nilsson notation $Nn_z\Omega - 1/2\Sigma$.

The energy and structure of the states $\Psi_i(1/2^-)$ and $\Psi_{i,f}(3/2^-)$ marked on the axis E_i and E_f are listed in Table where 1–2 of their main components are shown (in %).

$E_{i,f}$, MeV	$K^\pi = 3/2^-$ Structure(%)	E_i, MeV	$K^\pi = 1/2^-$ Structure(%)
0	$512\downarrow(97) + 510\uparrow\otimes Q_{2+1}(1)$	0.044	$510\uparrow(97) + 512\uparrow\otimes Q_{2+1}(1.5) +$ $+512\downarrow\otimes Q_{2+1}(1)$
0.849	$501\uparrow(28) + 503\uparrow\otimes Q_{2+1}(66) +$ $+512\downarrow(2) + 501\uparrow\otimes Q_{0+1}(1)$	0.982	$510\uparrow(1) + 512\downarrow\otimes Q_{2+1}(98)$
0.937	$512\downarrow(1) + 510\uparrow\otimes Q_{2+1}(98)$	1.316	$510\uparrow\otimes Q_{0+1}(99)$
1.367	$512\downarrow\otimes Q_{0+1}(99)$	1.736	$510\uparrow\otimes Q_{1+1}(99)$
1.735	$510\uparrow\otimes Q_{1+1}(99)$	1.737	$510\uparrow\otimes Q_{0+1}(99)$
1.793	$512\downarrow\otimes Q_{0+2}(99)$	1.782	$512\downarrow\otimes Q_{1+1}(99)$
1.898	$510\uparrow\otimes Q_{2+2}(99)$	1.834	$521\downarrow(78) + 521\downarrow\otimes Q_{0+1}(11)$
1.914	$514\downarrow\otimes Q_{2+1}(99)$	2.027	$521\downarrow(1) + 510\uparrow\otimes Q_{0+3}(98)$
1.951	$501\uparrow(48) + 503\uparrow\otimes Q_{2+1}(28) +$ $+501\uparrow\otimes Q_{0+1}(12)$	2.211	$512\uparrow\otimes Q_{2+1}(99)$
2.075	$501\uparrow(1) + 512\downarrow\otimes Q_{0+3}(98)$	2.215	$510\uparrow\otimes Q_{0+4}(99)$
2.183	$501\uparrow(2) + 503\uparrow\otimes Q_{2+2}(97)$		

The presented results are basically of methodical character. The calculations involved a limited number of one–particle states (about 30) and phonons (5 roots per each multipolarity with $\lambda \leq 3, \mu \leq 3$), which just explains a relatively simple structure of the states of an odd-A nucleus reported in Table. The whole picture in realistic calculations gets complicated.

The comparison of the left (M1 transitions) with right part of the Fig. 1 (E2 transitions) shows that there are no correlations between the above transitions.

7 Conclusion. Outlook

The presented analysis deals only with the simplest configurations of the wave function of an odd-A nucleus (4.3). In further studies, we suppose to take account of quasiparticle⊗2 phonons configurations in order to analyse transitions of the type $\alpha^+Q^+Q^+ \rightarrow \alpha^+Q^+$ that can happen to be as important as transitions of the type CD ($\alpha^+Q^+ \rightarrow \alpha^+$). Also, it is necessary to take the commutation relation for phonon operators (2.8) into account more accurately [5].

Let us discuss some prospects of application of the methods presented. Similar calculations can be carried out for strength functions of the cross sections of direct nuclear reactions of nucleon transfer. Comparison of these results with SF of the probabilities for electromagnetic transitions of different multipolarities will allow us to determine a preferable type of the reaction for nucleus excitation in a given energy interval and to simplify the analysis of experimental data. These methods will permit us to interpret decay characteristics of multipole giant resonances and results from other nuclear reactions. All this will make it possible to get more adequate results about complication of the structure of nuclear states at intermediate and high excitation energies . One will be able to compute with higher accuracy the lifetime of individual nuclear states, competition between $E\lambda$ and $M\lambda$ transitions and to compare the relation between valence and collective transitions (CC and CD). A similar study, when more complicated configurations as compared to (4.3) are included, opens a new possibility for investigating γ-ray transitions between nuclear states in the region of the nucleon binding energy and below it. It can be useful for further clarifying the problem of the so-called transition from order to chaos in nuclear spectra discussed recently in [14] and other papers.

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