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Introduction

Study of properties of one-dimensional models has become an important line in investigation of strong Langmuir (l) and whistler (w) turbulence. There exist /1–5/ soliton solutions in plane one-dimensional geometry. It was pointed out in papers /5–8/ that linear description of plasma density perturbations is not available in the range of near-sonic soliton velocities, $M \to 1$, and some analytical solutions (subsonic two-humped Langmuir solitons) have been found /7,8/. An important result is that small corrections due to nonlinearity and dispersion of ion-sound (s) perturbations turn out to be able to reconstruct solution for H.F. field.

We shall consider further some aspects of the problem, namely, stability of these peculiar (ls) solitons, and the dynamics of their interactions not only with each other, but with s-solitons and with rarefaction s-waves. Besides, two-humped ("camel") whistler solitons having positive density perturbation, will be described.

1. Dynamics of ls-solitons

The most rigorous description of nonlinearity and dispersion of s-waves, which are driven by H.F. field E of l-oscillations, is accomplished in dimensionless variables of papers /2–5/ by the set of hydrodynamics equations and Poisson equation:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{\partial \phi}{\partial x}, \qquad (1.1)$$

$$\frac{\partial n}{\partial t} + \frac{\partial (nv)}{\partial x} = 0, \qquad (1.2)$$

$$\frac{\epsilon}{3}\frac{\partial^2 \phi}{\partial x^2} = n_e - n, \quad n_e = \exp(\phi - \epsilon W). \qquad (1.3)$$

Here v is the dimensionless ion velocity in terms of $v_s = \sqrt{T/m_i}$, $T = T_e + T_i$, ϕ is the potential of L.F. perturbations in terms of T/e, $\epsilon = \frac{4}{3}\frac{m_e}{m_i}$, $W = |E|^2$, n, n_e are the ion and electron densities averaged over l-oscillation period in terms of unperturbed plasma density n_0.

In this notation an equation for $E(x,t)$ takes the form

$$i\frac{\partial E}{\partial t} + \frac{\partial^2 E}{\partial x^2} = \epsilon^{-1}(n_e - 1)E. \qquad (1.4)$$

From the set (1.1)-(1.3), considering L.F.nonlinearity and dispersion effect due to deviation from quasi-neutrality in L.F. motions as small corrections, one can obtain an equation for velocity

$$v_{xx} - v_{tt} + \frac{\epsilon}{3}v_{xxtt} - \epsilon(v^2)_{xt} = \epsilon W_{xt} + \epsilon(vW_x)_x. \qquad (1.5)$$

The second term in the right-hand side can be neglected in comparison with the first one, because of smallness of v in comparison with unity.

For density perturbations $\delta n = \epsilon^{-1}(n-1)$ in ls-solitons moving in an arbitrary direction of x-axis, an approximate equation takes the form /6,9/.

$$\delta n_{tt} - \delta n_{xx} - \epsilon(\delta n^2)_{xx} - \frac{\epsilon}{3}\delta n_{xxtt} = W_{xx}. \qquad (1.6)$$

Analytical solutions of papers /7,8/

$$E = \sqrt{48\epsilon\lambda^2}\cdot \mathrm{th}\, b\xi \cdot \mathrm{sech}\, b\xi \cdot \exp\left(i\frac{M}{2}x - i\Omega t\right),$$

$$\delta n = 6\lambda\, \mathrm{sech}^2 b\xi, \quad \xi = x - Mt, \quad \lambda = \frac{3}{20}\epsilon^{-1}(M^2 - 1), \qquad (1.7)$$

$$b = \sqrt{-\lambda}$$

with accuracy up to the terms corresponding to corrections of more high orders, are soliton solutions of sets (1.1)-(1.4) or (1.4), (1.6).

Stability of soliton solution (1.7) was verified by numerical experiment at $M = 0.95$ on the basis of sets (1.4), (1.6). Solution (1.7), being concerved with high accuracy, propagates up to distances many times longer than the size of soliton. Increase of density pit by the factor of 1.2 first leads to the increase of both $|E|^2$-maxima and to the transform of some part of l-plasmons into the front hump (acceleration of l-plasmons); then at $t \simeq 30$ both maxima of $|E|^2$ become again equal. Gradually density pit, being adjusted with the number of l-plasmons in initial packet, decreases, apparently, forming a soliton with the amplitude and velocity defined by the value of integral $S_1 = \int W dx$ stored in it. Formation is accompanied by the arising of sign-alternating train of s-waves behind soliton, which run in the direction opposite to that of l-packet. The distance at which ls-soliton is formed is much more than the soliton size.

When the initial density pit is equal to 80 per cent of that of ls-soliton, $|E|^2$-maxima first decrease and the back hump is enriched with l-plasmons. Note, that $|E|^2$-minimum ($|E|^2 = 0$) corresponds to maximum depth of the density pit.

Here let us consider the counter collision of two identical ls-solitons (1.7) at $M = 0.95$, solving the set (1.1)-(1.4) via computer (see *Fig. 1*). Such an interaction turns out to be inelastic like that in nonresonance region, when $M = 1 - \Delta$, $\Delta \lesssim 1$. Nonstationary solution is always

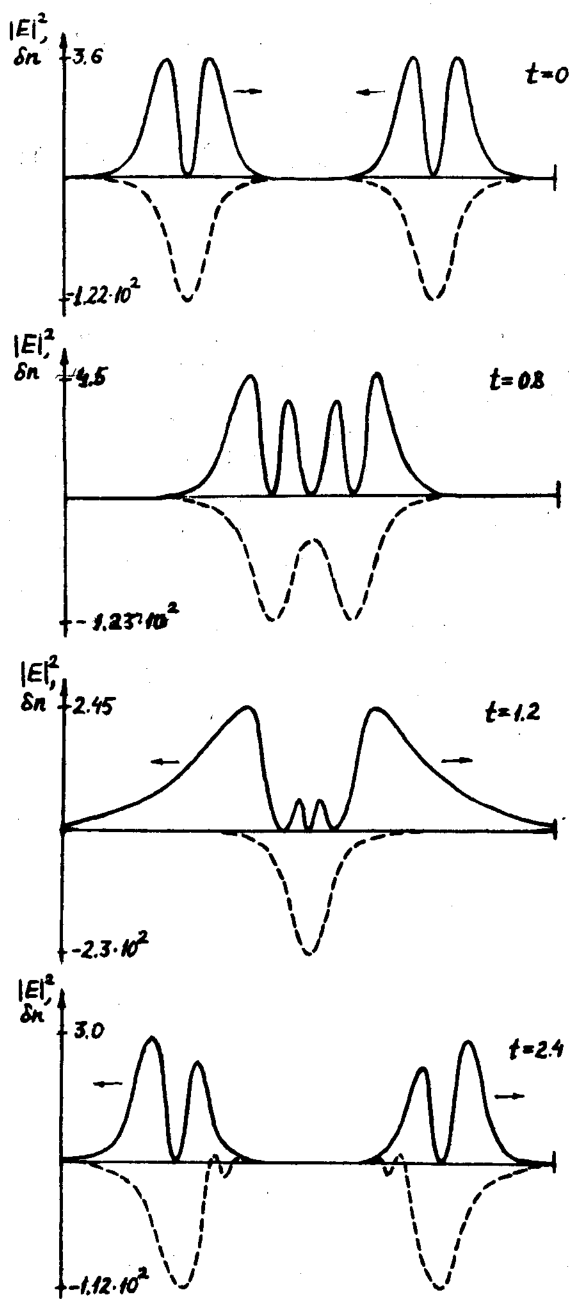

Fig. 1. Interaction of the counter moving l_s *-solitons at* M = *0.95.*

four-humped. Shortly after solitons contact each other the exchange of energy and momentum between plasmons occurs; the inside humps lose some part of plasmons and the outside ones are enriched by them; finally, there takes place the throwing-out of supersonic l -plasmons. A share of these accelerated supersonic plasmons is considerable - about 20 per cent. The main part of l -plasmons groups again in two diverging two-humped packets, which seem to turn into the less amplitude l_s -solitons.

The set of equations (1.1)-(1.4) allows us to consider head-on collisions between different type solitons, i.e., ion-sound and near-sonic (M= 0.95) Langmuir ones. At s-soliton amplitude $A_s = -0.2 A_\ell$ (A_ℓ - amplitude of density perturbation of l_s -soliton) their interaction is rather small. But yet at $A_s = -0.4 A_\ell$ s-soliton knocks about 40 per cent of l -plasmons out off l_s -soliton when going through the latter; and this part of plasmons gets supersonic velocities (see *Fig. 2*). Therefore, one can see again the essential acceleration of l -plasmons.

Results of interaction between l_s -soliton and rarefaction s -wave with $A_s = 0.4 A_\ell$ are less considerable. Only collision with rarefaction s -wave $A_s = A_\ell$ leads to a noticeable inelasticity (see *Fig. 3*). About one fourth of l -plasmon is concentrated in supersonic throwing-out. Oscillatory structure of density behind rarefaction s -wave can be explained mainly by the destroying of this wave due to nonlinearity and dispersion of s -waves.

We should underline here that the results of interaction of l_s -solitons with nonlinear compression and rarefaction s -waves differ qualitatively from those of interaction between nonresonance l -solitons and linear s -impulses[3].

2. Near-sonic Whistler Solitons

To describe density perturbation occuring in near-sonic whistler solitons (ws) with $M = 1 + \Delta$, $0 < \Delta \ll 1$, one should, as it was done previously, take into account corrections because of dispersion and nonlinearity of s -waves. It yields a nonhomogeneous equation:

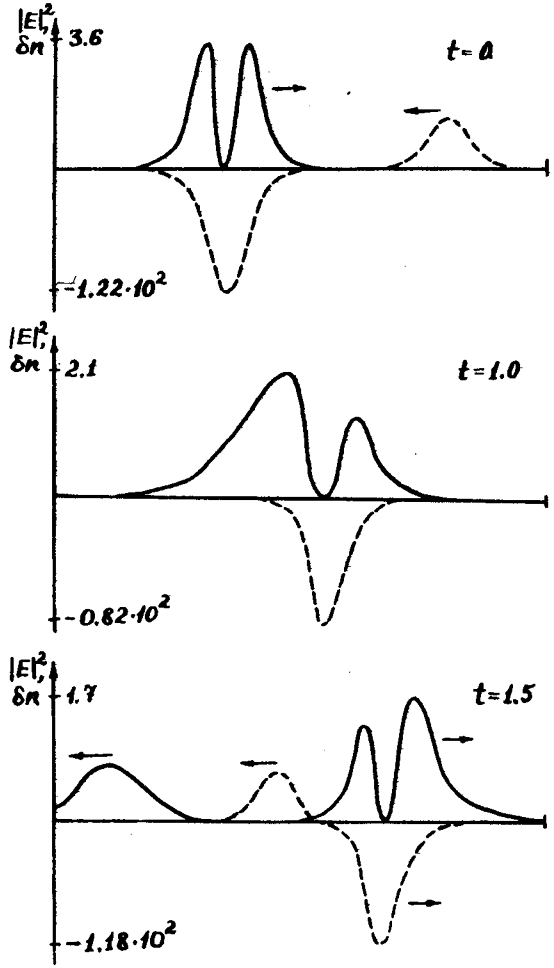

Fig. 2. Collision of ls *-soliton* (M = 0.95) *with* s *-soliton.*

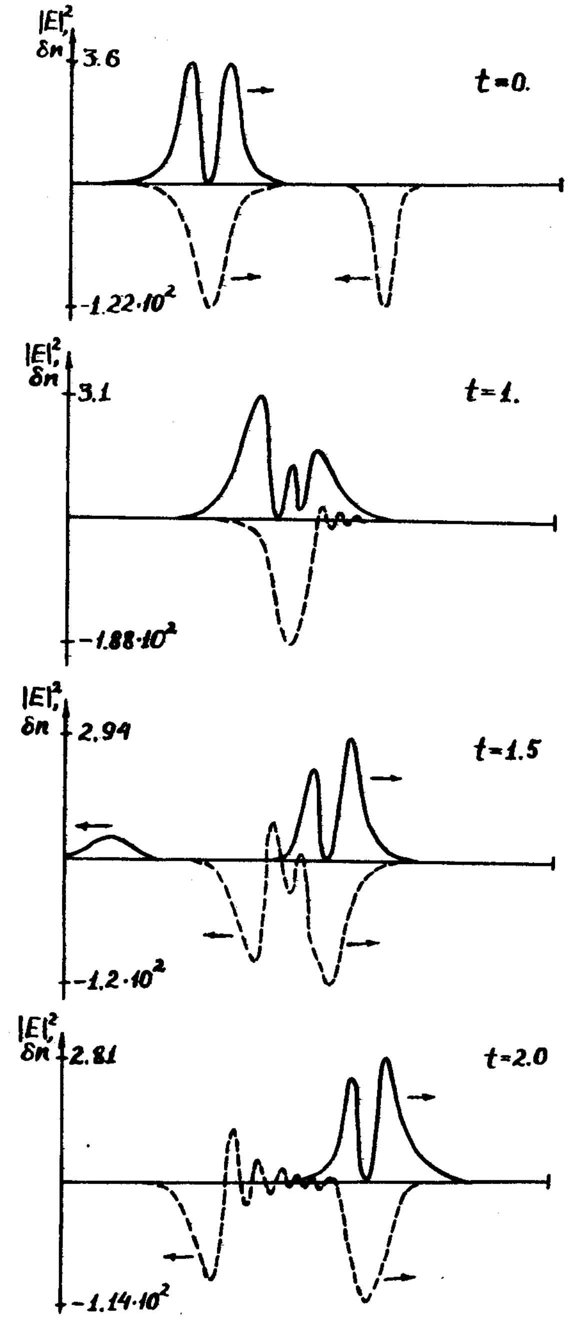

Fig. 3. Interaction of ls *-soliton* (M= 0.95) *with ion-sound rarefaction wave.*

$$V_{tt} - v_s^2 V_{zz} - v_s^2 (V^2)_{zz} - d_e^2 V_{zztt} = \frac{|h|^2_{zz}}{4\pi m_i n_0}. \quad (2.1)$$

Here we denote: $V = (n-1)/n_0$, h is the magnetic field of w-oscillations. Let us use dimensionless variables (d denotes dimensional ones):

$$z_d = \frac{\omega_{He}}{\omega_{pe}^2} \frac{c^2}{v_s} z, \quad t_d = \frac{\omega_{He}}{\omega_{pe}^2} \frac{c^2}{v_s^2} t, \quad V = \frac{\omega_{pe}^4 v_s^2}{\omega_{He}^2 k_0^2 c^2} \delta n,$$

$$|h_d|^2 = 4\pi n_0 m_i \frac{v_s^4 \omega_{pe}^4}{k_0^2 \omega_{He}^2} |h|^2, \quad k_d = \frac{\omega_{pe}^2}{\omega_{He}} \frac{v_s}{c^2} \kappa$$

(k_0 is an average wave-number which defines soliton velocity $v_g = 2\omega_{He} k_0 c^2/\omega_{pe} = M v_s$).

Using the Schrödinger equation for h field, we get the following set of equations:

$$i \frac{\partial h}{\partial t} + \frac{\partial^2 h}{\partial z^2} + \delta n h = 0, \quad (2.2)$$

$$\delta n_{tt} - \delta n_{zz} - \sigma(\delta n^2)_{zz} - \tilde{\beta}\, \delta n_{zztt} = -|h|^2_{zz}, \quad (2.3)$$

where

$$\sigma = \frac{\omega_{pe}^4 v_s^2}{\omega_{He}^2 k_0^2 c^4}, \quad \tilde{\beta} = \frac{\omega_{pe}^4 v_s^2 d_e^2}{\omega_{He}^2 c^4}.$$

In coordinate system, moving with the ws-soliton velocity, $x = z - Mt$, writing the field as $h = h(x)\cdot \exp(i\kappa_0 z - i\Omega t)$ and introducing $\lambda = \Omega - \kappa_0^2$, we get the following set of equations to search for a steady-state soliton ($M = 2\kappa_0$)

$$h_{xx} + \delta n \cdot h + \lambda h = 0, \quad (2.4)$$

$$\beta\, \delta n_{xx} + g\, \delta n + \sigma(\delta n)^2 = -h^2. \quad (2.5)$$

Here: $g = 1 - M^2$, $\beta = \tilde{\beta} M^2$, λ is an energy level to be obtained in self-consistent potential.

We shall look for the solution of set (2.4), (2.5) assuming that density perturbation in soliton has the form $\delta n = A\, \mathrm{sech}^2(\alpha x)$. Using such δn with indefinite parameters A and α from Eq. (2.4) one can get

$$h_{xx} + (\lambda + A\, \mathrm{sech}^2 \alpha x)\cdot h = 0.$$

Let us introduce a new variable $\xi = \mathrm{th}\, \alpha x$ /9/ . Equations (2.4), (2.5) take the form

$$\frac{d}{d\xi}\left[(1-\xi^2)\frac{dh}{d\xi}\right] + \left[s(s+1) - r^2(1-\xi^2)^{-1}\right] h = 0, \quad (2.6)$$

$$gA + \sigma A^2(1-\xi^2) - 2\beta A \alpha^2 (1 - 3\xi^2) = -h^2 (1-\xi^2). \quad (2.7)$$

Here we denote: $s(s+1) = A\alpha^{-2}$, $r = \sqrt{-\lambda}\, \alpha^{-1}$. Solution $h(\xi)$, finite at $\xi = \pm 1 (x = \pm\infty)$, can be obtained only when $s - r = n$, where $n = 0, 1, 2 \ldots$. At the same time Eq. (2.7) must be satisfied. It can be done if $n = 1$, $s = 2$. It this case the solution of (2.6) is given by the formula $h = B\xi\cdot(1-\xi^2)^{\frac{(s-1)}{2}}$, $B = \mathrm{const}$. Substituting this formal solution into Eq. (2.7) indefinite constants turn out to obey the following relations

$$(\beta - \sigma) A^2 = -B^2, \quad gA + \sigma A^2 - \frac{\beta}{3} A^2 = 0.$$

Therefore, the solution exists at $\sigma \geq \beta$, while

$$A = \frac{3(1-M^2)}{(\beta - 3\sigma)}, \quad B^2 = (\sigma - \beta) A^2 .$$

As a result we have

$$h = B \cdot \mathrm{th}\, \alpha x \cdot \mathrm{sech}\, \alpha x , \quad \delta n = A\, \mathrm{sech}^2 \alpha x ,$$

$$\alpha = \sqrt{-\lambda} , \quad \lambda = -\frac{A}{6} .$$

The condition $\sigma \geq \beta$ means that $k_{0w} d_e \geq 1$. The Schrödinger equation to be valid, it is necessary that the half-width of soliton in k-space is $\Delta \kappa \ll \kappa_0$ that yields

$$\left[\frac{4(M-1)}{(3\sigma - \beta)}\right]^{1/2} \ll 1 \quad \text{or} \quad A^{1/2} \ll 1 .$$

3. *Brief Conclusions*

The obtained results may be summarized as follows. First, the interaction of l_s-solitons with each other and with compression and rarefaction s-wave in the model under consideration differs qualitatively from that taking place for nonresonance l-solitons. In fact, the interactions of l_s-solitons can give the significant share of accelerated (supersonic) plasmons that, generally speaking, may slow down the creation of Langmuir condensate in $k \to 0$ region and, therefore, the dissipation of H.F. energy.

Secondly, the effect of interaction of l_s-solitons with compression s-wave turns out to be more essential than that for rarefaction s-wave, which qualitatively differs from the results obtained for slow l-solitons in [3]. It reveals an essential difference between soliton structures of one-parameter l_s-soliton family and that of two-parameter l-soliton family. Near-sonic l_s-solitons turn out to be less "conservative" at interactions than near-sonic l-solitons.

When $k_w d_e > 1$ near-sonic whistler ws-solitons are possible to exist. These ws-solitons are supersonic analogs of subsonic l_s-solitons, but they have density hump instead of l_s-soliton density pit.

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