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QUANTUM RETRODICTION AND CAUSALITY PRINCIPLE

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Квантовое ретросказание и принцип причинности

Квантовая механика является фактически предсказательной наукой. Но квантовое ретросказание тоже может оказаться нужным, например для экспериментальной проверки справедливости уравнения Шредингера для нахождения прошлой волновой функции, если задано настоящее состояние. Показано, что в ретросказательном аналоге предсказания квантовое измерение должно быть заменено другим физическим процессом, названным ретроизмерением. В этом процессе редукция волновой функции в собственные вектора измеряемой наблюдаемой должна происходить в обратном направлении во времени по сравнению с обычной редукцией. Примеры таких процессов неизвестны. Более того, можно показать, что они запрещены принципом причинности, утверждающим, что будущее событие не может влиять на более раннее. Поэтому принцип причинности приводит к нереализуемости квантового ретросказания. Показано, что подход к ретросказанию, предложенный ранее Ватанабе и Бельинфанте, должен рассматриваться только как неудовлетворительный эрзац ретросказания.

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Shirokov M.I. E4-94-4
Quantum Retrodiction and Causality Principle

Quantum mechanics is factually a predictive science. But quantum retrodiction may also be needed, e.g., for the experimental verification of the validity of the Schroedinger equation for the wave function in the past if the present state is given. It is shown that in the retrodictive analog of the prediction the measurement must be replaced by another physical process called the retromeasurement. In this process, the reduction of a state vector into eigenvectors of a measured observable must proceed in the opposite direction of time as compared to the usual reduction. Examples of such processes are unknown. Moreover, they are shown to be forbidden by the causality principle stating that the later event cannot influence the earlier one. So quantum retrodiction seems to be unrealizable. It is demonstrated that the approach to the retrodiction given by S.Watanabe and F.Belinfante must be considered as an unsatisfactory ersatz of retrodicting.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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1. Introduction

The existing quantum theory is factually the science which predicts the future, the past being given [1]. The determination of the past if the future is given (retrodiction) meets troubles in the framework of quantum theory, though the Schroedinger equation allows one to find the wave function $\psi(t)$, provided the state vector $\psi_0(t_0)$ is given, for both cases $t > t_0$ and $t < t_0$ [2,3]. As an example of troubles one can mention the Born probabilistic interpretation of the wave function. This interpretation deals with probabilities of future accidental events (the quantum event is the appearance of an eigenvalue of the measured observable). Meanwhile retrodiction should deal with the past accidental happenings.

Really, the quantum retrodiction should be based on some new additional postulates. For example, it is natural to take it for granted that quantum retrodiction must also be a statistical theory dealing with probabilities for happenings in the past (retroprobabilities). Some other suppositions will be introduced below in sect.3.

There exist several approaches to the quantum retrodiction problem, e.g., see [1,4,5,6,7]. They will be discussed in sections 3 and 4.

In order to avoid misconceptions let us stress that the problem is not directly related to the T or CPT reversibilities [4] because the latter are assertions about some predictive amplitudes.

A natural question may arise: is the quantum retrodiction really needed? The answer is that the retroexperiment can verify hypotheses about evolution backwards in time. The validity of the Schroedinger equation for the retrodiction is only one of the hypotheses of that type. There are suggestions [8,9,10] to use other microscopic equations which prefer a direction of time and can explain the origin of the «time arrow» [11,12,13]. They imply quantum irreversibility which is not related to the known irreversibility of the measurement process, e.g., see [14] and [1] ch.3.4.

A well-known example of statements determining the «time arrow» is the causality principle (CP). Its general formulation is «the later events cannot influence the earlier ones» or «the cause must preceed the effect», e.g., see [15] (for a more detailed form of the principle see sect.5 below). To verify (or falsify) the principle, one must realize the experimental situation when the cause (the event which can be varied at our will) is in the future with respect to the effect. If CP is valid, then the cause variations cannot result in variations of the earlier effect. This experimental situation means a retrodiction problem.

Let us note that CP has been used in ref. [15] when discussing the usual prediction (in → out) problems. It has been shown in [16] that in this case the mathematical consequences derived from CP in ref. [15] can be obtained starting from other preconditions not including CP. This means that dispersion relations can be obtained without using CP. So their verification does not imply CP verification.

The paper is organized as follows.

At first, the scheme of prediction is discussed in sect.2 because the analogy with the prediction is guiding for discussion of the retrodiction in sect.3. The main conclusion of the discussion is that the quantum retrodiction needs a physical process called the "retromeasurement" instead of the usual measurement. The examples of this process are unknown and some general principles (CP being the example) forbid its realization. But a realization of the quantum retrodiction is declared in the literature [1,4]. Sect.4 shows that this approach must be considered as an unsatisfactory ersatz of the retrodiction. Nevertheless, the ersatz gives an idea of the notion of retroprobability. Sect.5 gives an illustration of the retroexperiment which would be needed for the CP verification.

My conclusion is presented in sect.6.

2. Quantum Prediction

The quantum prediction problem may be separated into three stages, see Fig.1.

(a) The preparation of the initial state which takes place in the time interval (t_i', t_i), $t_i' < t_i$. In this interval the physical system S under consideration

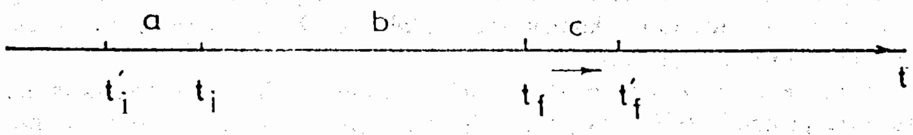

Fig.1. Prediction. (a) preparation; (b) evolution $\psi(t) = U(t, t_i)\psi_i$, $t > t_i$; (c) measurement that reduces $\psi(t_f)$ to $|f\rangle$, the arrow shows the time direction of the reduction

interacts with a preparing device. At the moment t_i, the interaction ceases, the S state becoming ψ_i at t_i.

(b) Evolution in the interval (t_i, t_f), $t_i < t_f$ according to the Schroedinger equation $i\,\partial_t \psi(t) = H\,\psi(t)$ under the condition $\psi(t = t_0) = \psi_i$. In the interval S is isolated in the sense that its Hamiltonian H does not depend upon variables describing the preparing and measuring (see below) devices.

(c) The measurement of an observable F at t_f. The system S begins to interact with the measuring device at t_f and this interaction ceases at t_f'. The result of the measurement is the reduction of $\psi(t_f)$ an eigenstate $|f\rangle$ of the observable.

Quantum mechanics postulates that the probability of observing $|f\rangle$ is equal to

$$P(f \leftarrow i) = |\langle f\,|\psi(t_f)\rangle|^2. \tag{1}$$

This prediction means that if one deals with the ensemble of N_i identical systems prepared in the state ψ_i, then the number of systems observed in $|f\rangle$ will be

$$N_i(f) = P(f \leftarrow i) N_i. \tag{2}$$

More precisely, $N_i(f)/N_i$ tends to $|\langle f\,|\psi(t_f)\rangle|^2$ when $N_i \to \infty$ in the sense of the law of large numbers, see, e.g., [17] ch.6.4. and ch.8.4.

The following comments will be of importance for the discussion of the retrodiction.

2.1. The preparation may be realized by a measurement of an observable I complemented by the selection of the systems S in a distinguished I eigenstate $|i\rangle = \psi_i$.

2.2. The reduction to $|f\rangle$ occurs after t_f. At t_f and immediately before t_f the system S is described by the vector $\psi(t_f) = U(t_f, t_i)\psi_i$. The state of the system in the interval (t_f, t_f') is determined by the interaction with the measuring device. The probability to find the state $|f\rangle$ depends on the wave function which S had *before* the measurement (e.g., on the state $|i\rangle$ if $t_f = t_i$).

2.3. Eq. (1) is valid under the natural assumption that a measurement does not discriminate some eigenvalues f of the observable F, i.e. the efficiency of f measuring is equal to 100%. If a certain eigenvalue f is not registered, then the measured number $N_i(f)$ would be zero irrespective of the value of $|\langle f\,|\psi(t_f)\rangle|^2$.

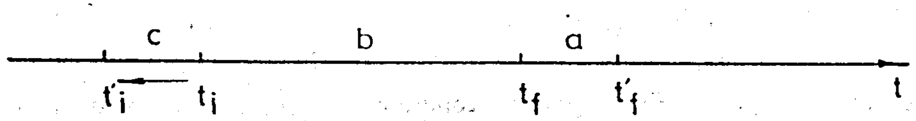

Fig.2. Retrodiction. (a) retroparation; (b) retroevolution $\psi(t) = U_r(t, t_f)\,\psi_f$, $t < t_f$; (c) retromeasurement that reduced $\psi(t_i)$ to $|i\rangle$, the arrow shows the time direction of the retroreduction

3. Quantum Retrodiction

By analogy with the prediction one may separate the retrodiction into the following stages:

a) Retroparing the state ψ_f at the moment t_f. This replaces the preparation in prediction, Belinfante [1] calls it the «postparing». The interaction of the related device with S takes place in the interval (t_f, t_f').

b) Retroevolution of the isolated system S from t_f to t_i according to an equation describing the backward-in-time evolution: $\psi(t) = U_r(t, t_f)\,\psi_f$, $t < t_f$.

c) Retromeasurement of an observable I. The system S interacts with the related device in the interval (t_i', t_i). The device pointer takes the definite position «i» before the moment t_i. This result of the retromeasurement means that the (retro)reduction has happened in the I eigenstate $|i\rangle$. The frequency (or retroprobability) of the result must be determined by the S state $\psi(t_i) = U_r(t_i, t_f)\,\psi_f$ at the moment t_i. In other words, the frequency is determined by *future* state of the system. Remind that the frequency of the usual measurement reduction is determined by the *previous* S state, see sect.2.2.

Before giving the definition of the probability of past events, I must make some preliminary notes.

3.1. Retroexperiment must have one important distinction from the prediction experiment. A prediction can be verified by future experiments (the observable measuring). They can be realized when an experimenter lives till the moment t_f. But the retrodiction needs fixing the later state ψ_f, the earlier S state being the subject of the retrodiction. Meanwhile, only the past and the present are available for us, we view the future as nonexisting yet. So one must consider both the later state ψ_f and the earlier $|i\rangle$ as being in our past. When retrodicting one must deal with the recording (or the protocol) of an experiment which has already been completed.

3.2. I suppose that the result of the retropreparation or retromeasurement is described by usual ket vector. It is a natural supposition if one uses the same Schroedinger equation (which is the equation for a ket vector) both for the forward-in-time and backward-in-time evolutions. Aharonov and Vaidman [7] adhere to another approach: they use a bra vector in order to describe the state determined by the measurement of an observable B at the moment t_f (as will be noted below in sect.3., this must be a retromeasurement).

3.3. The retromeasurement process (c) can be used for the retroparing (a). For this purpose, the retromeasurement of an observable F must be supplemented by a selection of a certain eigenstate $|f\rangle$ which would be the retropared state ψ_f.

The state $|f\rangle$ selected at the moment t_f must determined the past history of the system S, i.e., for times $t < t_f$. Meanwhile, the usual measurement does not determine the system's past state vector. On the contrary, the frequencies of the reduced states are determined by the past state, see sect.2.2.

This note has a direct relation to the series of papers by Aharonov et al. devoted to the discussion of the system S which is both preselected and postselected by ideal measurements of an observable A at the moment t_i and an observable B at the moment t_f, e.g., see [6,7]. As is stated above, the second measurement cannot determine the S state in the interval (t_i, t_f). It is just the retromeasurement which is needed for the postselection.

Let us mention that one cannot independently fix the S state vector $\psi(t)$ at the moment t_f if it was fixed at another moment t_i: $\psi(t_f)$ is determined by $\psi(t_i)$ and the Schroedinger equation.

3.4. Now let us give the experimental definition of the retroprobability $R(f \to i)$ by analogy with the definition (2) for the predictive probability.

Using the protocol of the retroexperiment (see section 3.1) one must determine the number N_f of the systems S which were postpared in the state $\psi_f = |f\rangle$. Then, one must pick out from this ensemble those systems S which have been retroreduced to the state $|i\rangle$ at t_i. This gives the number $N_f(i)$ (let us stress once more that retroreduction in the state $|i\rangle$ is not the preparation of the state). $R(f \to i)$ is defined as

$$R(f \to i) = N_f(i)/N_f. \tag{3}$$

One may postulate by analogy with (1) that the theoretical counterpart of the ratio (3) is given by

$$R_{\text{th}}(\dot{f} \to i) = |\langle i \,|\psi(t_i)\rangle|^2 = |\langle i \,|U_r(t_i, t_f)|f\rangle|^2, \tag{4}$$

where U_r is the operator of the evolution backward in time. If the retroevolution is supposed to be governed by the Schroedinger equation, then $U_r = \exp[-iH(t_i - t_f)]$.

3.5. In order to elucidate the approach to retrodiction under discussion, let us imagine a Being which belongs together with his experimental devices to the world the entropy of which increases in the direction from the (human) future to the (human) past [18]. For the Being his devices are usual measurement devices but they realize retromeasurements in our world. The Being perceives the system S evolution from t_f to t_i as evolution in the forward direction of his time arrow, so the human retroexperiment is the predictive experiment for the Being. By the way, Being's devices without the Being seem to be sufficient for the retroexperiment realization.

3.6. I cannot suggest a human realization of the physical process of retromeasurement. Moreover, the existence of a process like that is forbidden by some general principles, the causality principle being one of them. Indeed, the frequency $N_f(i)$ of the observable I eigenvalue «i» is determined in the retromeasurement by the future state vector $\psi(t_i) = U_r(t_i, t_f)\,\psi_f$. So the cause of the observed value of $N_f(i)$ (the value being the effect) is in future. With varying ψ_f, $N_f(i)$ varies. This is forbidden by CP, see the Introduction*.

The increasing entropy law may be another principle forbidding the retromeasurement. The usual measurement is an irreversible process [1,14], the retromeasurement is a process which is inverse in time and its realization needs something like Maxwell's demon.

So one may conclude that quantum retrodiction constructed by analogy with prediction is unrealizable.

4. An Ersatz of Quantum Retrodiction

Contrary to the conclusion of the previous section, it has been stated in [1,4] that quantum retrodiction is possible in some cases. I am going to argue that the statement is based on an approach which can be considered as an unsatisfactory ersatz of retrodiction. The approach can be presented as follows.

*Note that here I have applied CP to the evolution of the system S coupled with the retromeasurement device in the interval (t_i', t_i). Meanwhile when talking in the Introduction about the CP verification I had in mind its application to the (retro)evolution of the isolated system S.

Consider Neumann's ensemble of N systems S. Its state at the moment t_i is described by the density matrix

$$\rho(t_i) = \sum_i p_i \,|i\rangle\langle i\,|, \quad \sum_i p_i = 1, \tag{5}$$

where eigenstates $|i\rangle$ of an observable I constitute a complete set. The average number of the ensemble systems which are in the state $|i\rangle$ is $N_i = p_i N$. The observable F is measured at the moment t_f. Let N_f be number of cases when F assumes the eigenvalue f. One has

$$N_f = N \sum_i |\langle f\,|U(t_f, t_i)|\,i\rangle|^2 p_i = \sum_i N_i(f), \tag{6}$$

where $N_i(f)$ is defined by eq. (2). $N_i(f)$ is the number of systems reducing to $|f\rangle$ under the condition that initially these systems have been in $|i\rangle$. According to [4] and [1] ch.2.6, the retroprobability of the transition from $|f\rangle$ to $|i\rangle$ is defined as

$$R'(f \to i) = N_i(f)/N_f = N_i\, P(f \leftarrow i)\left[\sum_j N_j\, P(f \leftarrow j)\right]^{-1}. \tag{7}$$

Let us stress that $R'(f \to i)$ is defined by using predictive probability $P(f \leftarrow i)$ and the numbers $N_i = p_i N$. The retroprobability R, see eq. (3), coincides with R' only if one assumes that $N_f(i) = N_i(f)$, $N_f(i)$ being defined in sect.3.4.

So, R' is determined by the operator $U(t_f, t_i)$ of the evolution forward in time, while R is determined by the operator $U_r(t_i, t_f)$ of the backward-in-time evolution, see eq. (4). As has been stated in the Introduction, the purpose of retrodiction should be to verify hypotheses on U_r; R' cannot serve the purpose.

Watanabe has shown [5] that R' has another deficiency which does not allow one to consider R' as a satisfactory analog of the predictive probability. He has pointed out that $P(f \leftarrow i)$ is determined only by the choice of states $|i\rangle$, $|f\rangle$ and the Hamiltonian H of the Schroedinger equation. But $R'(f \to i)$ does not share this property; it depends not only on $|f\rangle$, $|i\rangle$ and the dynamics but also on p_i which can be varied arbitrarily. Watanabe has concluded that «quantum physics is irretrodictable», though his criticism refers only to the described ersatz of the retrodiction.

Let us illustrate his criticism by one example. Let all probabilities p_i be zero with the exception of one, p_j (which is then equal to 1). Then, it follows from eq. (7) that $R'(f \to j) = 1$ for any f, i.e. $R'(f \to j)$ does not depend on f as well as on the dynamics.

The analogue of R' from the classical probability theory may be of interest; R' is analogous to the so-called a posteriori probability which depends on the choice of a priori probability p_i. The assumed equality $N_f(i) = N_i(f)$ leads to the equation

$$R'(f \to i)N_f/N = P(f \leftarrow i)N_i/N$$

which is the known Bayes' equality for conditional probabilities, e.g., see [19]. So, the assumption $N_f(i) = N_i(f)$, which is the basis of the ersatz, can be formulated as follows. Consider $P(f \leftarrow i)$ as a conditional probability of f, given i. Then, the retroprobability R' is assumed to be the conditional probability of i, given f.

There is only one case when R' takes a reasonable value. The case can be described as follows. One must suppose at first that the retroevolution is determined by the usual Schroedinger equation

$$U_r(t_i, t_f) = \exp[-iH(t_i - t_f)] = U^{-1}(t_f, t_i) = U^{\dagger}(t_f, t_i). \qquad (8)$$

Then, the theoretical definition (4) of the retroprobability leads to the equality

$$R_{\mathrm{th}}(f \to i) = |\langle f | U(t_f, t_i) | i \rangle|^2 = P(f \leftarrow i). \qquad (9)$$

The equality is consistent with (7) only if $N_i = N_f = \sum_j N_j P(f \leftarrow j)$. This is realized only in the case when all p_i (and N_i) are supposed to be equal [1]. This second supposition was formulated by Watanabe as «a priori equal probability for each initial state» [4]. Belinfante calls it «the garbling condition» [1]. The supposition seems to be artificial. It has sense only if the sets of eigenvalues i and f are discrete and finite. If i assumes infinitely many discrete values, the supposition together with $\sum_i p_i = 1$ leads to a senseless consequence: $p_i = 0$ for all i.

5. Verification of the Causality Principle

Though retroexperiments seem unexecutable, see sect.3, I shall illustrate here how one would verify the causality principle CP. I have in mind the form of CP which has been used in [15]. The cause and effect are supposed to be localized in finite four-dimensional regions of the Minkowsky space. An external

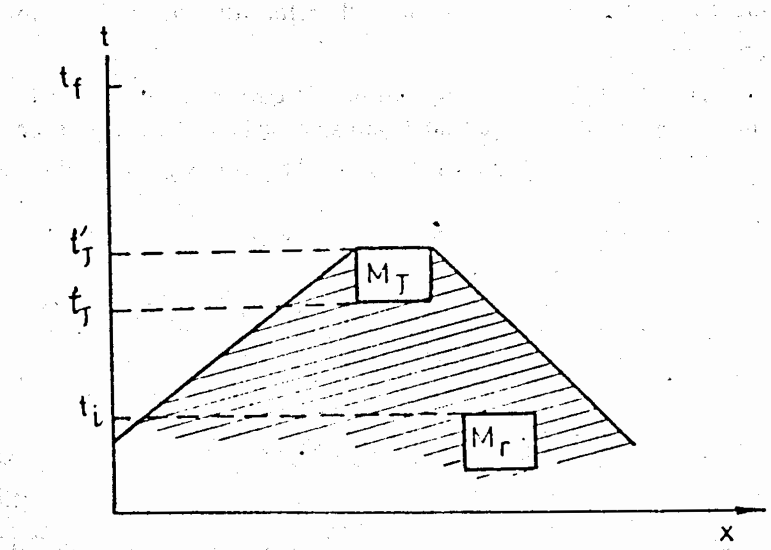

Fig.3. Retroexperiment. M_J is the four-dimensional region of the external current localization. M_r is the four-dimensional region of the retromeasurement localization. The past light cone of M_J is shaded

current J_μ, $\mu = 1,2,3,4$, plays the role of the «cause». It is localized in a three-dimensional volume V_J and is turned on at the moment t_J and is turned off at t_J', see fig.3. The current J_μ can emit and absorb photons. No-photon state $|o\rangle$ is fixed (retropared) at the moment t_f. A retromeasurement of the photon number («effect») is executed at the moment t_i. If the current J_μ is weak, then mainly one-photon states $|i\rangle = |\mathrm{k}, \varepsilon\rangle$ give contribution to the number (k and ε are photon momentum and polarization).

Let us describe the execution of the retroexperiment. At t_i the pointer of the retromeasurement device shows the photon presence. Later in the interval (t_J, t_J') the current J_μ acts. Only those cases are selected in which the photon state is $|o\rangle$ at the moment t_f.

The current J_μ cannot influence the future (moment t_f) state because the state is fixed to be $|o\rangle$. If J_μ is not turned on in (t_J, t_J'), then photons are absent at the moment t_i. The current turning on is therefore the cause of the photon possible appearing at t_i.

Let us suppose first that the retroevolution is governed by the electrodynamical Schroedinger equation and find ensuing consequences. The retroprobability $R = |\langle k\varepsilon | U(t_i, t_f)|o\rangle|^2$ to have the state $|k\varepsilon\rangle$ at t_i is given by the equation (interaction picture is used)

$$R = |\langle k\varepsilon | T\exp[-i \int_{t_f}^{t_i} dt \int d^3x\, J_\mu(x)\, A_\mu(x)|o\rangle|^2 \cong$$

$$\cong |\langle k\varepsilon | -i \int_{t_f}^{t_i} dt \int d^3x\, J_\mu(x) A_\mu(x)|o\rangle|^2. \quad (10)$$

Doubling J_μ gives four times increased R, i.e. in varying the cause the effect varies: $\delta R/\delta J_\mu \neq 0$. The mechanism of the cause action is that J_μ absorbs photons which were detected at t_i.

Now let us suppose that CP is valid. Then, $\delta R/\delta J_\mu$ must be zero: the cause (current) cannot influence the past effect (photon appearance). The photon state at t_i must be the same as in the case $J_\mu = 0$, i.e. it must be $|o\rangle$ and R must be zero at any J_μ value.

Variants of the retroexperiment are possible when the retromeasuring device is localized in a four-dimensional volume M_r, see Fig.3. (M_r must be localized in the past light cone of M_J.) For example, photons can be detected by means of a localized atom, which at t_f is unexcited and at t_i is detected to be in an excited state by means of a device which is localized near the atom.

6. Conclusion

I have drawn the conclusion that quantum mechanics is a predictive science not only factually but also because its retrodictive analogue seems to be unrealizable. My reasons are as follows. It has been shown in sect.3 that retrodiction needs a retromeasurement process which must replace the usual measurement. In this process, the reduction of the wave function to an observable A eigenstate $|a\rangle$ must proceed in time in the direction opposite to the time arrow: $|a\rangle \leftarrow \psi$. Meanwhile, in the usual measurement the reduction proceeds in the direction of

the time arrow: $\psi \rightarrow |a\rangle$. Frequency of the retroreduced state $|a\rangle$ must be determined by the future state vector of the quantum system, whereas frequency of the usual reduction $\psi \rightarrow |a\rangle$ is determined by the system's previous state.

Examples of such a retromeasurement process are unknown. Moreover, its realization is forbidden by the causality principle, see sect.3.6. Another trouble with the retromeasurement may be illustrated by the note that a usual measurement process is irreversible, whereas the retromeasurement must be a process inverse in time.

The irretrodictability of quantum mechanics has earlier been declared by S.Watanabe [6] but his conclusion refers to another approach to the retrodiction and was grounded on quite different reasons. This approach must be considered as an unsatisfactory ersatz of the retrodiction, see sect.4.

Quantum irretrodictability means that one cannot verify (falsify) hypotheses on laws of quantum evolution in the backward direction of time, the causality principle being the example of a hypothesis like that, see Introduction. The scientific status of the hypothesis may be then questioned. I have in mind K.Popper's principle stating that a hypothesis may be considered as being a scientific one (in contrast to some religious statements) only if it can be falsified [20].

Acknowledgements

I wish to thank Prof. R.Haag who encouraged this investigation noting that «it would be nice to say some more about the problem of «retrodictive» use of quantum theory» (private letter on the occasion of my paper [16]).

I am also grateful to Prof. M.I.Podgoretsky for useful comments.

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Note added in proof

Arguments have been presented recently by Y.Aharonov and L.Vaidman (Phys. Lett. A178, 38 (1993)) in favour of a Gedanken measuring process which does not lead to the wave function reduction. The process allows one to measure the system wave function (i.e. its module and phase), the function remaining the same before and after this measurement. For other examples of non-perturbing detectors see, e.g., papers by M.Scully et al. (Nature, 351, 111 (1989)) and S.Haroche (Europhys.News 24, 51 (1989)) and references therein.

This sort of measurement allows the setting of both the prediction and the retrodiction problems in a similar manner as in classical mechanics: find $\psi(t)$ at $t > t_0$ and $t < t_0$, $\psi(t_0)$ being given, and compare this $\psi(t)$ with the measured wave function. The wave function measurement is the same for both the prediction and retrodiction because the wave function does not alter. So this non-perturbing measurement would allow the realization of the retrodiction.

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