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EXISTENCE CRITERION OF SPURIOUS SOLUTIONS OF FADDEEV EQUATIONS

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Let H_0 and E be a free Hamiltonian and the total energy of a three-particle system with two-body interactions V_i, where $i = 1, 2, 3$. In Schrödinger theory [1], the wave function Ψ of this system obeys the equation

$$(H_0 - E)\, \Psi = (- \sum_{k=1}^{3} V_k)\, \Psi \tag{1}$$

and well-defined physical boundary conditions. In Faddeev theory [2], this problem is reformulated [3]: Ψ is represented as the sum

$$\Psi = \sum_{k=1}^{3} \Psi_k \tag{2}$$

of the Faddeev components Ψ_k obeying the set of three ($i = 1, 2, 3$) coupled equations

$$(H_0 - E)\, \Psi_i = -V_i\, \Psi = -V_i \sum_{k=1}^{3} \Psi_k \tag{3}$$

and the corresponding physical boundary conditions [4]. Then it is proved [3] that the Faddeev problem thus formulated is uniquely solvable and equivalent to the original Schrödinger problem.

In the Schrödinger eq. (1), unknown function Ψ is the total Faddeev sum (2), whereas in the Faddeev eqs. (3), unknown functions are the terms Ψ_i of this sum. In general, the sum of three nonzero functions may be identical zero. Therefore, if the set of eqs. (3) is not completed by any boundary conditions, it may have a nontrivial solution ($\Psi_i \neq 0$) for which the sum (2) is identical zero ($\Psi \equiv 0$). Unphysical solutions of this kind are called [5] the spurious solutions of eqs. (3). We denote the spurious solutions by S_i. This set of the functions, being inserted into eqs. (3), turns simultaneously both their right and left sides to zero:

$$(H_0 - E)\, S_i = 0\,, \tag{4}$$

$$\sum_{k=1}^{3} S_k = 0\,. \tag{5}$$

Therefore, the spurious solutions carry no information about two-body interactions, and all these solutions correspond to the trivial solution ($\Psi \equiv 0$) of the Schrödinger eq. (1). Hence, the spurious solutions do not correspond to any physical three-particle state.

Nonetheless, the spurious solutions are of special mathematical interest. The fact of the existence of the spurious solutions has been emphasized by a lot of authors (see reviews [5],[6] and original papers [7]-[13]). The completeness of physical and spurious solutions and spectral properties of the Faddeev differential equations have been studied in [7] and [8]. However, the spurious solutions are known in an explicit form only in particular cases. Namely, for a system of three identical bosons interacting via S-wave potentials and having the total angular momentum equal to zero [9],[10],[13] or unity [11],[12]. In general, the criterion of existence of the spurious solutions and a simple method for classification and analytical construction of these solutions are not yet at hand.

The main aim of this paper is to present the criterion and method of this sort for a system of three different particles interacting via spherically symmetric potentials.

We first briefly describe notation and present some formulae known in Faddeev theory [3], [4] and in the standard hyperharmonics approach [14], [15].

Let $(\vec{x}_i, \vec{y}_i)$ be three ($i = 1,2,3$) sets of the usual reduced Yacobi vectors [3] in the three-particle space $\mathcal{R}^6 = \mathcal{R}^3_x \oplus \mathcal{R}^3_y$ and define the corresponding hyperspherical coordinates [16] (r, Ω_i) by $r \equiv (x_i^2 + y_i^2)^{1/2}$ and $\Omega_i \equiv (\hat{x}_i, \hat{y}_i, \varphi_i)$, where r is the hyperradius, $\hat{c}$ stands for the two spherical angles [17] of some vector $\vec{c}$, and $\varphi_i \equiv \arctan(y_i/x_i)$.

By assumption, the potentials V_i are the functions of the corresponding distances x_i. Hence, the total energy E, the total angular momentum l ($\vec{l} = \vec{l}_{x_i} + \vec{l}_{y_i}$), its third component m, and parity $\sigma = \pm 1$ with respect to the inversion $(\vec{x}_i, \vec{y}_i) \to (-\vec{x}_i, -\vec{y}_i)$ are well-defined quantum numbers. Therefore, we are looking for the spurious solutions in the class $\mathcal{A}^\epsilon$ of the functions defined in $\mathcal{R}^6$, having continuous second-order derivatives with respect to all

their arguments (r, Ω_i) and possessing the set $\epsilon = (E, l, m, \sigma)$ of four conserved quantum numbers. For $\mathcal{A}^\epsilon$-class a complete and orthogonal basis on the unit sphere $\mathcal{S}^5$ in $\mathcal{R}^6$ is formed by the well-defined subset [16] of the standard hyperharmonics [18].

To present all our formulae in a more compact form we combine the indexes l_{x_i} and l_{y_i} into double index $\alpha \equiv \{l_{x_i}, l_{y_i}\}$ and write the hyperharmonics of that subset as

$$Y^{l,m}_{L,\alpha}(\Omega_i) \equiv N_{L,\alpha} \, (\sin\varphi_i)^{l_{y_i}} \, (\cos\varphi_i)^{l_{x_i}} \, P_n^{(l_{y_i}+1/2,\, l_{x_i}+1/2)}(\cos 2\varphi_i) \, \mathcal{Y}^{l,m}_\alpha(\hat{x}_i, \hat{y}_i) \,. \tag{6}$$

Here, $N_{L,\alpha}$ is the known [14] norm factor, $L = l_{x_i} + l_{y_i} + 2n$ with $n = 0, 1, \ldots$ is the hypermomentum, $\mathcal{Y}^{l,m}_\alpha$ is the standard bispherical harmonics [17], and, finally, $P_n^{(a,b)}$ is the Jacobi polynomial [19]. To ensure the conservation of parity σ hyperharmonics (6) should have the indexes L and α obeying the following conditions: the sum $l_{x_i} + l_{y_i}$ and L are even (odd) numbers for $\sigma = +1(-1)$. Further we imply this restriction.

The following properties of the hyperharmonics will be the key formulae for our study. First, hyperharmonics (6) are eigenfunctions,

$$\mathbf{L}^2 \, Y^{l,m}_{L,\alpha} = L(L+4) \, Y^{l,m}_{L,\alpha} \;, \tag{7}$$

of the squared hypermomentum or grand orbital operator $\mathbf{L}^2$, contained into the kinetic energy operator [14]

$$H_0(r, \Omega_i) = -r^5 \, \partial_r (r^{-5} \partial_r) + r^{-2} \, \mathbf{L}^2(\Omega_i) \,. \tag{8}$$

Second, hyperharmonics (6), written down in the different ($i \neq k$) representations $\langle \Omega_i \mid$ and $\langle \Omega_k \mid$, are connected by unitary transformation [20]

$$\langle \Omega_i \mid Y^{l,m}_{L,\alpha} \rangle \equiv Y^{l,m}_{L,\alpha}(\Omega_i) = \sum_{\alpha'} \langle \alpha \mid \alpha' \rangle_{l,L} \, Y^{l,m}_{L,\alpha'}(\Omega_k) \,. \tag{9}$$

Hereafter we imply that $i = 1,2,3$ and the indexes $\alpha \equiv \{l_{x_i}, l_{y_i}\}$ and $\alpha' \equiv \{l_{x_k}, l_{y_k}\}$ take all the values allowed at given l and L.

Transformation (9) conserves the quantum number l, m, σ and L and contains the Raynal-Revai coefficients [20]. They are defined as the overlap integrals:

$$\begin{aligned} \langle \alpha \mid \alpha' \rangle_{l,L}(\gamma_{k,i}) &\equiv \langle l_{x_i}, l_{y_i} \mid l_{x_k}, l_{y_k} \rangle_{l,L}(\gamma_{k,i}) \equiv \\ \langle Y^{l,m}_{L,\alpha}(\Omega_i) \mid Y^{l,m}_{L,\alpha'}(\Omega_k) \rangle &\equiv \int_{S^5} d\Omega_i \, (Y^{l,m}_{L,\alpha}(\Omega_i))^* \, Y^{l,m}_{L,\alpha'}(\Omega_k) \,. \end{aligned} \tag{10}$$

Therefore, these coefficients are real functions of the corresponding kinematical angles [15]

$$\gamma_{k,i} \equiv (-1)^p \arctan \left(\frac{m_j(m_1+m_2+m_3)}{m_i m_k} \right)^{1/2} , \tag{11}$$

where m_i are the particle masses, $p = 1$ for $k, i = 2,1; 1,3; 3,2$ and otherwise $p = -1$.

We now proceed to a study of the spurious solutions.

In Faddeev theory, each component Ψ_i and the equation, defining it in the set of eqs. (3), is written down in the corresponding representation $\langle r, \Omega_i \,|$. For this reason, conditions (4) and (5) imply that for any $i = 1,2,3$

$$(\, H_0(r,\Omega_i) - E \,)\, S_i(r,\Omega_i) = 0 \,, \tag{12}$$

$$\sum_{k=1}^{3} \langle\, r, \Omega_i \mid S_k(r,\Omega_k) \,\rangle = 0 \,. \tag{13}$$

We are seeking for the spurious solution as a series onto basis (6):

$$\langle\, r, \Omega_i \mid S_i \,\rangle = r^{-2} \sum_L \sum_\alpha \, S_{i,L,\alpha}(r)\, Y^{l,m}_{L,\alpha}(\Omega_i) \,. \tag{14}$$

Inserting (14) into (12) and using (7) and (8), we project the obtained equations onto basis (6) and then introduce a new variable $z = \sqrt{E} r$. As a result, for each unknown hyperradial component $S_{i,L,\alpha}(z)$ of series (14) we obtain the Bessel equation [19] with the index $\nu = L + 2$. This equation has the regular solution $J_{L+2}(z)$ only for $z > 0$, i.e. only if $E > 0$. All its regular solutions read

$$S_{i,L,\alpha} = D^L_{i,\alpha} \, J_{L+2}(\sqrt{E} r) \,, \tag{15}$$

where $D^L_{i,\alpha}$ is an arbitrary numerical factor.

To determine these factors we substitute series (14) with the found hyperradial components (15) in eqs. (13). We then project the equations obtained onto basis (6) and take into account (9) and (10). As a result, for each L we arrive at linear and homogeneous set of the desired equations

$$D^L_{i,\alpha} + \sum_{k \neq i} \sum_{\alpha'} \langle\, \alpha \mid \alpha' \,\rangle_{l,L} \, D^L_{k,\alpha'} = 0 \,. \tag{16}$$

Here, $i = 1,2,3$ and α and α' take all the values possible at prescribed l and L. That is why the matrix $\mathbf{M}^L$ of eqs. (16) has a finite dimension equal to triple number of hyperharmonics (6) with the same l and L. According to theory of matrix [21], eqs. (16) have N^L ($N^L \equiv dim\, \mathbf{M}^L - rank\, \mathbf{M}^L$) linearly independent solutions $\mathbf{D}^n \equiv D^{L,n}_{i,\alpha}$ ($n = 1, ..., N^L$) if and only if

$$det\, \mathbf{M}^L = 0 \,, \tag{17}$$

and a general solution of eqs. (16) is a linear combination

$$D^L_{i,\alpha} = \sum_{n=1}^{N^L} d^L_n \, D^{L,n}_{i,\alpha} \,. \tag{18}$$

of these solutions with arbitrary numerical coefficiens d^L_n.

Let $\mathcal{B}$ be a multitude of the values of L for which condition (17) holds. If this multitude is empty ($\mathcal{B} = \emptyset$), i.e. $det\mathbf{M}^L \neq 0$ for any L, then there are no nonzero functions (14) satisfying eqs. (13). In this trivial case, condition (5) is not fulfilled and the Faddeev eqs. (3) have no the spurious solutions, even though $E > 0$. Therefore, we assume that both the necessary and sufficient conditions ($E > 0$ and $\mathcal{B} \neq \emptyset$) of the existence of the spurious solutions are fulfilled. Note, that the first condition ($E > 0$) is known for a long time [5], [6], while the second condition ($\mathcal{B} \neq \emptyset$) is our original and main result. The next result is our classification of the spurious solutions. By using (14), (15), and (18), we classify the spurious solutions and present them in an explicit form.

So, for each $L \subset \mathcal{B}$ there are N^L linearly independent spurious solutions (14),

$$S_i^{L,n} = r^{-2} J_{L+2}(\sqrt{E}r) \sum_\alpha D_{i,\alpha}^{L,n} Y_{L,\alpha}^{l,m}(\Omega_i), \tag{19}$$

corresponding to the solutions $\mathbf{D}^n$ $(n = 1, ..., N^L)$ of eqs. (16) and possessing a good quantum number L in addition to the set ϵ. All the spurious solutions (14), corresponding to a general solution (18) of eqs. (16) and possessing good quantum numbers $L \subset \mathcal{B}$ and ϵ, are represented as

$$S_i^L = r^{-2} J_{L+2}(\sqrt{E}r) \sum_\alpha D_{i,\alpha}^L Y_{L,\alpha}^{l,m}(\Omega_i). \tag{20}$$

Finally, any spurious solution (14) with the set ϵ of well-defined quantum numbers can be represented as a linear combination

$$S_i = \sum_{L \subset \mathcal{B}} C^L S_i^L(r, \Omega_i) . \tag{21}$$

of the particular spurious solutions (20) and some numerical coefficients C^L.

For completeness we prove that all the spurious solutions possess one more good quantum number.

Let $\mathcal{S}^\epsilon$ and $\mathcal{U}^\epsilon$ be linear subspaces of the space $\mathcal{A}^\epsilon$, and let the hyperangular basises of $\mathcal{S}^\epsilon$ and $\mathcal{U}^\epsilon$ on S^5 in $\mathcal{R}^6$ be formed by hyperharmonics (6) with $L \subset \mathcal{B}$ and $L \not\subset \mathcal{B}$, respectively. The projectors on $\mathcal{S}^\epsilon$ and $\mathcal{U}^\epsilon$ then read

$$P_s \equiv \sum_{L \subset \mathcal{B}} \sum_\alpha | Y_{L,\alpha}^{l,m}(\Omega_i) \rangle \langle Y_{L,\alpha}^{l,m}(\Omega_i) | , \; P_u \equiv \sum_{L \not\subset \mathcal{B}} \sum_\alpha | Y_{L,\alpha}^{l,m}(\Omega_i) \rangle \langle Y_{L,\alpha}^{l,m}(\Omega_i) | . \tag{22}$$

Evidently, $\mathcal{S}^\epsilon$ is orthogonal to $\mathcal{U}^\epsilon$ in respect to the integration on S^5 over the set of the hyperangles Ω_i. As transformations (9) are unitary, representation (22) are invariant in reference to i. Taking into account this fact, we act on the spurious solutions (21) by projectors (22) and arrive then at a desired result:

$$P_s S_i = p_s S_i , \quad p_s = 1 ; \quad P_u S_i = 0 . \tag{23}$$

By virtue of (23), all the spurious solutions (21) are eigenfunctions of the operator P_s. Moreover, all of them correspond to the same eigenvalue $p_s = 1$ and belong to the subspace $\mathcal{S}^\epsilon$. Hence, in the subspace $\mathcal{U}^\epsilon$, eqs. (3) have no spurious solutions. According to the terminology of the paper [8], we call $\mathcal{S}^\epsilon$ and $\mathcal{U}^\epsilon$ the spaces of spurious and physical solutions of eqs. (3), respectively. It should be emphasized that the orthogonality of these spaces has been proved in [8] in another way.

To exemplify our results we present some simplest spurious solutions. Note that there are only two cases, namely, $l, L = 0$ and $l = 1, L = 2$, when the index α of hyperharmonics (6) can take only one value: $\alpha = \{0,0\}$ and $\alpha = \{1,1\}$, respectively. In these cases, there is only one nonzero Raynal-Revai coefficient: $\langle \alpha \mid \alpha \rangle_{l,L}(\gamma_{k,i}) = 1$. For this reason, relations (9) degenerate in the equalities

$$Y_{L,\alpha}^{l,m}(\Omega_i) = Y_{L,\alpha}^{l,m}(\Omega_k) , \quad l, L = 0 ; \quad l = 1, L = 2 , \tag{24}$$

and all the elements of the matrices $\mathbf{M}^0$ and $\mathbf{M}^2$ of the corresponding eqs. (16) are equal to unity. Therefore, $dim\, \mathbf{M}^L = 3$ and $rank\, \mathbf{M}^L = 1$ if $l, L = 0$ or $l = 1$, $L = 2$. In these both cases a general solution (18) of eqs. (16) has the following form: $D_{2,\alpha}^L$ and $D_{3,\alpha}^L$ are arbitrary numbers, whereas $D_{1,\alpha}^L = -D_{2,\alpha}^L - D_{3,\alpha}^L$. By formula (20), we find the Faddeev components

$$S_i^L = r^{-2} J_{L+2}(\sqrt{E}r) \; \left(D_{2,\alpha}^L (\delta_{i,2} - \delta_{i,1}) + D_{3,\alpha}^L (\delta_{i,3} - \delta_{i,1}) \right) \; Y_{L,\alpha}^{l,m}(\Omega_i) \tag{25}$$

of particular spurious solution with well-defined quantum numbers $l, m, L = 0$, $\sigma = 1$ and $\alpha = \{0,0\}$ or $l = 1$, $m = 0, \pm 1$, $\sigma = 1$, $L = 2$ and $\alpha = \{1,1\}$. The found solutions (25) do not depend on particle masses and vanish when all three particles are identical bosons or fermions. Let us prove the latter statement.

For any state of three identical bosons (fermions) the components Ψ_i of a physical solution of eqs. (3) obey also the conditions [3] ensuring a complete symmetry (antisymmetry) of the wave function Ψ in respect to the interchanging of particles in any pair.

Therefore, the components of the spurious solution should also satisfy the same conditions:

$$S_1(\vec{x}_1,\vec{y}_1) = \pm S_1(-\vec{x}_1,\vec{y}_1)\,, \quad S_2(\vec{x}_2,\vec{y}_2) = S_1(\vec{x}_2,\vec{y}_2)\,, \quad S_3(\vec{x}_3,\vec{y}_3) = S_1(\vec{x}_3,\vec{y}_3)\,, \tag{26}$$

where the sign $+(-)$ corresponds to boson (fermion) case. Substituting (25) in (26) and taking into account (24) give $D^L_{i,\alpha} = 0$ for any $i = 1,2,3$. Hence, only trivial ($S_i \equiv 0$) spurious solutions (25) satisfy conditions (26).

In basis (6), central potentials can be represented as operator series

$$V_i(x_i) = \sum_{\lambda_i=0}^{\infty} V^{\lambda_i}(\vec{x}_i)\,, \tag{27}$$

$$V_i^{\lambda_i}(\vec{x}_i) \equiv \sum_{L,L'} \sum_{l_{y_i}} |\, Y^{l,m}_{L,\alpha}(\Omega_i)\rangle\, V^{L,L'}_{i,\alpha}(r)\, \langle Y^{l,m}_{L',\alpha}(\Omega_i)\,|\,, \quad \alpha = \{\lambda_i, l_{y_i}\}\,. \tag{28}$$

Operators (28) have the following projecting properties:

$$\langle\, \Omega_i \mid V_i^{\lambda_i} \mid Y^{l,m}_{L',\alpha}(\Omega_i)\rangle = \delta_{\lambda_i,l_{x_i}} \sum_L V^{L,L'}_{i,\alpha}\, Y^{l,m}_{L,\alpha}(\Omega_i)\,, \quad \alpha = \{l_{x_i}, l_{y_i}\}\,. \tag{29}$$

Therefore, the potential operator $V_i^{\lambda_i}$ conserves the angular momentum λ_i of two particles in the pair i. These potentials are often used in low-energy nuclear physics [22]. When $\lambda_i = 0,1,2$ they are called S, P and D- wave potentials, respectively. Formulae (27) and (29) provide a possibility to consider the case of λ-wave potentials as a particular case of central potentials. To this end, in all the formulae (6), (7) and (14)-(22) we should forbid the indexes α and α' to take any values except $\{\lambda, l_{y_i}\}$ and $\{\lambda, l_{y_k}\}$.

To reproduce the known spurious solutions by using our method we consider the case of S-wave potentials. In these case $l_{x_i} = 0$ and $l_{y_i} = l$ are conserved as well as the set ϵ. For any l and L the indexes α and α' in eqs. (6) and (14)-(22) can take only one value: $\alpha = \alpha' = \{0, l\}$. Therefore, the dimension of the matrix $\mathbf{M^L}$ of eqs. (16) is equal to three. All nondiagonal elements of this matrix can be found by the known formulae [23]

$$M^L_{k,i} \equiv \langle\, 0, l \mid 0, l\,\rangle_{l,L}(\gamma_{k,i}) = (-\cos\gamma_{k,i})^l \;\; \frac{P^{(l+1/2,1/2)}_{(L-l)/2}(-\cos 2\gamma_{k,i})}{P^{(l+1/2,1/2)}_{(L-l)/2}(-1)}\,. \tag{30}$$

Since $dim\,\mathbf{M}^L = 3$, condition (17) is simplified and is reduced to the following equations:

$$det\,\mathbf{M}^L = 1 - (M^L_{2,1})^2 - (M^L_{1,3})^2 - (M^L_{3,2})^2 + 2\, M^L_{2,1}\, M^L_{1,3}\, M^L_{3,2} = 0\,. \tag{31}$$

Let $l = 0$ and $L = 2$. Then $\alpha = \alpha' = \{0, 0\}$. Using (30) and (31), we find that for any values of particle masses $M^2_{k,i} = \cos 2\gamma_{k,i}$, $rank\,\mathbf{M}^2 = 2$, and $det\,\mathbf{M}^2 = 0$. Using the relations $\gamma_{2,1}, \gamma_{1,3}, \gamma_{3,2} > 0$ and $\gamma_{3,2} = \pi - \gamma_{2,1} - \gamma_{1,3}$ for the kinematical angles (11), we solve eqs. (16). Substituting the found solution

$$D^L_{i,\alpha} = \sin\left(L\,(\gamma_{2,1} + \gamma_{1,3})\,\delta_{i,1} - (-1)^L L\,(\gamma_{1,3}\,\delta_{i,2} + \gamma_{2,1}\,\delta_{i,3})\right) \tag{32}$$

into (20) results in the corresponding particular spurious solution

$$\begin{aligned} S^L_i \;&=\; r^{-2}\, J_{L+2}(\sqrt{E}r)\; Y^{l,m}_{L,\alpha}(\Omega_i)\,\cdot \\ &\quad \sin\left(L\,(\gamma_{2,1} + \gamma_{1,3})\,\delta_{i,1} - (-1)^L L\,(\gamma_{1,3}\,\delta_{i,2} + \gamma_{2,1}\,\delta_{i,3})\right)\,. \end{aligned} \tag{33}$$

This solution possess good quantum numbers $l, l_{x_i}, l_{y_i} = 0$, $\sigma = 1$, and $L = 2$ and depends on particle masses via the kinematical angles.

Let $l, L = 1$. Then in all formulae $\alpha = \alpha' = \{0, 1\}$. Applying eqs. (30) and (31), we show that for any particle masses $M^1_{k,i} = -\cos\gamma_{k,i}$, $rank\,\mathbf{M}^1 = 2$, and $det\,\mathbf{M}^1 = 0$. We then find the solution of eqs. (16) and the corresponding particular spurious solution (20). These solutions are given by (32) and (33) with $l, L = 1$, $\sigma = -1$, and $\alpha = \{0, 1\}$.

Let l and L be arbitrary and particle masses be identical. Then $\alpha = \alpha' = \{0, l\}$ and, by definition (11), $|\,\gamma_{k,i}\,| = \pi/3$ for any k and i. Therefore, all nondiagonal elements (30) of the matrix M^L are equal to each other, and condition (31) is simplifyed. Namely, for each fixed l it is reduced to the equation defining L. We write this equation as

$$(-2)^{l-1}\, P^{(l+1/2,1/2)}_{(L-l)/2}(-1) = P^{(l+1/2,1/2)}_{(L-l)/2}(1/2)\,. \tag{34}$$

When $l = 0$ or $l = 1$ the value $L = 2$ or $L = 1$ satisfyes eq. (34) and the corresponding spurious solutions (33) take the following form:

$$S^L_i = (-1)^{L+1}\, (\sqrt{3}/2)\, r^{-2}\, J_{L+2}(\sqrt{E}r)\; Y^{l,m}_{L,\alpha}(\Omega_i)\,. \tag{35}$$

These functions satisfy conditions (26) for three identical bosons and reproduce the known spurious solutions [9]-[12].

Similarly we investigated the case of P-wave potentials. As we found, for $l = 0$, $L = 2$, $\alpha = \{1, 1\}$ or for $l, L = 1$, $\alpha = \{1, 0\}$ the solution of eqs. (16) and the corresponding spurious solution (20) are represented by (32) and (33), respectively. In the both considered cases, the spurious solution (33) obeys conditions (26) for a system of three identical fermions.

Exact solutions of the Faddeev eqs. (3) are known only for harmonic-oscillator potentials [9] and for the S-wave inverse square potentials [11], [24]. Equations (4) and (5) contain no potentials, and, therefore, the spurious solutions can be easily found analytically. These explicit solutions, for example, functions (25) and (33), can be then used as the reference solutions in testing the algorithms for numerical study of eqs. (3).

A method proposed in this paper is based on the well-developed hyperharmonics approach. Therefore, this method can be easily generalized for study of the spurious solutions in more realistic cases when particles possess spin and isospin. For such a generalization it is necessary to use the corresponding hyperharmonics [14] describing these degrees of freedom and repeat then all the constructions described above. Using the four-body hyperharmonics basis [14], [15], [25] one can also generalize our method for study of four-body spurious solutions. One of them has been recently found in [13].

In conclusion the main result of this paper may by formulated as follows:

Theorem. In $\mathcal{A}^c$-class the spurious solutions of eqs. (3) exist if and only if $E > 0$ and $B \neq \emptyset$. All the spuriuos solutions are represented as the sums (21) containing arbitrary coefficients C^L and the coefficients $D^L_{i,\alpha}$ obeying eqs. (16).

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Критерий существования ложных решений уравнений Фаддеева

Ложные решения дифференциальных уравнений Фаддеева для системы трех различных частиц, взаимодействующих посредством центральных парных потенциалов, исследуются в рамках метода гипергармоник. Доказывается критерий существования ложных решений. Предлагается простой способ для их классификации и построения в явном виде.

Работа выполнена в Лаборатории теоретической физики им.Н.Н.Боголюбова ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 1995

Pupyshev V.V. E4-95-152

Existence Criterion of Spurious Solutions of Faddeev Equations

The Faddeev differential equations for a system of three different particles interacting via central two-body potentials are investigated within the hyperharmonics approach. A simple method for classification and construction of these solutions is proposed.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 1995