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F.Šimkovic¹

MESON EXCHANGE CURRENTS
AND TWO NEUTRINO DOUBLE BETA DECAY

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¹On leave of absence from Department of Nuclear Physics, Comenius University, Bratislava, Slovakia

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Nuclear double beta decay is a subject of the current intensive activity, both experimentally and theoretically. The neutrinoless mode of this process ($2\beta 0\nu$ - decay) is related to physics beyond the standard model and requires the existence of the lepton number violating massive Majorana neutrinos [1, 2]. Up to now, experimentally only lower half life limits for $2\beta 0\nu$ - decay have been reported in the literature. Growing interest is also paid to the two-neutrino mode ($2\nu 2\beta$ - decay). It is the rarest process observed so far in nature. This mode being independent of the neutrino properties offers a sensitive test of nuclear structure calculations. The usual strategy has been first to try to reproduce the observed $2\beta 2\nu$ - decay half times in order to gain confidence in the calculated $2\beta 0\nu$ - decay nuclear matrix elements [3, 4, 5, 6].

In the present calculation of $2\beta 2\nu$ - decay we consider only the Gamow Teller nuclear matrix element

$$M_{GT} = \sum_n \frac{\langle 0_f^+ | A_k | 1_n^+ \rangle \langle 1_n^+ | A_k | 0_i^+ \rangle}{E_n - E_i + \Delta}, \quad (1)$$

where $|0_i^+\rangle$, $|0_f^+\rangle$ and $|1_n^+\rangle$ are respectively the wave functions of the initial, final and intermediate nuclei with corresponding energies E_i , E_f and E_n . Δ denotes the average energy $\Delta = \frac{1}{2}(E_i - E_f)$. A_k is the Gamow-Teller transition operator $A_k = \sum_i \tau_i^+ (\vec{\sigma}_i)_k$, $k=1,2,3$.

The quasiparticle random phase approximation (QRPA) is the nuclear structure method most widely used to calculate M_{GT} . However, the results are extremely sensitive to the details of the nuclear hamiltonian, in particular, to factor g_{pp} introduced to renormalize the particle-particle interaction strength [7, 8, 9, 10]. The magnitude g_{pp} consistent with the calculation of β^+ decay is not broad, nevertheless the value of M_{GT} calculated with g_{pp} within this interval crosses zero. The extreme sensitivity of QRPA to g_{pp} is the difficulty of making definite rate predictions. Several modification of QRPA have been proposed that might change that behavior as e.g. higher order RPA corrections [11], nuclear deformation [12] and particle number projection [13, 14]. However, none of these amendments inhibits the matrix element M_{GT} to pass through zero near the natural value of $g_{pp} = 1$.

The goal of the present paper is to analyse the $2\beta 2\nu$ - decay amplitude in field theory approach and to show that the calculation of the many body Green function M_{GT} in eq.(1) corresponds to the calculation of the contributions from a class of meson exchange current diagrams.

In the two nucleon mechanism of the $2\nu 2\beta$ - decay process the beta decay hamilt-

tonian acquires the form:

$$\mathcal{H}^\beta(x) = \frac{G_F}{\sqrt{2}} 2 [\bar{e}_L(x) \gamma_\alpha \nu_{eL}(x)] j_\alpha(x) + h.c., \quad (2)$$

where $e_L(x)$ and $\nu_{eL}(x)$ are operators of the left components of fields of the electron and neutrino, respectively. The strangeness conserving free charged hadron current takes the form:

$$j_\alpha(x) = \bar{p}(x) \gamma_\alpha (g_V + g_A \gamma_5) n(x), \quad (3)$$

where $p(x)$ and $n(x)$ are operators of the field of the proton and neutron, respectively, and $g_V = 1.0$ and $g_A = 1.25$.

Clearly $2\nu 2\beta$ - decay occurs in the second order perturbation theory of the weak interaction. For the matrix element of $2\nu 2\beta$ - decay process we have

$$\begin{aligned} \langle f | S^{(2)} | i \rangle = & \frac{(-i)^2}{2} \left(\frac{G_F}{\sqrt{2}} \right) N_{p_1} N_{p_2} N_{k_1} N_{k_2} \times \\ & \times \bar{u}(p_1) \gamma_\mu (1 + \gamma_5) u(-k_1) \bar{u}(p_2) \gamma_\mu (1 + \gamma_5) u(-k_2) J_{\mu\nu}(p_1, p_2, k_1, k_2) - \\ & - (p_1 \leftrightarrow p_2) - (k_1 \leftrightarrow k_2) + (p_1 \leftrightarrow p_2)(k_1 \leftrightarrow k_2), \end{aligned} \quad (4)$$

where

$$\begin{aligned} J_{\mu\nu}(p_1, p_2, k_1, k_2) = & \int e^{-i(p_1+k_1)x_1} e^{-i(p_2+k_2)x_2} \times \\ & \times_{out} \langle p_f | T(J_\mu(x_1) J_\nu(x_2)) | p_i \rangle_{in} dx_1 dx_2. \end{aligned} \quad (5)$$

Here, $N_p = (1/(2\pi)^{3/2})(1/(2p_0)^{1/2})$, p_1 and p_2 (k_1 and k_2) are four-momenta of electrons (antineutrinos), p_i and p_f are four-momenta of the initial and final nucleus, and the nuclear matrix element is

$${}_{out} \langle p_f | T(J_\mu(x_1) J_\nu(x_2)) | p_i \rangle_{in} = \langle p_f | T(j_\mu(x_1) j_\nu(x_2) e^{-i \int (\mathcal{H}^h(x') + \mathcal{H}^{h,\gamma}(x')) dx'}) | p_i \rangle, \quad (6)$$

where $J_\mu(x)$ is the weak charged nuclear hadron current in the Heisenberg representation [15, 16]. $\mathcal{H}^h(x)$ and $\mathcal{H}^{h,\gamma}(x)$ are respectively the strong interaction hamiltonian and the interaction hamiltonian of the electromagnetic and hadron fields. In this way, in eq. (5) the strong and electromagnetic interaction of the nucleons is taken into account exactly.

The matrix element in eq.(4) contains also the matrix element for two subsequent nuclear beta decay processes. In order to separate both processes we write T as a product of two hadron currents as follows [17]:

$$T(J_\mu(x_1) J_\nu(x_2)) = J_\mu(x_1) J_\nu(x_2) + \Theta(x_{2_0} - x_{1_0}) [J_\nu(x_2), J_\mu(x_1)]. \quad (7)$$

Then we have (henceforth the indices "in" and "out" will be omitted)

$$J_{\mu\nu}(p_1, p_2, k_1, k_2) = \int e^{-i(\vec{p}_1 + \vec{k}_1) \cdot \vec{x}_1} e^{-i(\vec{p}_2 + \vec{k}_2) \cdot \vec{x}_2} \times \quad (8)$$

$$\times (I_{\mu\nu}^{\beta\beta}(p_{10}, p_{20}, k_{10}, k_{20}, \vec{x}_1, \vec{x}_2) + I_{\mu\nu}^{2\beta 2\nu}(p_{10}, p_{20}, k_{10}, k_{20}, \vec{x}_1, \vec{x}_2)) d\vec{x}_1 d\vec{x}_2,$$

with

$$I_{\mu\nu}^{\beta\beta}(p_{10}, p_{20}, k_{10}, k_{20}, \vec{x}_1, \vec{x}_2) = \sum_n 2\pi\delta(E_f - E_n + p_{10} + k_{10}) \langle p_f | J_\mu(0, \vec{x}_1) | p_n \rangle \times \quad (9)$$

$$\times 2\pi\delta(E_n - E_i + p_{20} + k_{20}) \langle p_f | J_\nu(0, \vec{x}_2) | p_n \rangle,$$

and

$$I_{\mu\nu}^{2\beta 2\nu}(p_{10}, p_{20}, k_{10}, k_{20}, \vec{x}_1, \vec{x}_2) = 2\pi\delta(E_f - E_i + p_{10} + k_{10} + p_{20} + k_{20}) \times \quad (10)$$

$$\times \int e^{i(p_{20} + k_{20})t} \Theta(t) \langle p_f | [J_\nu(t, \vec{x}_2), J_\mu(0, \vec{x}_1)] | p_i \rangle dt.$$

Here $|p_n\rangle$ is an eigenvector of the intermediate nucleus with energy E_n and we used [16],

$$\langle p_f | J_\mu(x_1) J_\nu(x_2) | p_i \rangle = \quad (11)$$

$$\sum_n \langle p_f | J_\mu(0, \vec{x}_1) | p_n \rangle \langle p_n | J_\nu(0, \vec{x}_2) | p_i \rangle e^{i(E_f - E_n)x_{10}} e^{i(E_n - E_i)x_{20}}.$$

The symbol \sum_n means summation over the discrete states and integration over the continuum states of the intermediate nucleus. It includes the complete set of these states.

From the two delta functions in eq. (9), which have the meaning of energy law conservation, we see that the first term in the r.h.s. of eq. (8) corresponds to the two subsequent nuclear beta decay processes. This process is drawn in Fig.1a. For most of the nuclei in which the double beta decay is experimentally studied such transitions are energetically forbidden. For $E_n > E_i$ the argument of the second delta function in the r.h.s. of eq. (9) is always positive and this term is equal to zero.

The second term in the r.h.s. of eq. (8) corresponds to $2\nu 2\beta$ - decay process. We see that the $2\nu 2\beta$ - decay amplitude contains the nuclear matrix element of the non-equal-time commutator of the two hadron currents. We note that the commutator is non zero because the currents are in the Heisenberg representation. In the case of free hadron currents with the use of the anticommutation relations of the operators $\bar{p}(x)$ and $n(x)$ (see eq.(3)) we have

$$[j_\nu(x), j_\mu(y)] = 0. \quad (12)$$

The presence of the commutator of two currents in the $2\nu 2\beta$ - decay amplitude is telling us that the two single beta decays in the nucleus have to be correlated. The only possible correlations are of the type of meson and γ - exchanges which have their origin in the strong interaction hamiltonian $\mathcal{H}^h(x)$ and in the interaction hamiltonian of the electromagnetic and hadron fields $\mathcal{H}^{h\gamma}(x)$ (see eq. (6)). If we consider the Heisenberg current operator $J_\mu(x)$ in an approximative way in which $\mathcal{H}^h(x)$ is replaced by the pseudoscalar coupling pion-nucleon interaction hamiltonian [18], we just obtain the one pion exchange diagram of Fig.1b. From the above discussion it follows that it is possible to start with an appropriate S-matrix and deduce the most important meson and γ - exchange contributions to $2\beta 0\nu$ - decay amplitude. We maintain that the S- matrix approach is an alternative way for the calculation of the $2\nu 2\beta$ - decay amplitude, which does not need the construction of the intermediate nuclear states. It is only necessary to know the nuclear wave functions of the initial and final nucleus and to derive two body operators from the corresponding exchange diagrams. We shall discuss possible exchange mechanisms later. First, we shall show in a different way that nuclear exchange currents dominate the $2\beta 2\nu$ - amplitude in eq.(4)

To calculate the commutator in eq. (10) one can use first the well-known formula (e.g. [19]). The result is,

$$J_\nu(t, \vec{y}) = e^{itH} J_\nu(0, \vec{y}) e^{-itH} = \sum_{k=0}^{\infty} \frac{(it)^k}{k!} \overbrace{[H[H\dots[H, J_\nu(0, \vec{y})]\dots]]}^{k \text{ times}}, \quad (13)$$

where H is the nuclear hamiltonian. This formula has been first used in the Operator Expansion Method (OEM) in [20, 21, 22]. The summation on the right hand side of eq.(13) was evaluated by neglecting the kinetic energy part of nuclear hamiltonian and by considering only the central part of the effective nucleon-nucleon interaction. We note that OEM has been derived also in a different way by expanding the denominator of the many body Green function in eq.(1) into a Taylor series and by using the same approximations [23, 24, 25]. However, some questions arise about the convergence of such a power series expansion. By working in the time integral representation there are no such problems. Nevertheless the OEM is still a matter of contradicting discussions as there are more open questions. Starting with the form of the Green function in eq.(1) the assumption that the kinetic energy operator T can be ignored has been criticized recently [26]. In another paper by using the second quantization language it was argued that the single particle term of the nuclear hamiltonian plays an important role [27]. We shall prove the opposite. We shall

show explicit that if we approximate the nuclear hamiltonian H by a single particle hamiltonian $H^{s.p.}$ the $2\beta 2\nu$ - amplitude is equal to zero.

In the second quantization formalism we have

$$\hat{J}_\nu(0, \vec{y}) = \sum_{pm_p nm_n} \langle pm_p | J_\nu(0, \vec{y}) | nm_n \rangle c_{pm_p}^\dagger c_{nm_n}, \quad (14)$$

$$\hat{H}^{s.p.} = \sum_{pm_p} \epsilon_p c_{pm_p}^\dagger c_{pm_p} + \sum_{nm_n} \epsilon_n c_{nm_n}^\dagger c_{nm_n}. \quad (15)$$

Here, $c_{pm_p}^\dagger$ and $c_{nm_n}^\dagger$ (c_{pm_p} and c_{nm_n}) are creation (annihilation) operators of proton and neutron, respectively and ϵ_p and ϵ_n are single particle energies of proton and neutron states. Using eq.(12) we obtain

$$e^{iH^{s.p.}} \hat{J}_\nu(0, \vec{y}) e^{-iH^{s.p.}} = \sum_{p,n} \langle p, m_p | J_\nu(0, \vec{y}) | n, m_n \rangle e^{i(\epsilon_p - \epsilon_n)t} c_{pm_p}^\dagger c_{nm_n} \quad (16)$$

and

$$[e^{iH^{s.p.}} \hat{J}_\nu(0, \vec{y}) e^{-iH^{s.p.}}, \hat{J}_\mu(0, \vec{x})] = 0. \quad (17)$$

We have used only the anticommutation relations of the creation and annihilation operators of proton and neutron. We note that $2\nu 2\beta$ - decay is always equal to zero, if we approximate the nuclear current $\hat{J}_\nu(t, \vec{x})$ by a one body operator. In general $\hat{J}_\nu(t, \vec{x})$ is a sum of one- two- and more-body operators. Therefore, the commutator of two nuclear currents $[\hat{J}_\nu(t, \vec{y}), \hat{J}_\mu(0, \vec{x})]$ should be understood as a meson exchange current operator, the exchanges being induced by the residual two body interaction part of the nuclear hamiltonian. It proves that the single particle part of the nuclear hamiltonian plays a less important role.

We integrate over the time variable in eq. (10) using the standard procedure of the adiabatic switch-off of the interaction as $t \rightarrow \infty$, i.e.,

$$\int_0^\infty \exp^{-iat} dt \Rightarrow \lim_{\epsilon \rightarrow 0} \int_0^\infty \exp^{-i(a-i\epsilon)t} dt = \lim_{\epsilon \rightarrow 0} \frac{-i}{a - i\epsilon}. \quad (18)$$

We assume that the nuclear states and their corresponding energies can be considered as the eigenstates and eigenvalues of the nuclear hamiltonian H ,

$$H|p_i\rangle = E_i|p_i\rangle, \quad H|p_f\rangle = E_f|p_f\rangle, \quad H|p_n\rangle = E_n|p_n\rangle. \quad (19)$$

Then with the help of (11) we get

$$I_{\mu\nu}^{2\beta 2\nu}(p_{10}, p_{20}, k_{10}, k_{20}, \vec{x}_1, \vec{x}_2) = 2\pi\delta(E_f - E_n + p_{10} + k_{10} + p_{20} + k_{20})i \sum_n \quad (20)$$

$$\left\{ \frac{\langle p_f | J_\nu(0, \vec{x}_2) | p_n \rangle \langle p_n | J_\mu(0, \vec{x}_1) | p_i \rangle}{E_n - E_i + p_{10} + k_{10}} + \frac{\langle p_f | J_\mu(0, \vec{x}_1) | p_n \rangle \langle p_n | J_\nu(0, \vec{x}_2) | p_i \rangle}{E_n - E_i + p_{20} + k_{20}} \right\}.$$

We further assume the non-relativistic impulse approximation for the hadronic current $J_\nu(0, \vec{y})$ and neglect the contribution from the vector currents to $2\nu 2\beta^-$ decay amplitude. We also restrict our consideration to $s_{1/2}$ -wave states of the emitted leptons and consider only the energetically most favoured $0_{initial}^+ \rightarrow 0_{final}^+$ nuclear transition. We suppose $p_{10} + k_{10} \simeq p_0 + k_{20} \simeq (E_i - E_f)/2$. Then we have,

$$J_{\mu\nu}(p_1, p_2, k_1, k_2) = 2\pi\delta(E_f - E_i + p_{10} + k_{10} + p_{20} + k_{20})iM_{GT}\delta_{\mu k}\delta_{\nu k}, \quad k = 1, 2, 3, \quad (21)$$

where M_{GT} is defined in eq.(1). From the above derivation it follows that in calculations using the form M_{GT} of eq.(1) the meson exchanges are taken into account by the summation over the intermediate nuclear states 1_n^+ , which are constructed, e.g. by an RPA diagonalization.

We show that these calculations are sensitive to the truncation of the nuclear hamiltonian violating the condition of eq.(19). Following ref. [27] we express M_{GT} in the integral representation:

$$M_{GT} = \int_0^\infty \langle 0_f^+ | \hat{A} e^{-\hat{H}\tau} \hat{A} e^{\hat{H}\tau} | 0_i^+ \rangle e^{-\Delta\tau} d\tau. \quad (22)$$

If we rewrite the denominator of eq.(1) as $E_n - E_f - \Delta$ we have

$$M_{GT} = \int_0^\infty \langle 0_f^+ | e^{\hat{H}\tau} \hat{A} e^{-\hat{H}\tau} \hat{A} | 0_i^+ \rangle e^{\Delta\tau} d\tau. \quad (23)$$

The equivalence of both forms of M_{GT} in eq.(22) and in eq.(23) is evident and it can be proved with help of $e^{-\hat{H}\tau} | 0_i^+ \rangle = e^{-E_i\tau} | 0_i^+ \rangle$ and $e^{\hat{H}\tau} | 0_f^+ \rangle = e^{E_f\tau} | 0_f^+ \rangle$. Within the approximation $H \sim H_{s.p.}$ we obtain from eq.(22)

$$M_{GT}^{s.p.} = \sum_{pn} \sum_{\substack{\dot{p}\dot{i} \\ m_p m_n \quad m_p m_n}} \langle pm_p | A_k | nm_n \rangle \langle \dot{p}m_p | A_k | \dot{i}m_i \rangle \frac{\langle 0_f^+ | c_{pm_p}^+ c_{nm_n} c_{\dot{p}m_p}^+ c_{\dot{i}m_n} | 0_i^+ \rangle}{e_p - e_i + \Delta}. \quad (24)$$

However using eq.(23) the result is $(-M_{GT}^{s.p.})$. The relation $M_{GT}^{s.p.} = -M_{GT}^{s.p.}$ requires $M_{GT}^{s.p.} = 0$, which cannot be fulfilled numerically. The two different results come as a consequence of the violation of the assumption of eq.(19). This example shows how erroneous can become the use of the Green function in eq.(1) in respect to the approximation of the nuclear hamiltonian.

The quasiparticle nuclear hamiltonian used in QRPA calculations neither reproduces well the absolute values of the ground state energy of initial and final nucleus nor their relative values. In addition, it is the problem of the two vacua and two

independent normalizations, which have to be performed using two different representations of the nuclear hamiltonian. This can be hardly considered as a consistent development of meson exchange current matrix elements. To do that, one should start with the appropriate S-matrix and deduce the most important MEC contributions. Such an analysis however is rather involved.

Nevertheless, we shall advance some speculative arguments. We note that the energy release for these processes is very small, and this allows us to study meson exchange diagrams in the static limit. We can suppose that the dominant contributions to the $2\nu 2\beta$ - decay amplitude come from the pion exchange diagrams as $m_\pi^2 \ll m_\rho^2$ (m_π is the mass of the pion, m_ρ the mass of the rho-meson). In the case of pion exchanges we have smaller denominators in the amplitude. However, the pion exchange mechanisms in Fig.1b are expected to be strongly suppressed in comparison with another exchange mechanisms suggested by Ericson and Vergados [28] (see e.g. Fig.1c) because of the big masses of the virtual off-shell nucleons. The nucleon propagator can be approximated by $1/m_p(m_n)$ (m_p and m_n is the mass of proton and neutron, respectively). Ericson and Vergados constructed the effective two body operators from their exchange diagrams by using PCAC and soft pion theorems. If we suppose that their exchange diagrams give the main contribution to $2\nu 2\beta$ - decay amplitude, we can deduce, from ref. [28], for the $2\nu 2\beta$ - decay half life of ^{48}Ca the value 1.5×10^{25} years. The size of this value is independent of the nuclear structure of a given nucleus and a similar strong suppression of the value of $2\nu 2\beta$ - decay half time is also expected for other nuclei, with values about 10^{24-25} years. However, such results are in strong contradiction with the existing experimental data. We can hardly suppose that the difference of four - five orders from the experimental half lives have origin in the inaccuracy of the method. The S-matrix approach has been applied successful to study meson and gamma exchange effects in different nuclear processes, e.g. electron scattering [29] and compton scattering [30]. We note that values of the half lifes about $10^{24} - 10^{25}$ years do not contradict the QRPA calculations which give only lower limits on the value of the $2\nu 2\beta$ - decay half life [8, 9, 10]. It could mean that the mechanism considered at present is not dominant for the $2\nu 2\beta$ - decay process.

It is this motivation that stimulated us to study the electron-gamma exchange mechanism for $2\beta 0\nu$ - decay drawn in Fig.1d [31]. We note that in the two nucleon $2\beta 0\nu$ - decay mechanism studied at present only the electromagnetic interaction between electron and nucleon and between two nucleons has been included. The first interaction leads to a distortion of the electron wave function, which is taken

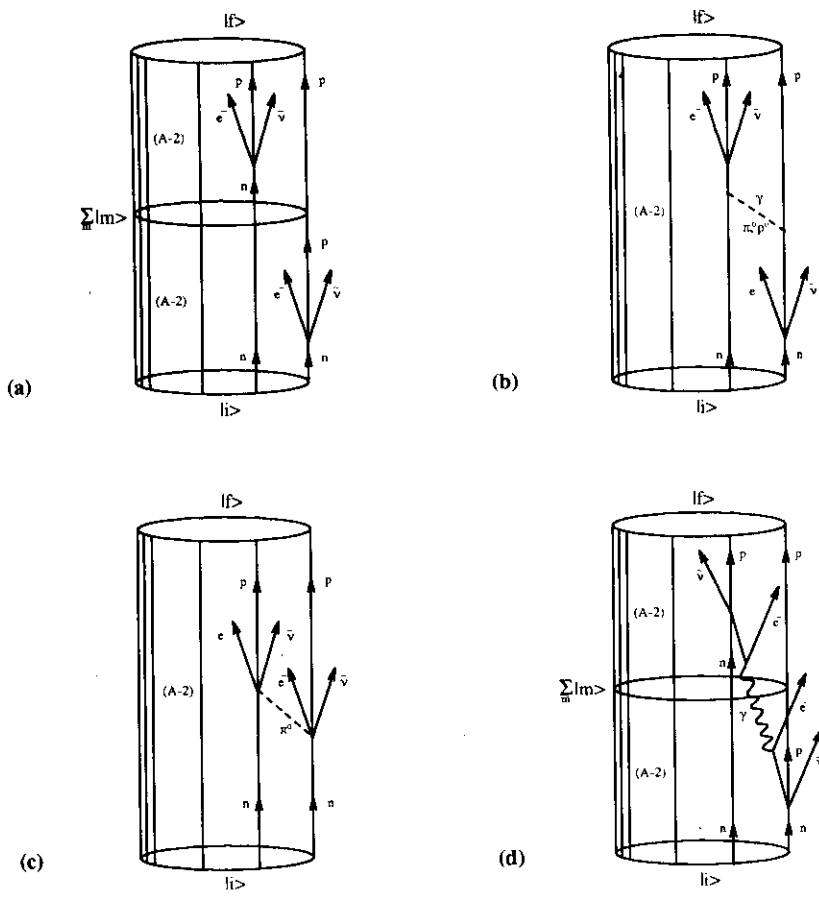


Figure 1: The Feynmann diagrams for the two subsequent nuclear beta decay processes and for the two neutrino nuclear double beta decay process. (a) The Feynmann diagram for two subsequent nuclear beta decays within the impulse approximation. (b) The Feynmann diagrams of the two neutrino double beta decay process of the two nucleon mechanism considered at present. (c) The pion exchange mechanism of the two neutrino double beta decay process of Ericson and Vergados. (d) The electron-gamma exchange mechanism of the two neutrino double beta decay process.

into account through the Coulomb distortion factor $F(Z, E)$ of the $s_{1/2}$ electron wave functions. If we include in the interaction hamiltonian of this process also the interaction hamiltonian of the electron $e(x)$ and the electromagnetic $A_\alpha(x)$ fields

$$\mathcal{H}^{e\gamma}(x) = ie\bar{e}(x)\gamma_\alpha e(x)A_\alpha(x), \quad (25)$$

we obtain the mechanism of Fig.1d. We see that we have two additional electromagnetic vertices, which can account for the suppression of the $2\beta 0\nu$ - decay amplitude by the factor $e^2 = 4\pi\alpha \sim 0.1$. On the other hand there are some arguments which favoured this mechanism. First, the nuclear currents can be approximated by one body operators (the first term on the r.h.s. of eq.(7) do contribute to $2\beta 0\nu$ - decay amplitude). Second, the exchange potential for this mechanism is favoured by the small denominator of the order E_n in comparison with the denominator of the pion exchange potential containing the mass of the pion m_π ($(E_n/m_\pi)^2 \sim 10^2 - 10^3$). Third, the corresponding Green function for this mechanism shall contain all possible intermediate states $|J_n^\pi \rangle$ (not only $|1_n^+ \rangle$ states) in the same way as in the case of the neutrinoless double beta decay. The calculations concerning this mechanism are in progress.

In summary, we have shown that the two nucleon mechanism considered at present can be described with a class of meson exchange current diagrams. It allows us to study this process in the S-matrix approach. It is only necessary to know the nuclear wave functions of the initial and final nucleus and to derive two body operators from the corresponding $2\nu 2\beta$ - decay exchange diagrams. A simple analysis of the pion exchange diagrams indicates that this mechanism is not the dominant one for two neutrino double beta decay process. Therefore, an alternative electro-weak exchange mechanism is introduced.

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