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V.G.Soloviev

NUCLEAR STRUCTURE AND ORDER-TO-CHAOS TRANSITION

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1 Nuclear many–body problem

We consider a general scheme of the nuclear many–body problem. The Hamiltonian of interacting nucleons in a nucleus has a general form

$$H = \sum_{1,2} T(1,2) a_1^+ a_2 - \sum_{1,2,2',1'} G(1,2;2',1') a_1^+ a_2^+ a_{2'} a_{1'}; \tag{1}$$

here a_1 and a_1^+ are the nucleon absorption and creation operators. The equation of motion for the density function

$$\rho(1,2) = < |a_1^+ a_2| > \tag{2}$$

has the following form:

$$i\frac{d\rho(1,2)}{dt} = < |[a_1^+ a_2, H]| > \tag{3}$$

and contains the distribution function $< |a_3^+ a_4^+ a_5 a_6| >$. The average value is taken over an arbitrary state $| >$. There is an exact relation

$$< |a_1^+ a_2^+ a_{2'}, a_{1'}| > = \sum_n \{ < |a_1^+ a_{1'}|n >< n|a_2^+ a_{2'}| > -$$

$$- < |a_1^+ a_{2'}|n >< n|a_2^+ a_{1'}| > + < |a_1^+ a_2^+|n >< n|a_{2'} a_{1'}| > \}. \tag{4}$$

According to the Hartree–Fock–Bogoliubov approximation

$$< |a_1^+ a_2^+ a_{2'} a_{1'}| > = < |a_1^+ a_{1'}| >< |a_2^+ a_{2'}| > -$$

$$- < |a_1^+ a_{2'}| >< |a_2^+ a_{1'} > + < |a_1^+ a_2^+| >< |a_{2'} a_{1'}| > . \tag{5}$$

It has been demonstrated in [1] that there is a representation where the density function has the diagonal form

$$\rho(1,2) = \rho(1)\delta_{1,2} \tag{6}$$

and the function $< a_1^+ a_2^+ >$ has the canonical form. In the mean field representation, the density function is diagonal for the ground state of the doubly closed shell or well–deformed nuclei. The mean field and interaction leading to superfluid pairing correlations are singled out from a general form of effective interactions in nuclei. This is the model of independent quasiparticles [2].

As has been shown in [3], a nuclear vibration is caused by the non–diagonal parts of the functions $\rho(1,2)$ and $< a_1^+ a_2^+ >$. In the mean field representation, the general equations are reduced to the RPA equations. It has been shown in [4] that the basic equations of the theory of finite systems [5] are identical to the equations obtained in [3].

All nuclear states are very complex. The wave functions of the ground and low-lying states have the simplest form in the mean field representation. If one uses the representation where the density function is diagonal for the fixed highly excited state, the wave function of this state has a very simple form but the wave functions of other states are very complex. It is very difficult to describe low-lying states in this representation.

There are several microscopic methods of describing nuclear excitations beyond the RPA. It is nuclear field theory [6,7] where one sums up the diagrams containing the collective and two-quasiparticle configurations. Calculations of giant resonances were performed in [8] and other papers with taking the *ph* (particle-hole) and $2p-2h$ configurations into account. The quasiparticle-phonon nuclear model (QPNM) [9,10] is used for a microscopic description of low-spin, small-amplitude vibrational states in spherical nuclei not far from the closed shells and in well-deformed nuclei. Many-phonon terms have been taken into account in the wave functions of excited states in a number of studies [11,12].

It is possible to state that none of the physical problems can be solved mathematically rigorously. Only simple models are integrable. The validity of the model is confirmed by the description of the relevant experimental data and the predictions. The Hartree-Fock-Bogoliubov approximation is very good; nevertheless it is an approximation in which very many terms in eq.(4) are not taken into account. Therefore, it appears that one should not be too worried about the self-consistency of the mean field and effective interactions. A very important step in nuclear theory is connected with an approximate, but not exact, treatment of the conservation law and with an approximate exclusion of spurious states. The mathematical basis for methods employing the violation of certain conservation laws was developed by Bogoliubov by using the concept of "quasiaverages".

2 Changing of nuclear structure with excitation energy

We describe low-lying states and giant resonances in terms of nuclear wave functions in the following form:

$$\Psi_n = \left\{ \sum_{1} b_1 Q_1^+ + \sum_{1,2} b_{1,2} Q_1^+ Q_2^+ + \sum_{1,2,3} b_{1,2,3} Q_1^+ Q_2^+ Q_3^+ + \cdots \right\} \Psi_0. \qquad (7)$$

Here, Q_i^+ denote the phonon creation operator, $|b|^2$ defines the contribution of the relevant components to the normalization of (7) and Ψ_0 is the wave function of the ground state. A two-quasiparticle state is treated as a specific case of a one-phonon state when the root of the RPA secular equation is very close to the relevant pole. The approximation consists in cutting off the series (7). In the QPNM, the expansion is restricted to two- or three-phonon terms.

According to the experimental information, the wave functions of low-lying states have a single dominating one-quasiparticle or one-phonon component. A reasonably good description of the low-lying states has been obtained with the dominant component alone. According to the calculation [13] of well-deformed nuclei in the rare-earth region within the QPNM, the contribution of the two-phonon components to the normalization of the wave functions of the states below 2.3MeV is smaller than 10%. The $K^\pi = 4^+$ double gamma vibrational states in ^{164}Dy, ^{166}Er and ^{168}Er are the exception. Most first $K^\pi = 4^+$ states are hexadecapole ones [14]. The centroid energy of the isovector giant dipole resonance, isoscalar and isovector quadrupole and octupole resonances, Gamow-Teller and other resonances are caused by the one-phonon components of their wave functions.

The quasiparticle-phonon interaction is responsible for the fragmentation of one-phonon states and for their mixing. The role of the quasiparticle-phonon interaction increases with the excitation energy. A reasonably good description of the fragmentation of one-quasiparticle and one-phonon states within the QPNM has been obtained in [13,15-18].

There are many experimental data on the double-quadrupole vibrational states in spherical nuclei. Information on double-octupole vibrational states is very scare. The double GDR has been observed in [19]. The experimental information on two-phonon states in well-deformed nuclei is limited by $K^\pi = 4^+$ double-gamma vibrational states in ^{164}Dy and $^{166,168}Er$ [20,21]. It is possible to state that most experimental data on the nuclear structure is an information on the one-quasiparticle and one-phonon components of the wave functions of excited states. The next step is an experimental study of fragmentation of two- and three-phonon states and quasiparticle $\otimes$ phonon and quasiparticle $\otimes$ two phonon states.

3 Order and chaos and order-to-chaos transition in terms of nuclear wave functions

Studies concerning the nearest-neighbour spacing distribution in nuclei usually identified chaos via the agreement with the Gaussian Orthogonal Ensemble statistics [22]. The nuclear wave functions of excited states with energies larger than 3 MeV have many components with isospin T_0 and T_0+1 containing few and many quasiparticles or/and phonons having also different K quantum numbers and so on. The wave functions are something like a superposition of several interacting ensembles. Therefore, it is necessary to study the order-to-chaos transition in terms of the properties of the nuclear wave functions [23]. The treatment of the order and order-to-chaos transitions in terms of the wave functions depends on the representation. We used the mean-field representation.

In the mean-field representation, the wave function of an excited state has the form (7). At high excitation energies, the wave function (7) has very many quasiparticle $\otimes$ phonon components. Due to the Pauli principle, the phonon operators are destroyed by many-quasiparticle operators in the wave function. The experimental data [24] on fast $E3$ transitions between excited states have shown that phonons survived among many quasiparticle configurations.

It has been stated in [23] that there is order in the large and chaos in the small few- or many-quasiparticle ⊗ phonon components of the nuclear wave functions. A quasiparticle-phonon interaction is responsible for the fragmentation of quasiparticle ⊗ phonon states and, therefore, for the order-to-chaos transition. It is necessary to stress that the full fragmentation of one-quasiparticle states cannot be considered as a transition to chaos. The many-quasiparticle configurations can give a large contribution to their wave functions.

The experimental data on the fragmentation of one-quasiparticle and one-phonon states do not allow one to establish the excitation-energy limit for the order-to-chaos transition. The study of the fragmentation of two- and three-phonon state and quasiparticle ⊗ phonon and quasiparticle ⊗ two-phonon states is the next step in investigating an order-to-chaos transition [25]. The fragmentation of three-quasiparticle states can be investigated in the one-nucleon transfer reaction on doubly odd targets. The fragmentation of five-quasiparticle states can be studied on the long-lived isomer $^{178m_2}Hf$ with $K^\pi = 16^+$ [26].

There are many experimental data on high spin isomers. Their wave functions contain many-quasiparticle and many-quasiparticle ⊗ phonon configurations. These levels demonstrate a regularity in the nuclear mean field up to an excitation energy of 10 MeV. The detailed experimental study of the gamma-ray de-excitation levels above the yrast line gives information on the fragmentation of many-quasiparticle and many-quasiparticle ⊗ phonon states. These strong and weak transitions allow one to establish at what energies the onset of chaos takes place [27].

4 Order against chaos in nuclei

Fluctuation properties, generic to all systems that show chaos, are independent of the specific properties of the system. Therefore, one does not need to study chaotic excited states. It is highly important to find nuclear-structure regularities at intermediate and high excitation energies.

To study the order-to-chaos transition, it is desirable to find a method of experimental observation of relatively large many-phonon configurations in nuclear wave functions. According to the calculation [28] within the QPNM, there are fast $E1$ and $M1$ transitions between excited states with the energy around 2.5 MeV. Fast $E1$ and $M1$ transitions take place if the wave function of the initial state has a relatively large two-phonon term consisting of the octupole phonon with $K^\pi = 0^-$ or 1^- or quadrupole phonon with $K^\pi = 1^+$ and another phonon, which is the same as the phonon of the final state wave function. Decay rates per second of these transitions are $10^2 - 10^4$ times as large as the transition rates to the ground state and $10^3 - 10^6$ times larger than transitions between relevant one-phonon states. It is possible to expect that the fast 2.5 MeV $E1$ and $M1$ transitions between large many-phonon components of the wave functions of the initial and final states differing by the octupole ($K^\pi = 0^-$ or 1^-) or quadrupole ($K^\pi = 1^+$) phonon should be observed independently of the excitation energy in deformed nuclei. Such fast $E1$ and $M1$ transitions do not exist in spherical nuclei.

The strong 2.5 MeV peak has been observed in the first-generation of gamma-ray spectra in the two-step cascades following thermal-neutron capture [29] and in the $^{163}Dy(^3He, \alpha)$ reaction at several excited energies [30]. These experimental data indicate relatively large many-phonon components in the wave functions in the excitation region up to 8 MeV in well-deformed nuclei. Therefore, one may expect that the order takes place up to excitation energies where the 2.5 MeV peak exists.

It is reasonable to introduce a new approach in the study of the nuclear-structure regularities against chaos at intermediate and high excitation energies. Instead of measuring and describing the energy and wave function of each individual state, one should investigate the strength distribution of few- and many-quasiparticle and one- and many-phonon states. This means that order in nuclei and order-to-chaos transition should be studied in terms of strength functions.

The experimental investigation of the strength distribution of the single-particle states in spherical nuclei allowed one to understand their properties while the single-particle state moves away from the Fermi level. The strength distribution of one-phonon states characterises the widths of GR and double GDR. The strength distribution of many-quasiparticle and many-phonon states reflects the regularity of the nuclear structure at intermediate and high excitation energies.

If the many-quasiparticle and many-phonon components of the wave functions are taken into account, a new region of regularity of nuclear states at higher excitation energies against chaos appears.

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