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RAMZAUER EFFECT IN TRIPLET NEUTRON-NEUTRON SCATTERING

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In the nuclear theory [1], the interaction between two magnetic moments $\vec{\mu}_1$ and $\vec{\mu}_2$ of nucleons is described by the long-range tensor potential

$$V^m = -\left[3(\vec{\mu}_1 \cdot \vec{r})(\vec{\mu}_2 \cdot \vec{r})r^{-2} - (\vec{\mu}_1 \cdot \vec{\mu}_2)\right]\left(\frac{1}{r^3}\right). \tag{1}$$

As it was shown in [2], the magnetic-moment interaction (1) for proton-proton scattering is negligible comparing to the interaction of the magnetic moment of one proton with the Coulomb field of another (the so-called Schwinger scattering [3]) at low energies. Since then the interaction (1) is usually not included into the pp and np phase-shift analyses [4]. The role of this interaction in nn scattering has been investigated neither theoretically nor experimentally [1,5]. The point is that the direct measurement of nn scattering characteristics remains an open problem for many years [1,6]. Recently in [7], a possible in-beam experiment on the direct measurement of nn scattering length has been discussed. This discussion stimulated our theoretical study of nn scattering at low energies. Evidently, in the case of two neutrons, Schwinger scattering is absent. But, the interplay of interaction (1) and a pure nuclear interaction may generate a local minimum in the total cross section of the spin-triplet nn scattering [8]. The main aim of this letter is to prove that this minimum should exist.

We analyze the behavior of the scattering characteristics at low energies in the three cases corresponding to various assumptions about the total potential V of nn interaction. In the first case $V = V^s$, where V^s is the usual short-range nucleon-nucleon potential containing center, spin-orbit and tensor terms. In the second case $V = V^m$, and V^m is defined by (1). It means, that we artificially switch off a pure nuclear interaction. Finally, $V = V^{ms} = V^m + V^s$. Thus, we include both the potentials V^s and V^m. This seems to be a more realistic assumption about the nn interaction at low energies. The magnetic tensor potential (1) vanishes in the spin-singlet nn states as the tensor term of a pure nuclear potential does. Therefore, we study only the spin-triplet nn scattering.
We use the natural system of units in which $\hbar = M = c = 1$ and the total energy E is equal to the squared scattering momentum k. For the phase shifts $\delta_{l,j}$ and mixing parameter ϵ_j we use the definition given in [9]. Note that the total momentum j of the spin-triplet nn state is conserved, while, due to the tensor interaction, the angular momentum l is a good quantum number for $j = 0$ and odd j, and l is not conserved for even $j \geq 2$. In the first case $l = 1, j = 0$ or $l = j = 1, 2, 3...$ and in the other case $l = j \pm 1$.

To analyze the low energy behavior of scattering parameters, we apply the variable phase approach [10]. In this approach, the phase shifts $\delta_{l,j}(k)$ and mixing parameter $\epsilon_j(k)$ are defined as the limits, when $r \to \infty$, of the corresponding phase functions $\delta_{l,j}(r;k)$ and $\epsilon_j(r;k)$ which vanish at $r = 0$ and obey the following equations:

$$\begin{aligned}
\partial_r \delta_{l,j} &= -k^{-1} \sec 2\epsilon_j \Big(V_{l,l} \left(\cos^4 \epsilon_j P_l^2 - \sin^4 \epsilon_j Q_l^2\right) \\
&- V_{l,l'} \sin 2\epsilon_j \big(\sin 2\epsilon_j (P_{l'}^2 - Q_{l'}^2)/4 + \cos^2 \epsilon_j P_l Q_{l'} - \sin^2 \epsilon_j P_{l'} Q_l\big)\Big), \\
\partial_r \epsilon_j &= -k^{-1} \Big(V_{l,l'} \left(\cos^2 \epsilon_j P_l P_{l'} + \sin^2 \epsilon_j Q_l Q_{l'}\right) - 2^{-1} \sin 2\epsilon_j \sum_{l''=j\pm1} V_{l'',l''} P_{l''} Q_{l''}\Big).
\end{aligned} \tag{2}$$

Here $P_l \equiv \cos \delta_{l,j}\, j_l - \sin \delta_{l,j}\, n_l$, $Q_l \equiv \sin \delta_{l,j}\, j_l + \cos \delta_{l,j}\, n_l$, and $V_{l,l'}$ are the potential matrix elements; if l is conserved, then $l = l'$ and $\epsilon_j = 0$, in the opposite case $l, l' = j \pm 1$, $l \neq l'$.

possess a local minimum at this energy if the nn states with $j \le 2$ give the main contribution to this cross section. Let us verify that this condition is fulfilled.

In the theory of polarization and triple-scattering for two-nucleon systems the scattering is described by a matrix M (see [9]). The matrix elements $M_{n,n'}$ are infinite series containing the α–matrix elements (see Table III in [9]). The elements $\alpha_{l,j}$ for $j < 0$ and α_j for $j \le 0$ are defined to be zero and in all the other cases

$$\alpha_{l,j} = \cos 2\epsilon_j(\exp 2i\delta_{l,j} - 1)\,, \quad \alpha_j = i\sin 2\epsilon_j \exp(i(\delta_{j+1,j} + \delta_{j-1,j}))\,, \tag{12}$$

where, also by definition, $\epsilon_j \equiv 0$ when l is conserved. In any case ($a = s, m, ms$) the scattering parameters vanish in the low-energy limit by the corresponding laws (4)-(8). Hence, the low-energy behavior of the α^a–matrix elements (12) is described by the first terms of their Taylor series:

$$\alpha^a_{l,j} \simeq 2i\delta^a_{l,j}\,, \quad \alpha^a_j \simeq 2i\epsilon^a_j\,, \quad k \to 0\,, \tag{13}$$

where the scattering parameters are replaced by their leading terms.

To find the leading terms of the nuclear matrix M^s we use (6) and (13) and in the infinite series for its elements $M^s_{n,n'}$ we omit all the terms vanishing more rapidly than $O(E)$. These terms are of the order of E^2 and are initiated by the nn states with $j > 2$. In the above approximation,

$$\begin{aligned} &M^s_{1,0} \simeq \sqrt{2}\exp(-i\varphi)E(A^s_{1,2} - A^s_{1,0})P^1_1(x)\,,\ M^s_{0,1} \simeq (3/\sqrt{2})\exp(i\varphi)E(A^s_{1,1} - A^s_{1,2})P^1_1(x)\,, \\ &M^s_{1,1} \simeq 3E(A^s_{1,1} + A^s_{1,2})P_1(x)\,,\ M^s_{0,0} \simeq 2E(A^s_{1,0} + 2A^s_{1,2})P_1(x)\,,\ M^s_{1,-1} = 0\,, \end{aligned} \tag{14}$$

where $P^n_l(x)$ are associated Legendre polynomials of the scattering angle $\theta = \arccos x$. Using (14) we establish the low-energy behavior of the total nuclear cross section:

$$\sigma^s(E) = 4\pi E^2 \sum_{j=0}^{2} (2j+1)(A^s_{1,j})^2 + O(E^4)\,, \quad E \to 0\,. \tag{15}$$

In the case $a = m$ we apply (5) and (13), and in the infinite series for the magnetic matrix elements $M^m_{n,n'}$ we neglect all the terms vanishing more rapidly than $O(bk)$. Thus we obtain the Born approximation $\tilde{M}^m_{n,n'}$ for the series $M^m_{n,n'}$.

It should be noted that the series $\tilde{M}^m_{n,n'}$ with $n = n' = 0, 1$ are exactly equal to finite sums of their first terms corresponding to $l = 1$ because all the other terms vanish. Hence, these series contain only the contributions from the nn states with $j \le 2$. The series $\tilde{M}^m_{1,-1}$ is infinite but rapidly converging. It contains small contribution from all the nn states with $j \ge 2$, and it is a tabulated series.

These facts allow us to obtain the following explicit representations:

$$\begin{aligned} &\tilde{M}^m_{1,0} = (\tilde{M}^m_{0,1})^* = -\sqrt{2}b\exp(i\varphi)P^1_1(x)\,, \\ &\tilde{M}^m_{1,1} = -bP_1(x)\,, \quad \tilde{M}^m_{0,0} = 2bP_1(x)\,, \quad \tilde{M}^m_{1,-1} = \exp(-2i\varphi)bP_1(x)\,. \end{aligned} \tag{16}$$

By using them we find the Born approximation $d\tilde{\sigma}^m = 2b^2$ and $\tilde{\sigma}^m = 8\pi b^2$ for the magnetic differential cross section $d\sigma^m$ and magnetic total cross section σ^m, respectively. This approximation shows that in the low-energy limit $d\sigma^m$ should be isotropical and σ^m cannot vanish in contrast with σ^s.

To find the magnetic-nuclear matrix M^{ms} we use (4)-(6), (8) and (13). In the infinite series for the elements $M^{ms}_{n,n'}$ we neglect all the terms vanishing rapidly than $O(bk)$, and we also omit all the terms generated by V^s and vanishing more rapidly than $O(E)$. As a result, we derive the simple approximate formulae

$$M^{ms}_{n,n'} \simeq M^s_{n,n'} + \tilde{M}^m_{n,n'} \tag{17}$$

with the terms (14) and (16). The formulae (14)-(17) allow us to obtain the low-energy approximation for the total magnetic-nuclear cross section:

$$\sigma^{ms}(E) \simeq \tilde{\sigma}^m(E) + (4\pi/3)\, bE\, (2A^s_{1,0} - 3A^s_{1,1} + A^s_{1,2}) + \sigma^s(E) \tag{18}$$

and to conclude that the contribution from the nn states with $j > 2$ to this cross section is small and is of an order of $o(bk)$.

Let us analyze the structure of the sum (18). Its first term is the constant $8\pi b^2$. The second term is a negative and linear function of E describing the interplay of V^s and V^m. And finally, the last term is the squared function of E. Due to this structure of the sum (18), there are three energy regions. If the energy is small enough ($E \le E^{lower}$), then $\sigma^{ms}(E) \simeq \tilde{\sigma}^m$. Hence, due to the potential V^m, the total cross section $\sigma^{ms}(E)$ has a non-zero limit $8\pi b^2$ as $E \to 0$. In the third region, where the energy is so large ($E \ge E^{upper}$) that $\sigma^{ms}(E) \simeq \sigma^s(E)$, one can neglect V^m. In this region $\sigma^{ms}(E)$ is a growing function of E. Clearly, the total cross section having the above-described behavior should possess a local minimum in the intermediate region $E^{lower} < E < E^{upper}$. This effect is caused by the interplay of the short- and long-range potentials V^s and V^m. Therefore, we call it the neutron Ramzauer effect by analogy with the atomic Ramzauer effect [15]. The latter is well-known in atomic physics [16] and is interpreted as the result of interference of a long-range electric polarization potential $\alpha_e r^{-4}$ and a short-range potential opposite in sign. The analytic connection between the parameter α_e and the position a local cross section minimum has been first found in [17].

To derive that connection in our case, we find a zero of the first derivative of the function (18). As a result, we express the position of the local minimum of $\sigma^{ms}(E)$,

$$E^{ms}_{min} \simeq (-b/2)(2A^s_{1,0} - 3A^s_{1,1} + A^s_{1,2}) / \sum_{j=0}^{2} (2j+1)(A^s_{1,j})^2\,, \tag{19}$$

in terms of the known constant b and coefficients $A^s_{1,j}$. Substituting their values (7) into (19) we find $E^{ms}_{min} \simeq 20\,\mathrm{KeV}$.

To check our qualitative conclusions about the role of the interaction (1), we have carried out a lot of calculations. Here we mention the most interesting of them, because a more detailed discussion will be given in our forthcoming paper.

For all $j = 0, 1, ..10$ we compared the magnetic scattering parameters calculated by solving eqs. (2) with their Born approximation (3). For any $E \le 10$ MeV the relative accuracy of this approximation is about 10^{-4}. Therefore, the approximate expressions (5),(8),(16) and (17) are also correct within the same relative accuracy.

We defined the bounds $E^{lower}_{l,j}$ and $E^{upper}_{l,j}$ as the maximal and minimal values of the energy for which the relations $|\, \delta^m_{l,j}(k)/\delta^{ms}_{l,j}(k) - 1 \,| \le 0.1$ and, respectively, $|\, \delta^s_{l,j}(k)/\delta^{ms}_{l,j}(k) - 1 \,| \le 0.1$ hold. Using this definition and the scattering parameters calculated by solving eqs. (2), we obtained the following estimates: $E_{l,j}^{lower} \simeq 2$ KeV and $E_{l,j}^{upper} \simeq 0.3$ MeV, if $l = 1$ and $j = 0,1,2$; while $E_{l,j}^{lower} \simeq 0.7$ MeV and $E_{l,j}^{upper} \simeq 7$ MeV, if $l = 3$ and $j = 2$. This discrepancy is evident physically and mathematically. In fact, the centrifugal barrier $l(l+1)r^{-2}$ screens the short-range potential V^s more effectively than the long-range potential V^m and, moreover, in the first case, the power of this screening increases with growing l. Mathematically, this fact is explained by formulae (4)-(6). Owing them, $\delta_{l,j}^m(k)$ is linear in k for all l and j, while $\delta_{l,j}^s(k)$ decreases by the k^{2l+1} law. Therefore, the energy region, where $\delta_{l,j}^m(k)$ dominate over $\delta_{l,j}^s(k)$, is expanded when l increases. In other words, $E_{l',j'}^{lower} > E_{l,j}^{lower}$, if $l' > l$, and an analogous relation is valid for the upper bounds.

Then we showed numerically that the approximations (6), (10), (11) and (14) are correct with a relative accuracy about 10^{-2}, if $l = 0,1$, $j = 0,1,2$ and $E < 7$ MeV.

Here we do not present the curves for the phase shifts $\delta_{l,j}^a$ with $j = 0,1$ and $a = s, m, ms$ because we have made this in [8].

Now we sketch our scheme for calculation of the cross sections. First we found the nuclear and magnetic nuclear parameters and the corresponding matrices α^a by numerical integrating of eqs. (2) and using formulae (12). Then we decomposed the infinite series for the M^a matrix elements as

$$\mathrm{M}_{n,n'}^a = \mathrm{M}_{n,n'}^a(j \leq 2) + \mathrm{M}_{n,n'}^a(j > 2)\,, \tag{20}$$

where the first and second terms stand for the finite subsums of these series containing the α^a matrix elements (12) only with the index $j \leq 2$ and, respectively, $j \geq 3$.

To calculate σ^s, we approximated $\mathrm{M}_{n,n'}^s$ by $\mathrm{M}_{n,n'}^s(j \leq 2)$, i.e. we neglected V^s in all the nn states with $j \geq 3$. Note that this is the standard low-energy approximation [4,12] and, according to (15), its accuracy is of the order of $O(E^4)$.

As we have shown before, the Born approximation is valid in the case $a = m$. Therefore, we calculated the matrix M^m, the terms of its decomposition (20) and the magnetic cross section within this approximation. In particular, we obtained the auxiliary formulae:

$$\begin{aligned} &\tilde{\mathrm{M}}_{1,1}^m(j > 2) = (-b/3)\,P_3(x)\,, \quad \tilde{\mathrm{M}}_{1,-1}^m(j > 2) = b\,\exp(-2i\varphi)\,\left(P_1^1(x) - P_3^2(x)\right)\,, \\ &\tilde{\mathrm{M}}_{0,1}^m(j > 2) = (-\sqrt{2}/9)\,b\,\exp(i\varphi)\,P_3^1(x)\,;\; \tilde{\mathrm{M}}_{n,0}^m(j > 2) = 0\,,\; n = 0,1. \end{aligned} \tag{21}$$

Then we used them to find the matrix M^{ms} within the following approximation

$$\mathrm{M}_{n,n'}^{ms} \simeq \mathrm{M}_{n,n'}^{ms}(j \leq 2) + \tilde{\mathrm{M}}_{n,n'}^m(j > 2)\,. \tag{22}$$

It means, that we neglected V^s in the nn states with $j > 2$, but we taken into account V^m in all these states within the Born approximation. To find the first terms of the sums (22), we used the calculated matrix elements $\alpha_{l,j}^{ms}$ with $j \leq 2$. As for the second terms, we calculated them by expressions (21). And finally, we found $d\sigma^{ms}$ and then σ^{ms}.

In Fig.1 we plotted three curves representing the total cross sections σ^a, $a = s, m, s$, calculated by the above-mentioned scheme. These curves coincide with those calculated by approximate formulae (15) and (18). These curves show that magnetic-nuclear cross section σ^{ms} coincides with magnetic cross section $\sigma^m = 8\pi b^2$ at $E = 0$, possesses a minimum at $E \simeq 20$ KeV, and tends to nuclear cross section σ^s with growing E in the third region, where $E > E^{upper} \simeq 0.3$ MeV.

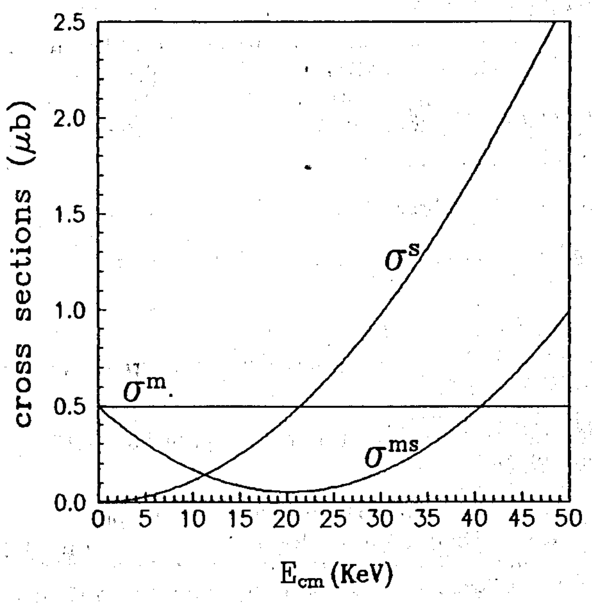

Fig. 1. The total nuclear σ^s, magnetic σ^m and magnetic-nuclear σ^{ms} cross sections of the triplet neutron-neutron scattering.

In conclusion, we summarize our main results. We have shown that the magnetic moment interaction (1) has to be taken into account in the region of low energies ($E <$ 0.3 MeV) because this interaction is responsible for the zeros of the magnetic-nuclear scattering parameters and the neutron Ramzauer effect. We have obtained the simple and explicit expressions (10) and (19) for these zeros and the position of the minimum of the magnetic-nuclear total cross section. We have derived the explicit formulae (8) and (18) ensuring a correct extrapolation of the nn scattering parameters and the total cross section to this region from the lowest experimentally available energy.

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Эффект Рамзауэра в триплетном нейтрон-нейтронном рассеянии

Мы показали, что в результате взаимодействия ядерного потенциала и потенциала магнитных моментов полное сечение триплетного рассеяния нейтрона на нейтроне должно иметь ненулевой предел при $E_{cm} = 0$ и локальный минимум при $E_{cm} \cong 20$ кэВ.

Работа выполнена в Лаборатории теоретической физики им.Н.Н.Боголюбова ОИЯИ.

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Pupyshev V.V., Solovtsova O.P. E4-95-121

Ramzauer Effect in Triplet Neutron-Neutron Scattering

As we show, due to interplay of pure nuclear and magnetic moment interactions, the total cross section of triplet neutron-neutron scattering should possess a non-zero limit at $E_{cm} = 0$ and a local minimum at $E_{cm} \approx 20$ keV.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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