

# 0БЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ 

## Дубна

E4-95-178

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THE MECHANISMS
OF THE $p D \rightarrow{ }^{3} H_{A} K^{+}$REACTION

Submitted to «Ядерная физика»

[^0]The $K^{+}$meson production in proton-nucleus collisions is of great interest as these reactions allow one to investigate the nuclear structure at short distances between nucleons [1]. The $p D \rightarrow{ }^{3} H_{\Lambda} K^{+}$reaction is a process with high momentum transfer. So, at the threshold of this reaction ( $T_{p}=1132 \mathrm{MeV}$ ) initial proton and deuteron have momenta $\sim 1 \mathrm{GeV} / \mathrm{c}$ in the $\mathrm{c} . \mathrm{m} . \mathrm{s}$. but in the final state all nucleons are at rest. At the proton kinetic energy in the laboratory system $T_{p}$ below 1580 MeV the $p+N \rightarrow N+\Lambda+K$ process on a free nucleon N at rest is forbidden by the energy-momentum conservation. Therefore the $p D \rightarrow{ }^{3} H_{\Lambda} K^{+}$reaction in this region occurs either through involving high momentum components of the deuteron wave function when incident proton collides with one of its nucleons (one-step mechanism, Fig. $1, a$ ) or by means of active interaction with two nucleons of the deuteron (two-step mechanism, Fig.1,b). It seems less obvious that in the last case the high momentum components of the wave function will be required. In this respect the $p D \rightarrow{ }^{3} H_{\Lambda} K^{+}$reaction is similar to $p D \rightarrow{ }^{3} H e \pi^{0}[2]$ and $p D \rightarrow{ }^{3} \mathrm{He} \mathrm{\eta}[3]$ reactions for which the two-step mechanism (called a three -body one in literature) was found to dominate [3, 4]. Indeed, the $p D \rightarrow{ }^{3} H_{\Lambda}+K^{+}$and $p D \rightarrow{ }^{3} \mathrm{He} \mathrm{\eta}$ reactions have deeper analogy in the framework of the two-step mechanism with subprocesses $p p \rightarrow d \pi^{+}$and $\pi^{+} n \rightarrow \Lambda K^{+}$or $\pi^{+} n \rightarrow p \eta$ respectively. The relation between masses of initial and final particles in these reactions is such that at the corresponding threshold of the reaction as well as for the angles $\theta_{c . m} \sim 90^{\circ}$ which determines the direction of the final meson momentum in respect to the incident beam, all intermediate particles ( $\pi$-meson, deuteron, nucleon) are near to on-mass-shell in a very wide energy range above the threshold [5]. For this reason the
two-step mechanism corresponding to the Feynman graph in Fig.1, $b$ seems to be the most realistic model of this reaction. It should be noted that for production of $\pi$-mesons and heavier mesons ( $\omega, \phi, \eta^{\prime}$ ) as well as for target-nuclei with $A \geq 3$ the above mentioned velocity matching does not take the place.

Another interesting aspect of the $p D \rightarrow^{3} H_{\Lambda} K^{+}$reaction is connected with formation of the hypertritium nucleus ${ }^{3} H_{\Lambda}$ in the final state. The ${ }^{3} H_{\Lambda}$ nucleus is a loosely bound system with the binding energy $\varepsilon \sim 2.35 \mathrm{MeV}$ which probably has a configuration of the ${ }^{3} H_{\Lambda} \rightarrow d+\Lambda[6]$. An investigation of the $p D \rightarrow{ }^{3} H_{\Lambda} K^{+}$reaction can give a new independent information about the wave function of the ${ }^{3} H_{\Lambda}$ nucleus.

In the framework of the two-step mechanism the amplitude $A^{\text {twost }}(p D \rightarrow$ ${ }^{3} H_{\Lambda} K^{+}$) of the $p D \rightarrow{ }^{3} H_{\Lambda} K^{+}$reaction can be written in the full analogy with the amplitude of the $p \mathrm{D} \rightarrow^{3} \mathrm{He} \mathrm{\eta}$ reaction [4]. As a result, we get
$A^{\text {twost }}\left(p D \rightarrow^{3} H_{\Lambda} K^{+}\right)=C \frac{\sqrt{3}}{2 m} A_{1}\left(p p \rightarrow d \pi^{+}\right) A_{2}\left(\pi^{+} n \rightarrow K^{+} d \Lambda\right) \mathcal{F}\left(P_{0}, E_{0}\right)$
where $A_{1}$ and $A_{2}$ are the amplitudes of the processes $p p \rightarrow d \pi^{+}$and $\pi^{+} n \rightarrow K^{+} \Lambda$ respectively, $m$ is the nucleon mass, $C=3 / 2$ is the isotopic spin factor allowing for the summation over isotopic spin indices in the intermediate state; the nuclear formfactor in exp. (1) is defined as

$$
\begin{equation*}
\mathcal{F}\left(P_{0}, E_{0}\right)=\int \frac{d^{3} q_{1}}{(2 \pi)^{3}} \frac{d^{3} q_{2}}{(2 \pi)^{3}} \frac{\Psi_{d}\left(\mathbf{q}_{1}\right) \Psi_{H}\left(\mathbf{q}_{2}\right)}{E_{0}^{2}-\left(\mathbf{P}_{0}+\mathbf{q}_{1}+\mathbf{q}_{2}\right)^{2}+i \epsilon} \tag{2}
\end{equation*}
$$

Here $\Psi_{d}\left(\mathrm{q}_{1}\right)$ is the wave function of the deuteron and $\Psi_{H}\left(\mathbf{q}_{2}\right)$ is the wave function of the ${ }^{3} H_{\Lambda}$ nucleus in the ${ }^{3} H_{\Lambda} \rightarrow d+\Lambda$-channel in
momentum space; $E_{0}$ and $\mathbf{P}_{0}$ are the energy and momentum of the intermediate $\pi$ - meson at zero momenta of nucleons in the nuclear vertices $\mathbf{q}_{1}=\mathbf{q}_{2}=0$ :

$$
\begin{equation*}
E_{0}=E_{K}+\frac{1}{3} E_{Y}-\frac{1}{2} E_{D}, \quad \mathbf{P}_{0}=\frac{2}{3} \mathbf{P}_{H}+\frac{1}{2} \mathbf{P}_{D} \tag{3}
\end{equation*}
$$

where $E_{j}$ is the energy of the $j$ th particle in the c.m.s., $\mathbf{P}_{D}$ and $\mathbf{P}_{H}$. are the momenta in the initial deuteron and the ${ }^{3} H_{\Lambda}$ nucleus in the c.m.s. respectively.

The amplitude (1) is connected to the differential cross section of the $p D \rightarrow{ }^{3} H_{\Lambda} K^{+}$reaction by the following expression

$$
\begin{equation*}
\frac{d \sigma}{d \Omega}=\frac{1}{64 \pi^{2}} \frac{1}{s_{p d}} \frac{\left|\mathbf{P}_{H}\right|}{\left|\mathbf{P}_{D}\right|} \overline{\left|A\left(p D \rightarrow^{3} H_{\Lambda} K^{+}\right)\right|^{2}} \tag{4}
\end{equation*}
$$

where $s_{p d}$ is the invariant mass of the initial $\mathrm{p}+\mathrm{d}$ state. The amplitudes $A_{1}\left(p p \rightarrow d \pi^{+}\right)$and $A_{2}\left(\pi^{+} n \rightarrow \Lambda K^{+}\right)$are related to the corresponding differential cross sections by analogous relations. One should note that the amplitudes $A_{1}$ and $A_{2}$ are factored outside the integral sign at the point $\mathbf{q}_{1}=\mathbf{q}_{2}=0$.

The amplitude of the one-step mechanism corresponding to the Feynman graph in Fig.1, a can be written as

$$
\begin{equation*}
A^{\text {onest }}\left(p D \rightarrow^{3} H K^{+}\right)=\sqrt{\frac{3}{m}} A_{3}\left(p N \rightarrow N \Lambda K^{+}\right) \Phi(Q) \tag{5}
\end{equation*}
$$

where $A_{3}$ is the $p N \rightarrow N \Lambda K^{+}$process amplitude which is factored outside the two-loop integration sign. The nuclear formfactor $\Phi(\mathbf{Q})$ is defined by

$$
\begin{equation*}
\Phi(Q)=\int d^{3} r \varphi_{d}(r) \varphi_{d}^{+}(r) \Psi_{H}^{+}\left(\frac{1}{2} \mathbf{r}\right) \exp (\mathbf{i Q r}) \tag{6}
\end{equation*}
$$

where

$$
\begin{equation*}
\mathbf{Q}=\frac{1}{3} \mathbf{P}_{H}-\frac{1}{2} \mathbf{P}_{D} \tag{7}
\end{equation*}
$$

One should note that integral (6) has a meaning of the deuteron elastic formfactor $F_{D}(2 Q)$ at the transferred momentum $\Delta=2 Q$ modified by the presence of the hypertritium wave function $\Psi_{H}\left(\frac{1}{2} r\right)$ in the integrand. It is obvious that the formfactor $\Phi(Q)$ decreases fast with growing $Q$.

The one-step amplitúde has been numericaly calculated here using both $S$ - and $D$-components of the deuteron wave function for the RSC potential in parametrisation [7]. Using the experimental data on the total cross section $\sigma_{N N \rightarrow K^{+}}[8]$ we estimated here the squared amplitude $\left|A_{3}\left(p N \rightarrow N \Lambda K^{+}\right)\right|^{2}$ as $\sim 250 \div 450 \mathrm{GeV}^{-2}$ in the initial proton energy range $1.6-3.0 \mathrm{GeV}$. The numerical calculations for the two-step mechanism are performed in the s-wave approximation for the deuteron wave function [7]. (As was shown by our calculations, the contribution of the deuteron D-component to the cross section is about $10 \%$ ). For the wave function of the ${ }^{3} H_{\Lambda}$ nucleus the $d+p$-model developed in Ref. [6] on the basis of separable $\Lambda N$-interaction is used. In this model the ${ }^{3} H_{\Lambda}$ wave function only contains the S-component. In the S-wave approximation the factor (2) takes the form

$$
\begin{equation*}
\mathcal{F}_{000}\left(P_{0}, E_{0}\right)=\frac{1}{4 \pi} \int_{0}^{\infty} j_{0}\left(P_{0} r\right) \exp \left(i E_{0} r\right) \varphi_{d}(r) \varphi_{H}(r) r d r \tag{8}
\end{equation*}
$$

For the differential cross section of the reaction $p p \rightarrow d \pi^{+}$the parametrisation of Ref. [9] is used here. For thè $\pi^{+} n \rightarrow \Lambda K^{+}$differential cross section the parametrisation of the total cross section from Ref. [10] is used and isotropic behaviour of the cross section is assumed.

We have investigated here numericaly the behaviour of the formfactor $\mathcal{F}_{000}\left(P_{0}, E_{0}\right)$ as a function of incident proton kinetic energy $T_{p}$ at different $K^{+}$-meson scattering angles $\theta_{\text {c.m. }}$. The momentum $P_{0}$ is
a rather fast decreasing function of $T_{p}$ at $\theta_{\text {c.m. }}=180^{\circ}\left(P_{0}=0.5-\right.$ $0.1 \mathrm{GeV} / \mathrm{c}$ in the range $\left.T_{p}=1.1-3.0 \mathrm{GeV}\right)$. On the contrary, at the scattering angles $\theta_{\text {c.m. }}=0^{\circ}$ and $90^{\circ}$ both the energy $E_{0}$ and momentum $P_{0}$ are increasing functions of $T_{p}\left(E_{0}, P_{0} \sim 0.5-1.2 \mathrm{GeV}\right)$. This behaviour of $P_{0}$ results in a large value of the formfactor $\left|\mathcal{F}_{000}\left(P_{0}, E_{0}\right)\right|^{2}$ at $\theta_{c . m .}=180^{\circ}$ in comparison to the ones at $\theta_{c . m .}=0^{\circ}$ and $90^{\circ}$. If one substitutes the wave function of the ${ }^{3} \mathrm{He}$ nucleus in the $d+p-$ channel [11] instead of the ${ }^{3} H_{\Lambda}$ hypernucleus in exp. (8) then the squared formfactor $\left|\mathcal{F}_{000}\left(P_{0}, E_{0}\right)\right|^{2}$ corresponds to the one for the $p D \rightarrow{ }^{3} \mathrm{He} \mathrm{\eta}$ reaction and it turns out to decrease faster with growing incident energy $T_{p}$ and its value at the threshold increases by a factor of 3-5.

The calculated differential cross sections of the $p D \rightarrow{ }^{3} H_{\Lambda} \mathrm{K}^{+}$reaction are presented in Fig.2 One can see from this picture that for any scattering angle the differential cross section has a sharp maximum at the proton energy $T_{p} \sim 1.2 \mathrm{GeV}$; which displays the corresponding sharp peak observed in the total cross section of the $\pi^{+} N \rightarrow \Lambda+K^{+}$reaction (see Ref. [10] and references therein). On the whole, the relations between differential cross sections at the angles $\theta_{\text {c.m. }}=0^{\circ}, 90^{\circ}$ and $180^{\circ}$ follow from corresponding relations between formfactors $\left|\mathcal{F}_{000}\left(P_{0}, E_{0}\right)\right|^{2}$.

The differential cross section of the $p D \rightarrow^{3} H_{\Lambda} \Lambda^{+}$reaction predicted by the two-step model differs from that for the $p D \rightarrow 3 \mathrm{He} \mathrm{\eta}$ reaction in two respects [4]. First, the maximum value of the $\Lambda^{-+}$-meson production cross section $\sim 1 \mathrm{nb} / \mathrm{sr}$ is about 50 times smaller than that for the $\eta-$ meson production. Secondly, the $p D \rightarrow 3 H_{\Lambda} K^{-+}$reaction cross section is a smoother decreasing function of incident proton
energy in comparison with the cross section of the $p D \rightarrow^{3} \mathrm{He} \mathrm{\eta}$ reaction. As follows from the behaviour of the formfactor $\left|\mathcal{F}_{000}\left(P_{0}, E_{0}\right)\right|^{2}$ both these peculiarities are in part connected to the form of the wave function of the ${ }^{3} H_{\Lambda}$ nucleus.

The results of calculation in the framework of the one-step mechanism are presented in Fig.3. One can see that the contribution of this mechanism is two - three orders of magnitude smaller than that following from the two-step model.

In conclusion, we note that the two-step mechanism of the $p D \rightarrow$ $3 H_{\Lambda} K^{+}$reaction is used owing to the velocity matching. In the case of $\eta$-meson production this mechanism explains qualitatively the energy dependence of the cross section above the threshold [4]. However, just at the threshold this model is in strong contradiction with the experimental data on the $p D \rightarrow^{3} H e \eta$ reaction [4]. One of a reason for it is probably a strong attractive interaction in the final $\eta-{ }^{3} \mathrm{He}$ state caused by an excitation of the nucleon $N^{*}(1535)$ resonance $[4,12]$. At present there are no experimental data pointing to the presence of strong coupling of the $K^{+}-$meson to any nucleon resonance in the resonance mass region of $1.2-2.0 \mathrm{GeV}$. Therefore one can suppose that final state interaction in the $p D \rightarrow{ }^{3} H_{\Lambda} K^{+}$reaction will not be of great importance in contrast to the $\eta$-production.

Authors is sincerely grateful to L.A. Kondratyuk and L. Mailing for useful discussion. This work was supported in part by grant $N^{o}$. 93-02-3745 of the Russian Foundation for Fundamental Researches.


Fig. 1 The one- step (a) and two-step (b) mechanisms of the $p D \rightarrow^{3}$ $H_{\Lambda} K^{+}$reaction.


Fig.2. The differential cross section of the $p D \rightarrow^{3} H_{\Lambda} K^{+}$reaction calculated for the two-step mechanism as a function of incident proton kinetic energy at different angles of $K^{+}$-meson $\theta_{\text {c.m. }}=0^{\circ}, 90^{\circ}, 180^{\circ}$


Fig.3. The same as in Fig. 3 but for the one-step mechanism

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