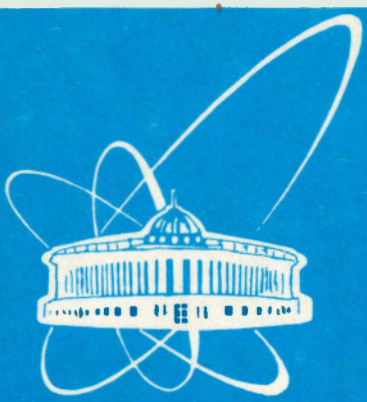

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ROLE OF THE LOW-LYING ISOSCALAR DIPOLE MODES IN THE POLARIZATION POTENTIAL

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Влияние низколежащих изоскалярных дипольных возбуждений
на поляризационный потенциал

Поляризационный потенциал для упругого рассеяния тяжелых ионов рассчитан с помощью формализма Фешбаха. Учтены изоскалярные коллективные возбуждения 0^+, 2^+, 3^- и 1^-. Для расчета соответствующих переходных плотностей используется метод моментов функции Вигнера. Показано, что вклад низколежащих изоскалярных 1^- возбуждений в поляризационный потенциал системы $^{208}Pb+^{208}Pb$ достигает 10-20%.

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Balbutsev E.B. et al.
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Role of the Low-Lying Isoscalar Dipole Modes
in the Polarization Potential

An analysis of the real and imaginary parts of the polarization potential in terms of the relative contributions of the single collective states for the $^{208}Pb+^{208}Pb$ system has been done. The polarization potential has been calculated within the Feshbach formalism taking into account the collective states calculated with the Wigner function moments method. The contribution of the isoscalar giant dipole resonance states has been estimated being of the order of 10-20% of the total at relatively low incident energy.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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1. INTRODUCTION

In the collisions between complex nuclei, the excitation of the internal degrees of freedom gives rise to a modification of the bare nucleus-nucleus potential, calculated for instance by the double folding procedure [1]. This modification is usually referred to as polarization potential. Many authors [2, 3, 4, 5], using different approaches and various prescriptions to calculate the form factors, have studied the relevance of the excitation of vibrational collective states (both low lying and giant resonances) for the determination of the polarization potential. The role played by the nucleon transfer process has also been investigated [5].

Very recently, by using $(\alpha, \alpha'\gamma)$ reactions, some experimental evidence has been found [6] on the existence of low lying isoscalar dipole modes in ^{208}Pb, ^{90}Zr, ^{58}Ni and ^{40}Ca. These modes are of compressional nature and are quite well described by the method of the Wigner Function Moments (WFM) [7]. Due to their isoscalar nature, it is interesting to investigate how much they contribute to the polarization potential. In order to calculate the latter within the same microscopic approach used in previous studies [3, 4] one needs the transition densities of the considered excited states. As shown in the next section, these can be extracted from the WFM results by using the linear response theory. Calculations performed for the $^{208}Pb + {}^{208}Pb$ system at various incident energies show that the contribution of these isoscalar compressional modes to the polarization potential amounts to 10-20 %.

2. DESCRIPTION OF WFM METHOD

The detailed description of the method and derivation of its main equations can be found in [8, 9]. Here we will remind briefly only the basic ideas, just to introduce the necessary definitions and notations.

The starting point for this method is the time-dependent Hartree-Fock equation for the density matrix $\hat{\rho}_q(\mathbf{r}_1, \mathbf{r}_2, t)$:

$$i\hbar \frac{\partial \hat{\rho}_q}{\partial t} = [\hat{H}^{(q)}, \hat{\rho}_q] \quad , \tag{1}$$

where $\hat{H}^{(q)}$ is a self-consistent single-particle Hamiltonian; q is the isospin index ($q = p$ for protons and $q = n$ for neutrons). A Skyrme-type effective force (SGII) is used as nucleon-nucleus interaction.

Equation (1) is transformed to an equation for the Wigner function

$$f_q(\mathbf{r},\mathbf{p},t) = \frac{1}{2\pi\hbar}\int e^{-i\mathbf{p}\cdot\mathbf{s}/\hbar}\,\rho_q(\mathbf{r}+\mathbf{s}/2,\mathbf{r}-\mathbf{s}/2,t)\,d\mathbf{s}\,:$$

$$\frac{\partial f_q}{\partial t} = \frac{2}{\hbar}\sin\,[\frac{\hbar}{2}\,(\nabla_r^H.\nabla_p^f - \nabla_p^H.\nabla_r^f)]\;H_W^{(q)} f_q \quad , \qquad (2)$$

where $H_W^{(q)}(\mathbf{r},\mathbf{p}) = \int e^{-i\mathbf{p}\cdot\mathbf{s}/\hbar}(\mathbf{r}+\mathbf{s}/2 \mid \hat{H}^{(q)} \mid \mathbf{r}-\mathbf{s}/2)\,d\mathbf{s}$ and $\mathbf{r} = (\mathbf{r}_1 + \mathbf{r}_2)/2$, $\mathbf{s} = \mathbf{r}_1 - \mathbf{r}_2$. The moments of f_q in momentum space define the nucleon densities $n_q(\mathbf{r},t)$, the mean velocities $u_q(\mathbf{r},t)$ and tensors of any rank $P_{q\,ij\ldots k}(\mathbf{r},t)$:

$$\begin{aligned} n_q(\mathbf{r},t) &= \int f_q(\mathbf{r},\mathbf{p},t)\,d\mathbf{p}, \\ m\cdot n_q\cdot \mathbf{u}_q(\mathbf{r},t) &= \int \mathbf{p}\, f_q(\mathbf{r},\mathbf{p},t)\,d\mathbf{p}, \\ P_{q\,i_1\ldots i_n} &= m^{1-n}\int (p_{i_1} - mu_{q\,i_1})\ldots(p_{i_n} - mu_{q\,i_n}) f_q(\mathbf{r},\mathbf{p},t)\,d\mathbf{p}\,, \end{aligned}$$

where m is the nucleon mass. The method consists in taking moments of equation (2) in phase space, i.e., in integrating Eq.(2) in $\mathbf{p}$ and $\mathbf{r}$ spaces with different weights, in order to obtain a set of dynamic equations (virial theorems) for some chosen moments of the Wigner function. For example, to describe the negative parity states $1^-, 2^-, 3^-$ one must integrate Eq.(2) in momentum space with the weights $1, p_i, p_ip_j, p_ip_jp_k$ (p_i are components of the momentum $\mathbf{p}$) as a first step. One arrives at a set of coupled time evolution equations for the density (continuity equation), for the mean velocity $u_{q\,i}(\mathbf{r},t)$ (Euler equation), for the pressure tensor $P_{q\,ij}(\mathbf{r},t)$ and for the third rank tensor $P_{q\,ijk}(\mathbf{r},t)$. Then the obtained equations, weighted with x_i and $x_ix_jx_k$ for n_q, with 1 and x_jx_k for $u_{q\,i}$, with x_q for $P_{q\,ij}$ and lastly with 1 for $P_{q\,ijk}$, are integrated in r-space. Thus one arrives at a closed system of coupled dynamical equations for different integral characteristics of the nucleus – Cartesian tensors of the first and third ranks: $\int n_q(\mathbf{r},t)\,x_i\,d\mathbf{r}$, $\int u_{q\,i}(\mathbf{r},t)\,d\mathbf{r}$, $\int n_q(\mathbf{r},t)\,x_ix_jx_k\,d\mathbf{r}$, $\int u_{q\,i}(\mathbf{r},t)\,x_jx_k d\mathbf{r}$, $\int P_{q\,ij}(\mathbf{r},t)\,x_k d\mathbf{r}$, $\int P_{q\,ijk}(\mathbf{r},t)\,d\mathbf{r}$.

In the case of simple interaction (a harmonic oscillator with a separable multipole-multipole residual interaction, for example) the integrals containing the interaction can be expressed in terms of the Cartesian tensors

mentioned above. As a result one obtains a system of nonlinear equations for the integral characteristics of different multipolarities. In the general case of a realistic interaction the integrals containing the interaction can not be expressed in terms of integral characteristics without any approximations. This problem is solved rather easily in the case of small amplitude motion. To obtain the corresponding equations we vary the virial equations taking $n_q(\mathbf{r},t) = n_q^{(0)}(\mathbf{r}) + \delta n_q(\mathbf{r},t)$, $u_{q\,i}(\mathbf{r},t) = u_{q\,i}^{(0)}(\mathbf{r}) + \delta u_{q\,i}(\mathbf{r},t)$, $P_{q\,ij}(\mathbf{r},t) = P_{q\,ij}^{(0)}(\mathbf{r}) + \delta P_{q\,ij}(\mathbf{r},t)$, $P_{q\,ijk}(\mathbf{r},t) = P_{q\,ijk}^{(0)}(\mathbf{r}) + \delta P_{q\,ijk}(\mathbf{r},t)$ and ignoring terms quadratic in the variations δ. So we arrive at a set of linearised equations, which are a very convenient means to study the collective small amplitude motion. These equations are simple differential equations. In the case of harmonic oscillations they become algebraic equations. The coefficients in these equations depend only on the ground state properties. They are linear combinations of integrals over the nuclear volume of different powers of ground state particle densities $n_q^{(0)}$, kinetic energy densities $P_{q\,ii}^{(0)}$ and of their space derivatives.

3. TRANSITION DENSITY

The transition density is defined as the matrix element $\langle 0|\hat{\rho}(\mathbf{r})|\alpha\rangle$ of the density operator

$$\hat{\rho}(\mathbf{r}) = \sum_{i=1}^{N}\delta(\mathbf{r}-\hat{r}_i) \quad , \qquad (3)$$

where $\hat{r}_i$ is the position operator of particle i and $|0\rangle$ and $|\alpha\rangle$ are the ground and excited state, respectively [10]. The problem of WFM method consists in finding this matrix element without calculating the wavefunctions of $|0\rangle$ and $|\alpha\rangle$. This can be solved with the help of linear response theory. The linear response to the perturbation operator $\hat{O}(t) = \hat{F}\exp(-i\omega t) + \hat{F}^\dagger\exp(+i\omega t)$ can be written as

$$\begin{aligned} \rho_{kl}^{(1)} &= \frac{1}{\hbar}\sum_{pq\nu}\left\{\frac{\langle 0|a_l^\dagger a_k|\nu\rangle\langle\nu|a_p^\dagger a_q|0\rangle}{\omega-\Omega_\nu} - \frac{\langle 0|a_p^\dagger a_q|\nu\rangle\langle\nu|a_l^\dagger a_k|0\rangle}{\omega+\Omega_\nu}\right\} f_{pq} \\ &= \frac{1}{\hbar}\sum_{\nu}\left\{\frac{\langle 0|a_l^\dagger a_k|\nu\rangle\langle\nu|\hat{F}|0\rangle}{\omega-\Omega_\nu} - \frac{\langle 0|\hat{F}|\nu\rangle\langle\nu|a_l^\dagger a_k|0\rangle}{\omega+\Omega_\nu}\right\} \quad , \qquad (4) \end{aligned}$$

where $a_l^\dagger, a_k$ are creation and annihilation operators, Ω_ν the eigenfrequencies of the system, and $\hat{F} = \sum_{pq} f_{pq} a_p^\dagger a_q$. Using the second quantization representation for the density operator

$$\hat{\rho}(\mathbf{r}) = \sum_{qp} d_{pq}(\mathbf{r}) a_p^\dagger a_q \,, \tag{5}$$

where $d_{pq}(\mathbf{r}) = \langle p|\delta(\mathbf{r}-\hat{r})|q\rangle = \phi_p^*(\mathbf{r})\phi_q(\mathbf{r})$ and $\phi_q(\mathbf{r})$ being a basis of single particle wavefunctions, we can write the following equation for the change of density

$$\begin{aligned} \delta n(\mathbf{r}) &= \sum_{kl} \rho_{kl}^{(1)} d_{lk}(\mathbf{r}) \\ &= \frac{1}{\hbar} \sum_\nu \left\{ \frac{\langle 0|\hat{\rho}(\mathbf{r})|\nu\rangle\langle\nu|\hat{F}|0\rangle}{\omega - \Omega_\nu} - \frac{\langle 0|\hat{F}|\nu\rangle\langle\nu|\hat{\rho}(\mathbf{r})|0\rangle}{\omega + \Omega_\nu} \right\}, \end{aligned} \tag{6}$$

which we calculate with the help of the WFM. By varying the continuity equation

$$\frac{\partial n_q}{\partial t} = -div\left\{ n_q \mathbf{u}_q + \eta n_q n_{q'} (\mathbf{u}_q - \mathbf{u}_q') \right\} \tag{7}$$

we express the change of density in terms of small displacements $\xi_{q\,i}$

$$\delta n_q = -\sum_{s=1}^{3} \frac{\partial}{\partial x_s} \left\{ n_q \xi_{q\,s} + \eta\, n_q n_{q'} (\xi_{q\,s} - \xi_{q'\,s}) \right\}, \tag{8}$$

where $\frac{\partial}{\partial t}\xi_{q\,i}(\mathbf{r},t) = \delta u_{q\,i}(\mathbf{r},t)$. These displacements are parametrized in the following way

$$\xi_{q\,i}(\mathbf{r},t) = L_{q\,i}(t) + \sum_{j=1}^{3} L_{q\,i,j}(t)\, x_j + \sum_{j,k=1}^{3} L_{q\,i,jk}(t)\, x_j x_k \tag{9}$$

The tensors $L_{q\,i}(t)$, $L_{q\,i,j}(t)$, $L_{q\,i,jk}(t)$ are found by solving the system of coupled dynamical equations involving moments of the Wigner function. Their derivation was described in ref. [7, 9]. It is evident that one can extract the information about the transition density from eq. (6) taking the limit

$$\lim_{\omega\to\Omega_\alpha} \hbar(\omega - \Omega_\alpha)\delta n(\mathbf{r}) = \langle 0|\hat{\rho}(\mathbf{r})|\alpha\rangle\langle\alpha|\hat{F}|0\rangle \,, \tag{10}$$

Thus, to determine the transition density $\langle 0|\hat{\rho}(\mathbf{r})|\alpha\rangle$ from this expression it is necessary to know the matrix element $\langle\alpha|\hat{F}|0\rangle$. By using eqs. (6) and (10) one can get the square modulus of the required matrix element :

$$\lim_{\omega\to\Omega_\alpha} \hbar(\omega - \Omega_\alpha) \int d\mathbf{r}\, f(\mathbf{r})\delta n(\mathbf{r}) = \langle 0|\hat{F}|\alpha\rangle\langle\alpha|\hat{F}|0\rangle = \left|\langle\alpha|\hat{F}|0\rangle\right|^2, \tag{11}$$

which is true for any hermitian single particle operator $\hat{F}$. In the case of a real operator one can write

$$\langle\alpha|\hat{F}|0\rangle = \pm\Big[\lim_{\omega\to\Omega_\alpha} \hbar(\omega - \Omega_\alpha) \int d\mathbf{r}\, f(\mathbf{r})\delta n(\mathbf{r}) \Big]^{\frac{1}{2}}. \tag{12}$$

Then the final expression for the transition density is

$$\langle 0|\hat{\rho}(\mathbf{r})|\alpha\rangle = \frac{\pm \lim_{\omega\to\Omega_\alpha} \hbar(\omega - \Omega_\alpha)\delta n(\mathbf{r})}{\left[\lim_{\omega\to\Omega_\alpha} \hbar(\omega - \Omega_\alpha) \int d\mathbf{r}\, f(\mathbf{r})\delta n(\mathbf{r})\right]^{\frac{1}{2}}}. \tag{13}$$

Combining eqs. (13) and (8) we get the explicit expressions for the transition densities of the various multipolarities

$$\langle 0|\hat{\rho}(\mathbf{r})|2^+\rangle = r\frac{\partial n(r)}{\partial r}(A_1 + A_2 n(r))Y_{2\mu}\,, \tag{14}$$

$$\langle 0|\hat{\rho}(\mathbf{r})|3^-\rangle = r^2\frac{\partial n(r)}{\partial r}(B_1 + B_2 n(r))Y_{3\mu}\,, \tag{15}$$

$$\begin{aligned} \langle 0|\hat{\rho}(\mathbf{r})|1^-\rangle = &\Big\{ r\, n(r)(C_1 + C_4 n(r)) + \\ &\frac{\partial n(r)}{\partial r}[C_3 + C_2 r^2 + n(r)(C_5 + C_6 r^2)] \Big\} Y_{1\mu}\,, \end{aligned} \tag{16}$$

$$\begin{aligned} \langle 0|\hat{\rho}(\mathbf{r})|0^+\rangle = &\Big\{ D_1\big(n(r) + \frac{r}{3}\frac{\partial n(r)}{\partial r}\big) + \\ &D_2\big(n^2(r) + \frac{2}{3} r\, n(r)\frac{\partial n(r)}{\partial r}\big) \Big\} Y_{00}\,, \end{aligned} \tag{17}$$

where the constants A and D are combinations of the tensors $L_{q\,i,j}$; B, of tensors $L_{q\,i,jk}$; C, of tensors $L_{q\,i}$ and $L_{q\,i,jk}$. These expressions are too complicate, to be written here. So we describe the derivation of formulae for the most simple constants (A_1, A_2) in the Appendix, just to give a general idea of the procedure.

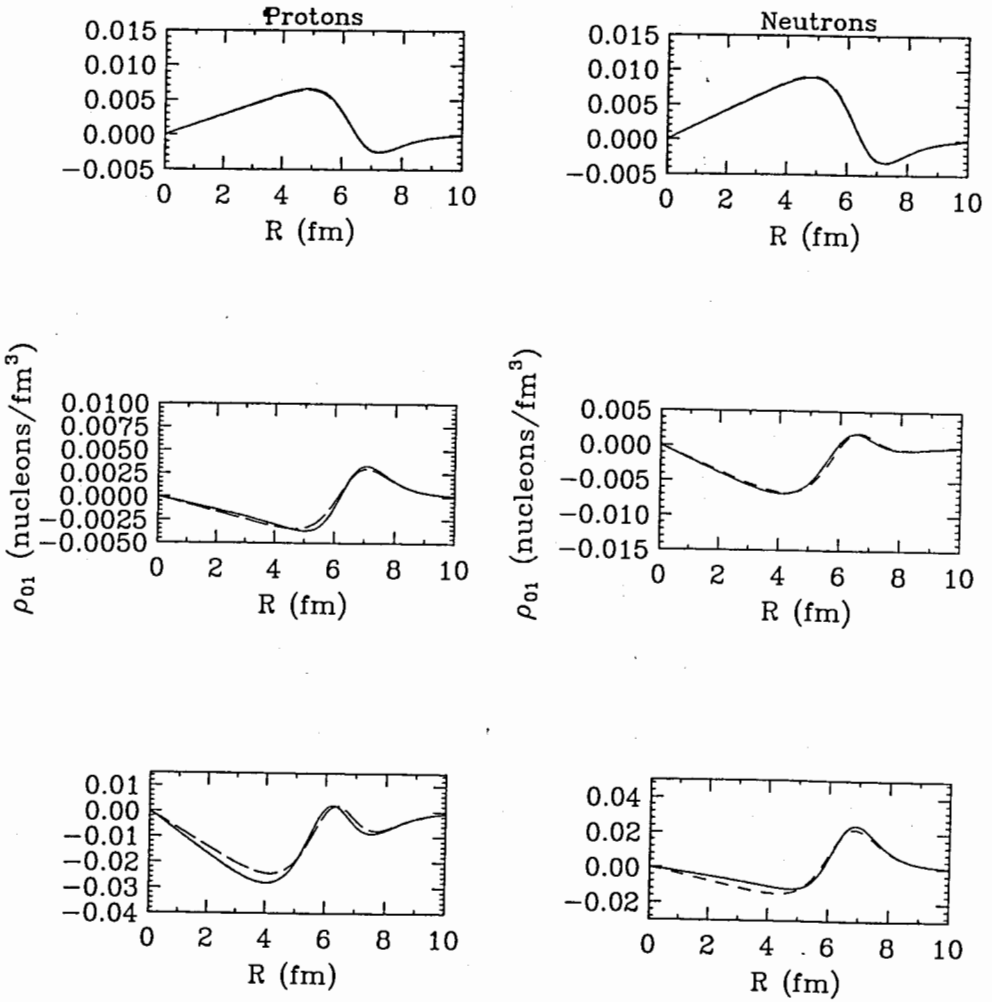

Fig.1.
Proton (left) and neutron (right) transition densities for low-lying dipole excitations in ^{208}Pb with the energies $E_1 = 7.65MeV$, $E_2 = 8.99MeV$ and $E_3 = 10.01MeV$, from top to bottom. The dashed curves are the results of calculations with only the linear density terms (formulae (14)-(17)).

The transition density of Giant Octupole Resonance (GOR) differs from that of Low Energy Octupole Resonance (LEOR) by the values of the constants B. It is necessary to note that in our calculations LEOR includes also the contribution from the lowest 3^- state, because the WFM method gives the centroid of all 3^- states lying below GOR. The expressions (14), (15) for the transition densities of 2^+ and 3^- excitations differ from that of Tassie model by terms quadratic in the density (terms in A_2 and B_2). Numerically these terms turn out to be small. The formula for the transition density of 0^+ practically coincides with the one derived in [13]. The only difference is the term proportional to D_2, which however is numerically very small. The expression (16) can be compared with formula of Harakeh and Dieperink [12], which was derived supposing that all energy weighted sum rule (EWSR) is exhausted by one state. All the three terms linear in the density have the same radial dependence, but the corresponding coefficients C_1, C_2, C_3 have nothing in common with that of ref. [12]. Moreover, the terms proportional to $n(r)^2$ give a noticeable contribution in this case. So the total radial dependence of 1^- transition density may be different from that of [12]. We have three low-lying 1^- states contributing to the isoscalar EWSR. Their proton and neutron transition densities are shown in Fig.1, left and right side, respectively. The dashed lines give the transition densities calculated by using only the linear terms in the density, while the solid lines include also the quadratic ones. It is seen that the transition densities of all three excitations are rather large inside the nucleus demonstrating their compressional nature. The highest excitation has the largest contribution quadratic in the density and has a rather big admixture of isovector mode.

4. POLARIZATION POTENTIAL

The most elegant way to calculate the polarization potential is to follow the Feshbach formalism [2, 4]. In this approach it is very easy to take explicitly into account the contribution of some definite collective states. Here we will briefly describe the method which has been extensively described in ref. [2, 4]. In the Feshbach approach the effective heavy ion interaction is written as

$$V(\mathbf{R},\mathbf{R}') = \langle 00|v(\mathbf{R})|00\rangle\delta(\mathbf{R}-\mathbf{R}')$$

$$+ \sum_{K_1 K_2} \langle 00|v(\mathbf{R})|K_1 K_2\rangle\, G_{K_1 K_2}(\mathbf{R},\mathbf{R}')\langle K_1 K_2|v(\mathbf{R}')|00\rangle$$
$$= V_F(\mathbf{R}) + \Delta\mathcal{V}(\mathbf{R},\mathbf{R}') \quad . \tag{18}$$

In the first term the nucleon-nucleon interaction $v(\mathbf{R})$ is double folded with the ground state densities of the two nuclei. In the second term the sum is over all the non-elastic channels, and $v(\mathbf{R})$ is double folded with the transition densities of the two nuclei: it describes the coupling of the elastic channel to the non-elastic ones. This term is the so-called dynamical polarization potential. Its physical meaning is transparent from eq. (18): The interaction acting at the distance $\mathbf{R}'$ takes the system in one of the non-elastic channels, then it is propagated at another distance $\mathbf{R}$ where the interaction, acting again, brings back the system into the elastic channel. Then the polarization potential is non-local, and if one or more of the non-elastic channels is open it is also complex and its absorptive part describes the loss of flux from the elastic channel. The couplings make $\Delta\mathcal{V}$ also energy dependent because of the appearance of the energy in the propagator $G_{K_1K_2}(\mathbf{R},\mathbf{R}')$.

The calculation of eq. (18) is a very difficult task because of the presence of the propagator, then we calculate it in the WKB approximation:

$$G_{K_1K_2}(\rho,s) \simeq -\frac{\mu}{2\pi\hbar^2}\frac{exp(iM_{K_1K_2}(\rho)s)}{s} \tag{19}$$

with

$$M^2_{K_1K_2}(\rho) = \frac{2\mu}{\hbar^2}\left[E_{cm} - E_{K_1} - E_{K_2} - V_L(\rho) - V_C(\rho)\right], \tag{20}$$

where

$$\rho = \frac{1}{2}(\mathbf{R}+\mathbf{R}')\,; \quad \mathbf{s} = |\mathbf{R}-\mathbf{R}'| \tag{21}$$

The local optical potential $V_L(\rho)$ is given by

$$V_L(\rho) = V_F(\rho) + \Delta\mathcal{V}_L(\rho) \tag{22}$$

while V_C is the Coulomb potential between the two nuclei. A procedure to define the local polarization potential $\Delta\mathcal{V}_L$ will be given later on in this paper.

Within the double folding approach the form factors can be written as

$$F_{K_10}(\mathbf{R}) \equiv \langle 00|v(\mathbf{R})|K_10\rangle = \int d\mathbf{r}_1 d\mathbf{r}_2\, \rho_{K_10}(\mathbf{r}_1)v(|\mathbf{r}_1-\mathbf{r}_2+\mathbf{R}|)\rho_{00}(\mathbf{r}_2) \tag{23}$$

where ρ_{K_10} and ρ_{00} are the transition density and the ground state density, respectively. These are calculated within the WFM method, as described in section 3.. The details of the calculation of $\Delta\mathcal{V}(\rho,\mathbf{s})$ are reported in ref. [4].

The potential so obtained is non-local, but since the range of non-locality of $\Delta\mathcal{V}$ is small with respect to its radius [4] we can use a standard procedure [11] to obtain a local potential from a non-local one. Then

$$\Delta V(\rho) = 4\pi \int j_0(ks) Re\Delta\mathcal{V}(\rho,s)s^2 ds \tag{24}$$
$$W(\rho) = 4\pi \int j_0(ks) Im\Delta\mathcal{V}(\rho,s)s^2 ds\,, \tag{25}$$

where j_o is the Bessel function and

$$k^2 = \frac{2\mu}{\hbar^2}\left[E_{cm} - V_F(\rho) - V_C(\rho)\right] \quad . \tag{26}$$

5. RESULTS AND DISCUSSION

We have done calculations for the system $^{208}Pb + ^{208}Pb$ at several incident energies. The levels used in the calculations are reported in the Table : they were obtained with the WFM method. We have parametrized the ground state density with a Fermi distribution $n(r) = n_0/(1+\exp[(r-R)/a])$ whose parameters are the following [14]: $R = 1.115\,A^{1/3} - 0.53\,A^{-1/3}\ fm$, $a = 0.568\ fm$, while n_0 is fixed by the condition $4\pi\int_0^\infty n(r)r^2\,dr = A$. As already stated in section 3., the transition densities have the advantage to have an analytical form from which one can see that their first term corresponds to the Tassie model in the case of 2^+, 3^- and 0^+ multipolarities. The corrections with respect to the Tassie ones are not important. Formula for 1^- is new and has nothing common with Tassie model, because it takes into account the center of mass movement. The bare potential has been obtained by double folding the nucleon - nucleon effective interaction M3Y [1] with the ground state density of the two nuclei. In the same way we have obtained the form factors eq. (23) by double folding the M3Y with the ground state density of one nucleus and the transition density of the other one. Examples of form factors are shown in Fig. 2: at the right side we show the one related to the LEOR, while at the left side are reported the ones concerning the three

Table
Isoscalar collective states of ^{208}Pb used in the calculations

J^π	E, MeV	$EWSR$, %
1^-	7.65	5.79
	8.99	2.94
	10.01	1.13
2^+	11.82	33.35
3^-	4.65	15.14
	22.21	16.20
0^+	13.63	34.28

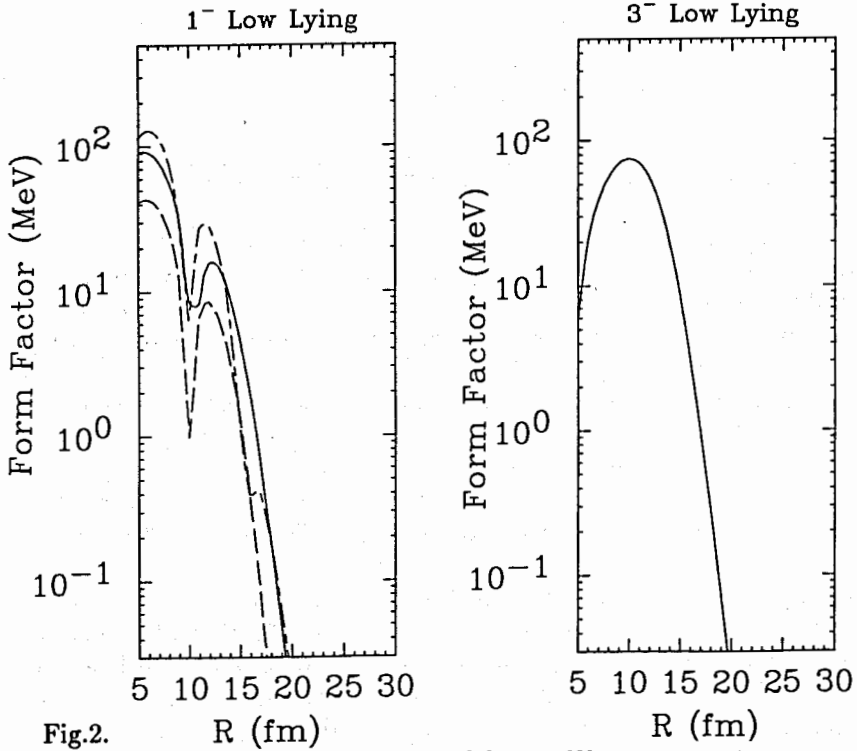

Fig.2.
Form factors for ^{208}Pb for the reaction $^{208}Pb + {}^{208}Pb$ for two different multipolarities. The three curves in the left part correspond to the isoscalar GDR states with excitations energies $E_1 = 7.65\ MeV$ (solid curve), $E_2 = 8.99\ MeV$ (dashed curve) and $E_3 = 10.01\ MeV$ (long - short dashed curve).

isoscalar GDR states. In the latter case the oscillations are due to the fact that the form factors change sign around $10fm$.

In order to show the relative contribution of the states included in the calculations we have computed the polarization potential for various energies as a function of the relative distance R. In fig. 3 we show the contribution due to different multipolarities(as indicated in the figure) to the real (left)

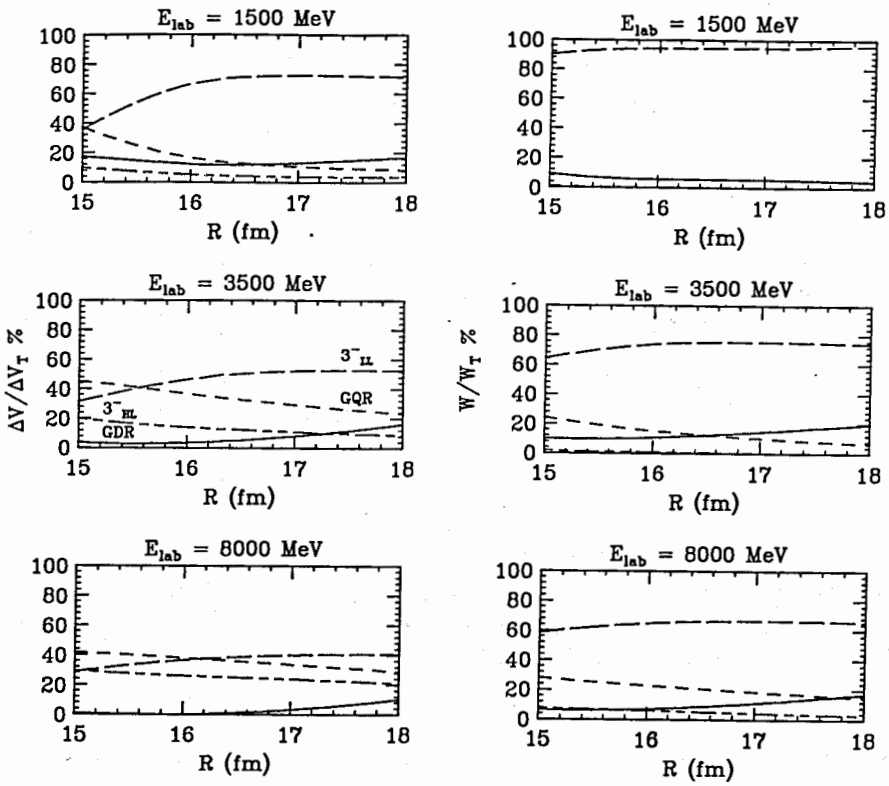

Fig.3.
Contribution of the different multipolarities (as indicated in the figure) to the real (left) and imaginary (right) parts of the local polarization potential plotted in percentage of the total potential as function of the relative distance R for three different values of incident energies for the reaction $^{208}Pb + {}^{208}Pb$.

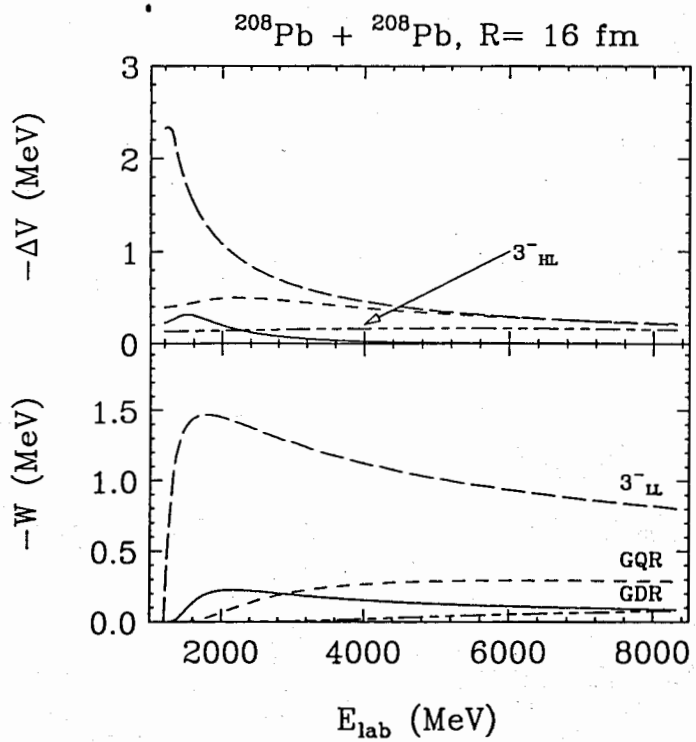

Fig.4.
Contribution of the different multipolarities (as indicated in the figure) to the real (upper part) and imaginary (lower part) part of the local polarization potential plotted as function of incident energy for a fixed relative distance $R = 16fm$ for the reaction $^{208}Pb + {}^{208}Pb$.

and imaginary (right) part of the polarization potential plotted in percentage of the total potential as a function of the relative distance R. We notice that the most important contribution to both real and imaginary parts is due to the low lying 3^- state. As the incident energy is increasing the relative contribution of the GQR becomes more important. This different behaviour has already been found in previous studies [3, 4]. The novel result is represented by the contribution of the isoscalar GDR states which lies between 10-20 %, that is to say comparable with the contribution of the GQR at least for the imaginary part.

We have also calculated the polarization potential at a fixed relative distance ($R = 16\ fm$) as a function of the incident energy taking into account the explicit contribution of the different multipolarities. The results are shown in fig. 4 where the different curves correspond to the contributions of the different states as indicated in the figure. Again we notice the importance of the LEOR, which is overwhelming with respect to the other states in a large range of incident energy for the case of the imaginary part, while for the real part the contribution of the GQR, at higher energies, is essentially the same as the one of the low lying 3^- state. The contribution of the isoscalar GDR to both real and imaginary parts of the polarization potential is more important (at this distance) at low energy, and it decreases with the energy slower for the absorptive part than for the real part.

The interplay between the LEOR and the GQR states seems to contradict the findings of ref. [3, 4]. In particular the contribution of the LEOR to the absorptive part is very big with respect to the one of the GQR. As explicitly stated in the second paper of ref. [4], this is due to the fact that we are using transition densities (hence different form factor) different from the one used in ref. [4] which were calculated in the framework of the Random Phase Approximation.

6. CONCLUSION

We have done an analysis of the real and imaginary parts of the polarization potential in terms of the relative contributions of the single collective states for the system $^{208}Pb + {}^{208}Pb$. The polarization potential has been calculated within the Feshbach formalism taking into account the collective states calculated with the WFM method. Within this method it is possible to obtain also isoscalar GDR states whose contribution to the polarization potential has been estimated being of the order of 10-20 % of the total at relatively low incident energy. Then this contribution should in principle show up in some experimental observable, like for instance the elastic cross section. The other multipolarities give a contribution which has been analyzed in great detail in the previous work [3, 4] where slightly different results were obtained. This can be ascribed to the different form factors we have used in the two cases.

APPENDIX

Transition density for 2^+ excitation

Multiplying (8) by $r^2 Y_{2\mu}(\theta,\varphi)$ and integrating over angles we find the radial dependence for the change of density

$$\delta n_q(r)_{2\mu} = -\frac{4\pi}{15}\{r\frac{\partial n_q}{\partial r}\cdot\mathcal{L}_{q\,2\mu} + \eta r\frac{\partial}{\partial r}(n_q\cdot n_{q'})\cdot(\mathcal{L}_{q\,2\mu}-\mathcal{L}_{q'\,2\mu})\}\,, \qquad (A1)$$

where the components of the irreducible tensors $\mathcal{L}_{q\,2\mu}$ are linear combinations of $L_{q\,ij}$. In the case of a spherical nucleus we can study any component of $\mathcal{L}_{q\,2\mu}$, for example $\mathcal{L}_{q\,22}$:

$$\mathcal{L}_{q\,22} = L_{q\,11} - L_{q\,22} + 2iL_{q\,12}, \quad L_{ij} = L_{i,j} + L_{j,i}. \qquad (A2)$$

For a spherical nucleus $L_{q\,11} = L_{q\,22}$ so we will concentrate our attention on $L_{q\,12}$. Following the prescriptions of chapter 2 and using formulae (8,9) one can derive the system of coupled dynamical equations for $L_{q\,12}$ and $\int \delta P_{q\,12}d\mathbf{r} \equiv \pi_{q\,12}$ (see ref. [9]):

$$\ddot{L}_{q\,12} - 2b_{q\,1}L_{q\,12} + 2b_{q\,2}L_{q'\,12} - 2b_{q\,3}\beta\pi_{q\,12} =$$

$$-i\beta\int r\frac{\partial n_q}{\partial r}\,Y_{2\mu}\cdot W(t)\,d\mathbf{r}\,, \qquad (A3)$$

$$\beta\dot{\pi}_{q\,12} + d_{q\,1}\cdot\dot{L}_{q\,12} - d_{q\,2}\cdot\dot{L}_{q'\,12} = 0\,,$$

where $\beta = \frac{3A}{4\pi m Z_q \alpha_1^4}$ and $W(t) = e_p r^2(Y_{2\mu}e^{-i\omega t} + Y^*_{2\mu}e^{i\omega t})$ is the external field.The corresponding system for the other kind of nucleons is obtained by changing $q \leftrightarrow q'$. The following notations have been introduced here:

$$b_{q\,1} = \frac{z'}{m\alpha_1^4}\mathcal{A} + \gamma\frac{t_+}{4\hbar^2\alpha_1^4}\mathcal{E}_{q'} + 2T_{3q} + z'(7T_1 + 2T_{2q}) + \delta_{q,p}\cdot\frac{8}{15}\varphi(1+\eta n_0 z')\,,$$

$$b_{q\,2} = \frac{z}{m\alpha_1^4}\mathcal{A} + \gamma\frac{t_+}{4\hbar^2\alpha_1^4}\mathcal{E}_q + z(7T_1 + 2T_{2q}) + \delta_{q,p}\cdot\frac{8}{15}\varphi\eta n_0 z\,,$$

$$b_{q\,3} = 1 + \frac{2\pi m}{\hbar^2 A}\alpha_2^2\cdot(t_+ - zt_-/2), \quad d_{q\,2} = \frac{15}{8}\gamma\eta z\frac{\alpha_{8/3}^2}{m\alpha_1^4}\,,$$

$$d_{q\,1} = \frac{3\gamma}{m\alpha_1^4}(\alpha_{5/3}^2 + \frac{5}{8}\eta z'\alpha_{8/3}^2)\,,$$

$$\mathcal{A} = \frac{t_0}{5}(1+\frac{x_0}{2})(S_2^4 + 2\eta\mathcal{R}_4^1) + \frac{t_3}{120}\{(\sigma+1)[(\sigma(1-x_3)+2(2+x_3)] +$$
$$+2\sigma(\sigma-1)(1+2x_3)zz'\}\cdot(\mathcal{R}_4^\sigma + 2\eta\mathcal{R}_4^{\sigma+1})\,,$$

$$\mathcal{E}_q = z(\mathcal{R}_4^{2/3} + 2\eta\mathcal{R}_4^{5/3}), \quad \varphi = \pi n_0\frac{e_p^2}{m}\frac{Z_p}{A}\,,$$

$$T_1 = \frac{\hbar^2}{4m\alpha_1^4}\frac{t_-}{35}[4S_2^2 + Q_0 + 2\eta(4\mathcal{R}_2^1 + Q_1 - \frac{4}{3}S_3^3)]\,,$$

$$T_{q\,2} = \frac{\hbar^2}{4m\alpha_1^4}\eta(\frac{z}{2}t_+ - t_-)(\mathcal{R}_2^1 + \frac{1}{5}S_3^3)], \quad T_{q\,3} = \frac{\hbar^2}{8m\alpha_1^4}(\frac{z}{2}t_+ - t_-)S_2^2\,,$$

$$\alpha_\mu^\nu = \int_0^\infty n(r)^\mu r^\nu\,dr, \quad \mathcal{R}_\nu^\mu = \int_0^\infty n(r)^\mu r^\nu(\frac{\partial n}{\partial r})^2\,dr$$

$$S_\nu^\mu = \int_0^\infty r^\mu(\frac{\partial n}{\partial r})^\nu\,dr, \quad Q_\mu = \int_0^\infty n(r)^\mu r^4(\frac{\partial^2 n}{\partial^2 r})^2\,dr\,,$$

$$n(r) = n_p(r) + n_n(r), \quad z = \frac{Z_q}{A}, \quad z' = \frac{Z_{q'}}{A}, \quad \eta = \frac{mt_+}{2\hbar^2}, \quad t_+ = t_1 + t_2,$$

$t_- = t_1 - t_2$. The quantities t_0, t_1, t_2, t_3, σ, x_0, x_3 are the parameters of Skyrme forces; e_p, the proton charge; m, the nucleon mass. Deriving these formulae for $b_{q\,i}$, $d_{q\,i}$ we used the Thomas-Fermi approximation for the ground state pressure tensor $P^0_{q\,ij}(\mathbf{r}) = \delta_{ij}\gamma n_q(r)^{5/3}$ with $\gamma = \frac{\hbar^2}{5m}(3\pi^2)^{2/3}$.

The system of differential equations (A3) becomes algebraic one taking into account the evident time dependence of variables $L_{q\,12}(t) = \tilde{L}_{q\,12}\,e^{i\omega t}$ and $\pi_{q\,12}(t) = \tilde{\pi}_{q\,12}\,e^{i\omega t}$. the solution of this system is :

$$\tilde{L}_{q\,12} = \Delta_q/\Delta, \quad \tilde{L}_{q'\,12} = \Delta_{q'}/\Delta\,, \qquad (A4)$$

where Δ is the determinant of system (A3). The determinant Δ_q is obtained by substituting the corresponding column of Δ by the right-hand side column of (A3). Δ is a biquadratic polynomial and has two roots Ω_s^2 and Ω_v^2 which are interpreted as isoscalar and isovector giant quadrupole resonances. Writing the determinant in the form $\Delta = c\cdot(\omega^2 - \Omega_s^2)(\omega^2 - \Omega_v^2)$, where c is a constant and putting (A4) into (A1) one can easily calculate the limits in (13) and find the constants A_1, A_2.

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