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THE INTEGRAL EQUATION FOR THE WAVE FUNCTION OF THREE IDENTICAL PARTICLES IN THE BOUNDARY CONDITION MODEL

IV. The One-Dimensional Equation for the Channel Wave Function

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1. Introduction

In previous papers[1,2] (henceforth references to formulas of these two papers are denoted by (E1,...) and (E2,...), respectively) the Schrödinger equation has been considered for the bound state of three identical spinless particles. It has been assumed that the particles interact only in the s-states and that the interaction itself is described by the boundary condition model (BCM). It has been shown that in this case the determination of the total three-particle wave function (E2,4), which follows from the Schrödinger equation (E1, 22), is reduced in a correct way to the solution of a one-dimensional integral equation (E2,28) for a function of one variable. By means of this equation the total wave function of the system of three identical bosons can be determined in a unique way using the symmetry conditions (E1,21).

Below it will be shown that it is also possible to reduce the equation for the channel wave function (E2,5) given in the space representation to a one-dimensional integral equation in a correct way using the method developed in ref.[2] for the BCM under consideration of s-wave only. However, the circumstance connected with the fact that the channel wave function must not fulfil the symmetry condition of the type (E1, 21) leads to the fact that the obtained one-dimensional equation will determine this function in a non-unique way. Thus, we get the result that the determination of the total wave function (E2,4) becomes completely unique, while the occurrence of the non-uniqueness in the channel wave function does not influence in any way the one-dimensional integral equation.

2. The equation for the channel wave function in the BCM

As in ref. [2] we will consider a special problem of the bound state of three identical spinless particles interacting only in s-states. At the beginning we also assume that the pair interaction is described by a normal potential. In this case we can express the total three-particle wave function $\Psi(\vec{r}_1,\vec{\rho}_1)$ by means of the channel wave function in the following way [3]:

$$\Psi(\vec{r}_1,\vec{\rho}_1) = \psi(r_1,\rho_1)+\psi(r_2,\rho_2)+\psi(r_3,\rho_3), \tag{1}$$

where $\vec{r}_i$ and $\vec{\rho}_i$ are the corresponding Jacoby coordinates defined in (E1,20) and (E2,24), respectively. The channel wave function $\psi(r,\rho)$ fulfils under consideration of (E2,1) and (E2,2) the equation [4]:

$$\frac{1}{r_1\rho_1}\left[\frac{\partial^2}{\partial r_1^2}+\frac{3}{4}\frac{\partial^2}{\partial \rho_1^2}+E\right]r_1\rho_1\,\psi(r_1,\rho_1) = V(r_1)\psi(r_1,\rho_1)+ \tag{2}$$

$$+V(r_1)\frac{1}{4\pi}\int d\Omega_{\vec{r}_1}\left[\psi(r_2,\rho_2)+\psi(r_3,\rho_3)\right].$$

From this latter equation the equation follows for the Fourier-component $\psi(r,q)$ of the variable ρ of $\psi(r,\rho)$:

$$\frac{1}{r_1}\left[\frac{d^2}{dr_1^2}+E_q-V(r_1)\right]r_1\psi(r_1,q) = V(r_1)S(r_1,q), \tag{3}$$

where

$$S(r_1,q)=\frac{1}{4\pi}\int d\Omega_{\vec{r}_1}\, d\vec{\rho}_1\, e^{-i\vec{q}\vec{\rho}_1}\left[\psi(r_2,\rho_2)+\psi(r_3,\rho_3)\right], \tag{4}$$

$$E_q = E-\frac{3}{4}q^2. \tag{5}$$

The solution of eq. (3) might be written in terms of the Green-function $H(r,r',E)$ in the form:

$$\psi(r,q)=\int_0^\infty r'^2dr'\,H(r,r',E_q)V(r')S(r',q). \tag{6}$$

For a potential $V(r)$ of finite range c ($V(r)=0$ for $r>c$) and under consideration of the explicit expression of the Green function $H(r,r',E)$ given by (E2,11) it follows from (4) and (6) that the channel wave function reads

$$\psi(r,\rho)=\frac{1}{4\pi}\theta(c-r)X(r,\rho)+\theta(r-c)\chi(r,\rho), \tag{7}$$

where

$$\chi(r,\rho)=\frac{1}{2\pi^2}\int_0^\infty q^2dq\, j_0(q\rho)\, i\sqrt{E_q}\, h_0^{(1)}(r\sqrt{E_q})\,Y(q), \tag{8}$$

$$Y(q)=\frac{1}{4\pi}\int d\vec{r}_1\, d\vec{\rho}_1\, g(r_1,E_q)e^{-i\vec{q}\vec{\rho}_1}\left[\psi(r_2,\rho_2)+\psi(r_3,\rho_3)\right], \tag{9}$$

$$g(r,E)=\frac{2}{\pi}\int_0^\infty \rho^2d\rho\, j_0(\rho r)\, t_0(\rho,\sqrt{E},E). \tag{10}$$

Here $j_0(x)$ and $h_0^{(1)}(x)$ are the spherical Bessel and Hankel functions, respectively, and $t_0(\rho,\sqrt{E},E)$ is the half-on-shell S-component of the two-particle t-matrix given in (E1,1). The step function is defined by $\theta(X)=1$ for $X>0$ and $\theta(X)=0$ for $X<0$.

Now as in ref.[5] we will assume that the expression (7) for the channel wave function, which does not contain the potential in an explicit form, is also correct in the BCM. Then the determination of the channel wave function in the BCM, i.e., actually the solution of eq.(2) is reduced to the determination of the two-dimensional function $X(r,\rho)$ and the one-dimensional function $Y(q)$. To determine correctly the functions $X(r,\rho)$ and $Y(q)$ we need two equations. As the first equation relation (9) can be used, because the function $g(r,E)$ has in (9) a well defined value (E2,21):

$$g(r,E)=-\frac{1}{r}\,\frac{e^{-ic\sqrt{E}}}{f_0-ic\sqrt{E}}\left[f_0\,\delta(r-c)+c\,\delta'(r-c)\right], \tag{11}$$

which follows from the fact that in BCM the half-on-shell t-matrix $t_0(\rho,\sqrt{E},E)=t_0(\sqrt{E},\rho,E)$ is defined in a unique way by expression (E1,9). Therefore the first equation has the form:

$$Y(q)=8\pi\int_0^\infty r\,dr\int_0^\infty \rho\,d\rho\left[\frac{1}{4\pi}\theta(c-r)X(r,\rho)+\theta(r-c)X(r,\rho)\right]\times$$
$$\times\int_{|\frac{1}{2}r-\rho|}^{\frac{1}{2}r+\rho} r'dr'\,g(r',E_q)\,j_0(q\rho'), \tag{12}$$

where

$$\rho'=\frac{\sqrt{3}}{2}\sqrt{r^2+\frac{4}{3}\rho^2-r'^2}\ . \tag{13}$$

The second equation for the function $X(r,\rho)$ and $Y(q)$ follows from condition (E2,18), i.e., follows from the fact that for $r_1<c$ the s-component $\Psi_0(r_1,\rho_1)$ of the total wave function $\Psi(\vec{r}_1,\vec{\rho}_1)$ is identically equal to zero. Taking into account (7) and introducing a new function

$$A(r,\rho)=r\rho X(r,\rho) \tag{14}$$

we obtain result from eq. (E2, 18):

$$\theta(c-r)A(r,\rho)+2\theta(c-r)\int_{|\frac{1}{2}r-\rho|}^{\frac{1}{2}r+\rho}\frac{dr'}{\rho'}\theta(c-r')A(r',\rho')=$$
$$=-8\pi\theta(c-r)\int_{|\frac{1}{2}r-\rho|}^{\frac{1}{2}r+\rho} r'dr'\,\theta(r'-c)X(r',\rho'), \tag{15}$$

where ρ' is defined by expression (13).

3. The solution of the equation for the function $A(r,\rho)$.

Neglecting in (15) the right-hand side we see that remaining equation for $A(r,\rho)$ is of the same form as eq. (E2,35). Then it is possible to solve this equation by means of the method used in ref.[2] in an analytical way. This method has been proposed for the solution of a similar problem in ref.[6] x). In solving eq. (15) we introduce as

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x) The author of paper [6] is V.Efimov, from the Leningrad Institute of Nuclear Physics, having the same name as the author of this paper.

in ref.[6] new variables R, α, α' defined by

$$
\begin{aligned}
&\tau = R\sin\alpha, \qquad \rho = \frac{\sqrt{3}}{2} R\cos\alpha,\\
&\tau' = R\sin\alpha', \qquad \rho' = \frac{\sqrt{3}}{2} R\cos\alpha',\\
&R^2 = \tau^2 + \frac{4}{3}\rho^2 = \tau'^2 + \frac{4}{3}\rho'^2 .
\end{aligned}
\tag{16}
$$

In new variables eq. (15) reads for $\tau < C$:

$$
\mathcal{A}(R,\alpha) + \frac{4}{\sqrt{3}} \int\limits_{|\frac{\pi}{3}-\alpha|}^{\min(\frac{\pi}{3}+\alpha,\, \frac{2\pi}{3}-\alpha)} d\alpha'\, \theta(c - R\sin\alpha')\, \mathcal{A}(R,\alpha') = -F(R,\alpha),
\tag{17}
$$

where

$$
F(R,\alpha) = \int\limits_{|\frac{\pi}{3}-\alpha|}^{\min(\frac{\pi}{3}+\alpha,\, \frac{2\pi}{3}-\alpha)} d\alpha'\, \theta(R\sin\alpha' - c)\, \Phi(R,\alpha'),
$$

$$
\Phi(R,\alpha) = \frac{8}{\pi\sqrt{3}} \int\limits_0^{\infty} q\,dq\, Y(q)\, e^{i\sqrt{E_q}\, R\sin\alpha} \sin\left(\frac{\sqrt{3}}{2} q R\cos\alpha\right).
\tag{18}
$$

The variable R in eq. (17) can be considered as a parameter and depending on R the whole region $\tau < C$ can be divided into some subregions as is shown in fig. 1 in accordance with that the right-hand side and the integration limits of (17) have now different values. Using the methods given in ref.[6] the solution for $\mathcal{A}(R,\alpha)$ can be found in the whole region $\tau < C$ and has for the different subregions the form:

$$
\mathcal{A}_1(R,\alpha) = -\int_{\frac{\pi}{3}-\alpha}^{\frac{\pi}{3}+\alpha} d\alpha'\, \Phi(R,\alpha'),
\tag{19}
$$

$$
\begin{aligned}
\mathcal{A}_2(R,\alpha) = {}& \int_{\alpha_1}^{\alpha} d\alpha'\, \mathcal{G}(\alpha,\alpha')\, \Phi\left(R, \frac{\pi}{3}+\alpha'\right) + \\
&+ \frac{1}{\sin(\gamma_0 + \pi/4)} \left[H_1(R)\sin\left(\gamma - \frac{\pi}{4}\right) - \sqrt{2}\, H_2(R) \sin\frac{4}{\sqrt{3}}(\alpha - \alpha_1) \right],
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
\mathcal{A}_3(R,\alpha) = {}& \int_{\alpha}^{\alpha_0} d\alpha'\, \mathcal{G}(\alpha,\alpha')\, \Phi\left(R, \frac{2\pi}{3}-\alpha'\right) + \\
&+ \frac{1}{\sin(\gamma_0 + \pi/4)} \left[-H_1(R)\cos\left(\gamma + \frac{\pi}{4}\right) + \sqrt{2}\, H_2(R) \cos\frac{4}{\sqrt{3}}(\alpha_0 - \alpha) \right],
\end{aligned}
\tag{21}
$$

$$
\begin{aligned}
\mathcal{A}_4(R,\alpha) = {}& \int_{\alpha_2}^{\alpha} d\alpha'\, \mathcal{G}(\alpha,\alpha')\, \Phi\left(R, \frac{\pi}{3}+\alpha'\right) + \\
&+ \frac{\sqrt{3}}{2} \frac{H_3(R)}{Q(R)} \left[\sin 4\alpha_2 \cos\frac{4}{\sqrt{3}}(\alpha - \alpha_2) - \sin 4\alpha_3 \sin\frac{4}{\sqrt{3}}(\alpha - \alpha_2) \right],
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
\mathcal{A}_5(R,\alpha) = {}& \int_{\alpha}^{\alpha_3} d\alpha'\, \mathcal{G}(\alpha,\alpha')\, \Phi\left(R, \frac{2\pi}{3}-\alpha'\right) + \\
&+ \frac{\sqrt{3}}{2} \frac{H_3(R)}{Q(R)} \left[\sin 4\alpha_2 \sin\frac{4}{\sqrt{3}}(\alpha_3 - \alpha) + \sin 4\alpha_3 \cos\frac{4}{\sqrt{3}}(\alpha_3 - \alpha) \right],
\end{aligned}
\tag{23}
$$

$$
\mathcal{A}_6(R,\alpha) = \mathcal{A}_7(R,\alpha) = \frac{\sqrt{3}}{2} \frac{H_3(R)}{Q(R)} \sin 4\alpha,
\tag{24}
$$

where $\mathcal{G}(\alpha,\alpha') = \sin\frac{4}{\sqrt{3}}(\alpha - \alpha') - \cos\frac{4}{\sqrt{3}}(\alpha - \alpha')$,

$$
H_1(R) = \int_{\alpha_0}^{\alpha_3} d\alpha\, \Phi(R,\alpha),
$$

$$
H_2(R) = \int_{\alpha_3}^{\pi/2} d\alpha\, \sin\frac{4}{\sqrt{3}}\left(\frac{\pi}{2} - \alpha\right) \Phi(R,\alpha),
$$

$$
H_3(R) = \int_{\alpha_0}^{\pi/2} d\alpha\, \sin\frac{4}{\sqrt{3}}\left(\frac{\pi}{2} - \alpha\right) \Phi(R,\alpha),
$$

$$
Q(R) = \frac{\sin\left(4 + \frac{4}{\sqrt{3}}\right)\varkappa_0}{4 + \frac{4}{\sqrt{3}}} - \frac{\sin\left(4 - \frac{4}{\sqrt{3}}\right)\varkappa_0}{4 - \frac{4}{\sqrt{3}}},
$$

$$\gamma=\frac{4}{\sqrt{3}}\left(\alpha-\frac{\pi}{6}\right),\qquad \alpha_0=\arcsin\frac{c}{R},$$

$$\gamma_0=\frac{4}{\sqrt{3}}\left(\alpha_0-\frac{\pi}{6}\right),\quad \alpha_1=\frac{\pi}{3}-\alpha_0,\quad \alpha_2=\alpha_0-\frac{\pi}{3},$$

$$\alpha_3=\frac{2\pi}{3}-\alpha_0,\quad \varkappa_0=\frac{\pi}{2}-\alpha_0=\arccos\frac{c}{R}.$$

In the region 8 (see fig. 1) the equation (17) takes the form:

$$A(R,\alpha)+\frac{4}{\sqrt{3}}\int\limits_{\left|\frac{\pi}{3}-\alpha\right|}^{\min\left(\frac{\pi}{3}+\alpha,\ \frac{2\pi}{3}-\alpha\right)} d\alpha'\,A(R,\alpha')=0$$

and its solution is

$$A_8(R,\alpha)=F(R)\sin 4\alpha, \tag{25}$$

where $F(R)$ is an arbitrary function of R. Therefore in correspondence with (14) the channel wave function (7) can be written as

$$\begin{aligned}\psi(r,\rho)&=\theta(c-R)\,\varphi(r,\rho)+\\&+\theta(R-c)\,\theta(c-r)\frac{1}{4\pi r\rho}A(r,\rho)+\theta(r-c)\,\chi(r,\rho),\end{aligned} \tag{26}$$

where in accordance with ref./6/ the function $\varphi(r,\rho)$ is an arbitrary function of mixed symmetry belonging to the two-dimensional representation of the group of the permutation for three identical particles and fulfils the condition

$$\varphi(r_1,\rho_1)+\varphi(r_2,\rho_2)+\varphi(r_3,\rho_3)=0. \tag{27}$$

The non-uniqueness of the type $\theta(c-R)\,\varphi(r,\rho)$ comes out also in ref./2/ in accordance with (E2,50) at the definition in the BCM of the product $V(r)\,\Psi_0(r,\rho)$ (E2,14) from the three-particle Schrödinger equation (E2,3). However, in ref./2/ it has been shown that if the BCM is considered as the limit case of the potential (E1,17) then it follows from symmetry condition (E1, 21) of the total wave function that we have to require $\varphi(r,\rho)\equiv 0$ which leads to a unique definition of the channel wave function and the total wave function (1). A similar consideration for removing the non-uniqueness in the channel wave function cannot be applied. Therefore, eq. (2) has no unique solution in the BCM.

Note, that the non-uniqueness of the type $\theta(c-R)\,\varphi(r,\rho)$ of the channel function (26) in the region 8 of fig. 1 has no influence on the form of eq. (12). Actually, from the explicit form (11) of the function $g(r,E)$ it follows that the integration in (12) over a term containing $\chi(r,\rho)$ happens in the plane (r,ρ) defined by the conditions:

$$r<c,\quad \left|\tfrac{1}{2}r-\rho\right|\le c\le \tfrac{1}{2}r+\rho. \tag{28}$$

This region of the plane (r,ρ) is the region given by the subregions 2-5 (see fig. 1) and does not contain the region 9 which includes the non-uniqueness (25) of the function $A(R,\alpha)$. Inserting the explicit expressions (20)-(23) for $A(R,\alpha)$ into (12) and taking into account (18) and (8) give a one-dimensional equation for the function $Y(q)$. Its solution will determine in accordance with (8) and (19)-(24) the channel wave function (26) in a unique way for all values of r and ρ besides those defined by the subregion 8 in fig. 1. However from condition (27) it follows that in spite of the fact that the channel wave function $\psi(r,\rho)$ is non-uniquely defined at $R<c$ the total three-particle function $\Psi(\vec{r},\vec{\rho})$ defined in accordance with (1) is completely unique. The fact that the total wave function $\Psi(\vec{r},\vec{\rho})$ is defined in the BCM in a unique way from eq. (2) and relation (1) is not in contradiction with the non-uniqueness of the channel wave function and its definition (E2,5) because in accordance with (E2,5) for the definition of $\psi(r,\rho)$ it is necessary to know the product $V(r)\Psi_0(r,\rho)$. From eq. (2) and relation (1) it is possible to determine at least in unique way $\Psi_0(r,\rho)$ but in the BCM in accordance with (E2,18) and (E1,17) at $r<c$ $\Psi_0(r,\rho)=0,\ V(r)=\infty$ and therefore in (E2,5) there arises the non-uniqueness of the type $0\cdot\infty$. As it has been shown in ref. [2] it is possible to determine the product $V(r)\Psi_0(r,\rho)$ and therefore also $\psi(r,\rho)$ in a unique way on the basis of the three-particle Schrödinger equation and by means of the symmetry condition of the total three-particle wave function $\Psi(\vec{r},\vec{\rho})$.

4. Conclusion

In our specific model of three identical bosons interacting only in s-states the channel wave function in the momentum space fulfils the Faddeev equation[3]

$$\psi(K,q)-[2\pi^2(K^2-E_q)]^{-1}\int d\vec{q}\,'\big[t_0(K,|\tfrac{1}{2}\vec{q}+\vec{q}\,'|,E_q)\times \qquad (29)$$

$$\times\psi(|-\vec{q}-\tfrac{1}{2}\vec{q}\,'|,q')+t_0(K,|-\tfrac{1}{2}\vec{q}-\vec{q}\,'|,E_q)\psi(|\vec{q}+\tfrac{1}{2}\vec{q}\,'|,q')\big]=0,$$

where $t_0(K,\rho,E)$ is the s-component of two-particle t -matrix and E_q is defined by (5). It has been mentioned above that in the BCM the solution of eq. (2), which contains in contradiction with eq. (29) the interaction potential in an explicit form, is non-unique. Therefore, of course, the question arises about the uniqueness of the solution of eq. (29) in the BCM. If we consider the BCM as a limit case of the potential (E1,17) then the question is reduced to the explanation of the uniqueness of the solution of the Faddeev equation at presence of a hard core only in the two-particle s-component. In ref.[7,8] it has been shown, that in the case when the hard core acts in all two-body partial components, the Faddeev equations[3] have no unique solution. Using the explicit form of the above obtained non-uniqueness of the channel function $\psi(r,\rho)$ (26) in the coordinate representation it is possible to demonstrate that eq. (29), while it takes only the s-components of the interaction into account, will not have a unique solution in the BCM.

Thus, when $\psi(K,q)$ fulfils eq. (29) then a solution of even this equation in the BCM is also function

$$\bar{\psi}(K,q)=\psi(K,q)+\Gamma(K,q), \qquad (30)$$

where

$$\Gamma(K,q)=\int d\vec{r}_1 d\vec{\rho}_1\, e^{-i\vec{K}\vec{r}_1-i\vec{q}\vec{\rho}_1}\,\theta(c-R)\,\varphi(r_1,\rho_1) \qquad (31)$$

and $\varphi(r_1,\rho_1)$ is an arbitrary function of mixed symmetry fulfilling the condition (27), $R^2 = r_i^2 + \frac{4}{3}\rho_i^2$ and $\vec{r}_i, \vec{\rho}_i$ are respectively the Jacoby coordinates defined by (E1, 20) and (E2, 24). The partial s-component of the two-particle t-matrix takes in the BCM in accordance with (E1.10) the form

$$t_0(K,\rho,E) = -(K^2-E)F_0(K,\rho) + \\ +[\cos Kc - ic\sqrt{E}\, j_0(Kc)]e^{ic\sqrt{E}}\, t_0(\rho,\sqrt{E},E), \tag{32}$$

where

$$F_0(K,\rho) = \int_0^c r^2 dr\, j_0(Kr)\, j_0(\rho r) \tag{33}$$

and $t_0(\rho,\sqrt{E},E) = t_0(\sqrt{E},\rho,E)$ is the s-component of the half-on-shell t-matrix (E1,9). Taking into account the explicit form (32) of $t_0(K,\rho,E)$ and $F_0(K,\rho)$ (33) and using identity $\theta(c-R) = \theta(c-R)\theta(c-r_1)$ and writing (31) in the form

$$\Gamma(K,q) = \frac{1}{4\pi}\int d\Omega_{\vec{K}}\, \Gamma(\vec{K},\vec{q}) =$$

$$= \int d\vec{r}_1 d\vec{\rho}_1\, j_0(Kr_1)\, e^{-i\vec{q}\vec{\rho}_1}\, \theta(c-R)\varphi(r_1,\rho_1)$$

it can be shown that the insertion of $\Gamma(K,q)$ into (29) leads to the result

$$\Delta(K,q) = \Delta_1(K,q) + \Delta_2(K,q), \tag{34}$$

$$\Delta_1(K,q) = \int d\vec{r}_1 d\vec{\rho}_1\, j_0(Kr_1)\, e^{-i\vec{q}\vec{\rho}_1}\, \theta(c-R)[\varphi(r_1,\rho_1) + \\ + \varphi(r_2,\rho_2) + \varphi(r_3,\rho_3)], \tag{35}$$

$$\Delta_2(K,q) = -\frac{2(4\pi)^2}{K^2-E_q}[\cos Kc - ic\sqrt{E_q}\, j_0(Kc)]e^{ic\sqrt{E_q}} \times \\ \times \int_0^\infty r dr \int_0^\infty \rho d\rho\, \theta(c-R)\varphi(r,\rho) \int_{|\frac{1}{2}r-\rho|}^{\frac{1}{2}r+\rho} r' dr'\, g(r',E_q)\, j_0(q\rho'). \tag{36}$$

Here $g(r,E)$ and ρ' are defined by (11) and (13), respectively. From the condition (27) and from eq. (35) it follows, that $\Delta_1(K,q) = 0$, but from the explicit expression (11) for $g(r,E)$ in accordance with (36) it follows that $\Delta_2(K,q) = 0$. Therefore, in correspondence with (34) we have $\Delta(K,q) = 0$ and the function $\bar{\psi}(K,q)$ (30) is also as $\psi(K,q)$ solution of eq. (29) with the t-matrix (32). This latter fact shows the non-uniqueness of the solution of the Faddeev equation in the case of three identical bosons under consideration of a hard core only in the s-components. In contradiction with that the one-dimensional equation for the function $\Upsilon(q)$ following from (12) does have a completely unique solution. The non-uniqueness of the solution of eq. (29) in the BCM givesreason to doubt about the correctness of the value E_0 =12.69 MeV obtained in ref. /9/ for the binding energy of three identical bosons by solving the two-dimensional eq. (29) with the t-matrix (32). This value is quite different from that value E_0 =7.70 MeV obtained in ref./10/ by solving one-dimensional equation which has been deduced in ref./2/ from the Schrődinger equation for the wave function $\Psi(\vec{r}_1,\vec{\rho}_1)$ (1) of the same system of three identical bosons interacting via the BCM as in ref./9/. To explain the difference between the results of refs.[9,10] it should be of use to calculate E_0 by means of the one-dimensional equation (12), which comes from eq. (2) for the channel wave function. The explicit form of this equation and its solution will be published in the next report.

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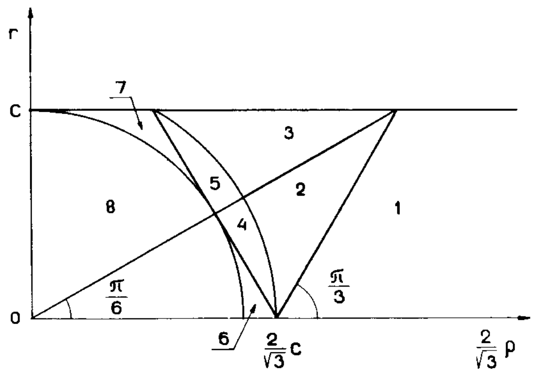

Fig. 1. The regions in defining the function $\mathcal{A}(R,\alpha)$.

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