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HEAVY-ION ONE-NUCLEON TRANSFER REACTIONS IN QUASI-CLASSICS AT HIGH ENERGIES

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1 Introduction

The standard way to consider one-nucleon transfer reactions is to use the DBWA-method where the basic ingradient of the theory is the distorted waves introduced in *in*- and *out*-channels of a reaction. In the case of heavy ion collisions at comparatively high energies $E \gg |V|$, several simplifications can be done from the beginning. The first one is the quasi-elastic consideration which uses the same elastic channel parameters for *in*- and *out*-distorted waves. It comes true if the energy loss in a reaction occurs very small as compared with kinetic energies of colliding nuclei, e.g. $E_\alpha \approx E_\beta = E, \Delta E \ll E$. The other approximation is the quasi-classical calculation of distorted waves. Indeed, the main condition of quasi-classics $kR \gg 1$ is usually working well for heavy-ion scattering. However, the traditional methods utilise the QC-distorted waves, expanding them in sets of partial waves and then applying the WKB-method to calculate every partial phase. This way is not convenient at high energies since one needs to take into account hundreds of partial waves. This arises difficulties in numerical calculations of a large number of matrix elements with further summing them up in the reaction amplitude and troubles in searching for the physics of the reaction mechanism. This latter is because of the hidden dependence of an amplitude on input parameters of potentials in the case of numerical calculations operating with a lot of partial waves. To avoid these difficulties, we apply the HEA-method developed for calculations of the three-dimensional quasi-classical wave functions and for the corresponding matrix elements with these functions included [1,2]. The method can be applied under the conditions $kR \gg 1$, $E \gg V$ and $\theta > \theta_c \simeq |V|/E$, where θ_c is the classical deflection angle. This latter is introduced to include distortion of the straight-line trajectories of motion, the important point in investigating heavy-ion collisions. On the whole, this gives us the possibility to avoid complicated numerical calculations and to obtain, in the framework of the DWBA, analytical expressions for qualitative physical estimations and for a quantitaive comparison with experimental data.

2. Differential cross section

We consider the reaction $a + A \to b + B$ where $a = x + b, B = A + x$ and the transferred particle x is proposed to be spinless. The corresponding cross section and the amplitude in the zero-range approximation are as follows:

$$\frac{d\sigma}{d\Omega} = \frac{m_\alpha m_\beta}{(2\pi\hbar^2)^2}\frac{k_\beta}{k_\alpha}\frac{2J_B+1}{(2J_A+1)(2J_a+1)}\sum_l \frac{S_l}{2l+1}|\tilde{T}_l^{tr}|^2, \tag{1}$$

$$\tilde{T}_l^{tr} = -D_0 \int d\vec{r}\,\Psi_\beta^{(-)*}(\vec{r})\Psi_\alpha^{(+)}(\vec{r})\Re_l(r)Y_{l0}^*(\hat{r}), \tag{2}$$

where

$$D_0 = 8\pi\sqrt{(m_a\hbar^2/2m_x m_b)^3\epsilon_{xb}}$$

depends on the structure of an incident particle, ϵ_{xb} is the seperation energy of a nucleon x in the incident a and $\Re_l(r)$ is the radial wave function of the x-particle in the final nucleus B. This function has the asymptotic behaviour $\exp(-\kappa_l r)/r$ and goes to the constant as $r \to 0$. A slope in asymptotics is determined by κ_l depending on a seperation energy ϵ_l :

$$\kappa_l = \sqrt{2m_x\epsilon_l/\hbar^2}\,.$$

. We have emphasized that the main effect in heavy-ion reactions comes from the region near the interaction radius. This means that the behaviour of the function $\Re_l$ at $r < R$ is of no importance, and one can select it in the form

$$\Re_l(r) = N_l\frac{1}{r}\frac{df_s(r,R,a_l)}{dr}, \tag{3}$$

where

$$f_s = \frac{\sinh\frac{R}{a_l}}{\cosh\frac{R}{a_l}+\cosh\frac{r}{a_l}} \tag{4}$$

is the symmetrised Fermi-function having the asymptotics $\exp(-a_l r)/r$ and being a constant at $r = 0$. The "diffuseness" parameter of the transition region is to be taken $a_l = \kappa_l^{-1}$. The constant N_l can be obtained by chaging variables $z = 1 + \cosh(r/a_l)/\cosh(R/a_l)$ in the normalization condition:

$$\int_0^\infty \Re_l^2(r)r^2dr = N_l^2\int_0^\infty \left(\frac{df_s}{dr}\right)^2 dr = \frac{N_l^2}{a_l}\frac{\sinh^2(R/a_l)}{\cosh^2(R/a_l)}\int_0^\infty \frac{\sinh^2(r/a_l)}{\cosh^2(R/a_l)}\frac{1}{z^4}dr = 1. \tag{5}$$

Neglecting here the terms $\cosh^{-1}(R/a_l)$ as compared with 1, we reduce the latter integral to the table one:

$$\int_1^\infty \frac{x-1}{x^4}dx = \frac{1}{6}.$$

So,we get $N_l = \sqrt{6a_l} = \sqrt{6/\kappa_l}$.

The symmetrized Fermi-function has the same behaviour as the usual Fermi-function in the region $r \sim R$ and we use it in our further consideration.

In formula (1) we include only one term with $m = 0$ since the other terms with $m \neq 0$ may be neglected because of additional fast oscillations in integrands as compared with the first one. Inserting (3) into (2), we get an amplitude of the typical form inherent in HEA. Moreover, here we can use the quasi-elastic approximation because the loss of energy in the reaction is comparatively small and $E_\alpha \simeq E_\beta$. Thus, the QC-distorted waves in our case are calculated as in the elastic channel and have the form [1]:

$$\Psi_\alpha^{(+)} = \exp[+i(\vec{k}_\alpha - \frac{\vec{q}_{c\alpha}}{2})\vec{r} - \frac{i}{\hbar v}\int_{-\infty}^{z} V(\sqrt{\rho^2+\lambda^2})d\lambda + \frac{1}{\hbar v}\int_{-\infty}^{z} W(\sqrt{\rho^2+\lambda^2})d\lambda], \qquad (6)$$

$$\Psi_\beta^{(-)*} = \exp[-i(\vec{k}_\beta + \frac{\vec{q}_{c\beta}}{2})\vec{r} - \frac{i}{\hbar v}\int_{z}^{\infty} V(\sqrt{\rho^2+\lambda^2})d\lambda + \frac{1}{\hbar v}\int_{z}^{\infty} W(\sqrt{\rho^2+\lambda^2})d\lambda]. \qquad (7)$$

In the quasi-elastic kinematics it is convenient to select the coordinate system so that the axes were directed as follows: $oz \| \vec{q}$ and $ox \| \vec{K} = \vec{k_\alpha} + \vec{k_\beta}$. This allows us to write the product $\Psi_\beta^{(-)*}\Psi_\alpha^{(+)}$ in the following form [1,2]:

$$\Psi_\beta^{(-)*}\Psi_\alpha^{(+)} = \exp(i\tilde{\Phi}), \qquad (8)$$

where

$$\tilde{\Phi} = 2\tilde{a}_0 + \tilde{\beta}\mu + n_1\mu^2 + c_1\mu^3 + n_2(1-\mu^2)\cos^2\tilde{\varphi} + c_2\mu(1-\mu^2)\cos^2\tilde{\varphi}. \qquad (9)$$

Here $\tilde{\beta}$, c and n are the known functions expressed through r, parameters of the potentials, $\alpha = \sin(\theta/2)$, and $\alpha_c = \sin(\theta_c/2) \simeq \frac{1}{2E}[V(R_t) + V_C(R_t) + iW(R_t)]$, taken at the radius $R_t = r_{0t}(A_a^{1/3} + A_A^{1/3})$ of the external limited trajectory of motion. For example,

$$\tilde{\beta} = \tilde{q}r = q_{ef}r + 2k_t\alpha r; \quad q_{ef} = 2k(\alpha - \alpha_c); \quad k_t = -[B^V + iB^W + B^C(3 - \frac{r^2}{R_C^2})], \qquad (10)$$

where

$$B^V = \frac{V_0}{\hbar v}, \quad B^W = \frac{W_0}{\hbar v}, \quad B^C = \frac{Z_1Z_2 2e^2}{R_C\hbar v}$$

and $V_0 = -|V_0|$, $W_0 = -|W_0|$ are the depths of the real and imaginary parts of the potentials.

We can see that now the integrand (2) contains in the exponent a typical power dependence on the variables r and μ. Keeping in mind that $d\vec{r} = -r^2 dr d\mu d\bar{\varphi}$, we first integrate in (2) over $d\mu$ by parts neglecting in it the term having the smallness $(kR)^{-2}$. Then, the result can be presented as follows:

$$I_l = \int_{-1}^{+1} d\mu \exp(i\bar{\Phi}) Y_{l0} \simeq -i\left(\frac{\exp(i\bar{\Phi})}{\partial\bar{\Phi}/\partial\mu}\Big|_{+1} - (-)^l \frac{\exp(i\bar{\Phi})}{\partial\bar{\Phi}/\partial\mu}\Big|_{-1} \right) Y_{l0}(1) =$$

$$= -i\sqrt{\frac{2l+1}{4\pi}} \exp(2i\bar{a}_0 + in_1)[I^{(+)} - (-)^l I^{(-)}], \tag{11}$$

where

$$I^{(\pm)} = \frac{\exp[\pm i(\tilde{\beta}_n + c_1)]}{\Delta_{(\pm)} \mp \delta_{(\pm)} \cos^2\bar{\varphi}}, \quad \Delta_{(\pm)} = \tilde{\beta}_n + 3c_1 \pm 2n_1, \quad \delta_{(\pm)} = 2(n_2 \pm c_2). \tag{12}$$

Then, the integration over $d\bar{\varphi}$ is performed with the help of a table integral. Thus, we can write the amplitude (2) in the form of a one-dimensional integral

$$\tilde{T}_l^{tr} = -iD_0\sqrt{6\pi a_l(2l+1)} e^{i2\bar{a}_0} \int_0^\infty \frac{df_s}{dr}\left\{ F^{(+)}(r) - (-)^l F^{(-)}(r) \right\} dr, \tag{13}$$

where

$$F^{(\pm)}(r) = \frac{\exp[\pm i\phi^\pm]}{L^\pm} = \exp[\pm i\Phi^\pm], \tag{14}$$

$$\Phi^\pm = \phi^\pm - \ln L^\pm, \quad \phi^\pm = f_1 r \pm f_2 r^2 + f_3 r^3, \quad L^\pm = \sqrt{(f_1 \mp f_4 r + f_5 r^2)(f_1 \pm f_6 r + f_7 r^2)} \tag{15}$$

with f_i the functions of the parameters of the potentials, α and α_c:

$$\begin{gathered} f_1 = 2k(\alpha - \alpha_c) - 2(B^V + iB^W + 3B^C)\alpha; \quad f_2 = (\frac{B^V}{R_V} + i\frac{B^W}{R_W})(1-\alpha^2); \\ f_3 = \frac{2B^C}{R_C^2}(1 - \frac{2}{3}\alpha^2)\alpha; \quad f_4 = 2(\frac{B^V}{R_V} + i\frac{B^W}{R_W})\alpha^2; \quad f_5 = \frac{2B^C}{R_C^2}(1-2\alpha^2)\alpha; \\ f_6 = 2(\frac{B^V}{R_V} + i\frac{B^W}{R_W})(1-2\alpha^2); \quad f_7 = \frac{2B^C}{R_C^2}(5-6\alpha^2)\alpha. \end{gathered} \tag{16}$$

Integrals of the type (13) can be calculated in the analytical form if one uses on the complex r-plane the second order poles of the derivative df_s/dr displayed in the region of the nuclear surface at $r_n^\pm = R \pm i\pi(2n+1)a_l$, where $n = 0, 1, 2...$ It is easy to show that the main contribution to (13) is coming from two poles closest to the real r-axis. Then, the final expression for the differential cross section is derived as follows:

$$\frac{d\sigma}{d\Omega} = 6\pi a_l^3 S_l D_0^2 \frac{2J_B+1}{(2J_A+1)(2J_a+1)} e^{4B^W R_W} \left| \left[\frac{d}{dr}\exp(i\Phi^{(+)})\right]_{r_0^+} + (-)^l \left[\frac{d}{dr}\exp(-i\Phi^{(-)})\right]_{r_0^-} \right|^2 \tag{17}$$

To the aim of a qualitative consideration, we can rewrite this expression keeping only the main terms in the real and imaginary parts of the amplitude. In the case of heavy ion reactions we have a large value kR, and the terms depending on this parameter influence mainly the form of the differntial cross section. On the other hand, the other terms depending on the parameters of the potentials and α_c determine mainly the absolute value of the cross section. So, to present the result more clearly, we separate these dependences in the following obvious form:

$$\frac{d\sigma}{d\Omega} \sim \exp[-\psi(\alpha,\alpha_c)]\exp[4B^W R_W]\times$$

$$\left| e^{i2kR(\alpha-Re\alpha_c)} - (-)^l e^{-i2kR(\alpha-Re\alpha_c)}\exp[4\pi a_l B^V R/R_V + 4R(kIm\alpha_c - B^W\alpha)]\right|^2, \tag{18}$$

where

$$\psi(\alpha,\alpha_c) = 4\pi a_l k(\alpha - Re\alpha_c) + 4kRIm\alpha_c. \tag{19}$$

If W_0 is large so that $\exp[4\pi a_l B^V R/R_V + 4R(kIm\alpha_c - B^W\alpha)] \ll 1$, then only the first term gives contribution to (18). Then

$$\frac{d\sigma}{d\Omega} \sim \exp^{-4\pi a_l k(\alpha-Re\alpha_c)} e^{-4kRIm\alpha_c} e^{4B^W R_W}. \tag{20}$$

One can see that the cross section decreases with the scattering angle as an exponential function, which falls down depending on a thickness parameter a_l. The magnitude of the latter is determined by the binding energy of the transferred nucleon in the final nucleus B. We see that the absolute value of the cross section increases with the classical deflection angle.

If W_0 is comparatively small, both the terms in vertical brackets in (18) become important, so we have

$$\frac{d\sigma}{d\Omega} \sim \exp[-\psi(\alpha,\alpha_c)]\exp[4B^W, R_W]\times$$

$$\begin{cases} [\sin^2(2kR(\alpha - Re\alpha_c)) + \sinh^2(2\pi a_l B^V R/R_V + 2R(kIm\alpha_c - B^W\alpha))], & for\ even\ l \\ [\cos^2(2kR(\alpha - Re\alpha_c)) + \sinh^2(2\pi a_l B^V R/R_V + 2R(kIm\alpha_c - B^W\alpha))], & for\ odd\ l. \end{cases} \tag{21}$$

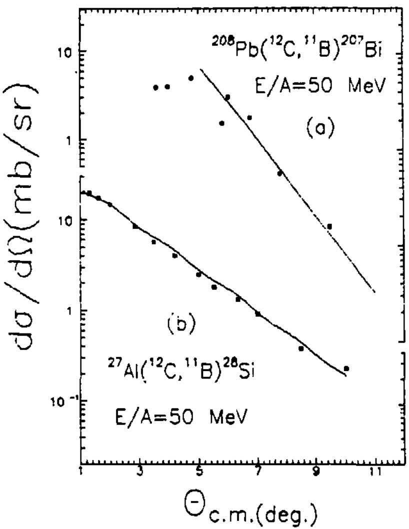
In this case, the cross section decreases as an exponential function and simultaneously oscillates with a freqnency depending on the radius parameter R, α and α_c. We have already analyzed the dependence of the transfer reactions on the imaginary part of the nuclear potential in the previous paper [3]. Now we paid more attention to a very interesting dependence on the classical deflection angle α_c, which is really observed in heavy-ion experiments, as the so-called limited angle of a Coulomb deflection.

3. Numerical Calculations and Conclusion

We have calculated the differential cross section (17) for the reaction of the proton stripping from ^{12}C to the ground state of ^{209}Bi and ^{28}Si and also stripping from ^{18}O to the ground state of ^{29}Si as well as for the pick-up reaction of one neutron from the ground state of ^{208}Pb to the hole state $(2f_{7/2})$ of ^{207}Pb. Solid lines in Fig. 1 show the calculated differential cross sections as functions of the angle θ in comparison with experimental data from [4] for the reactions (a) $^{12}C +^{208} Pb \Rightarrow {}^{11}B +^{209} Bi$ at $E = 600MeV$ and (b) $^{12}C +^{27} Al \Rightarrow {}^{11}B +^{28} Si$ at $E = 600MeV$. In Fig.2 the comparison is made for the reactions (c) $^{18}O +^{28} Si \Rightarrow {}^{17}O +^{29} Si$ at $E = 352MeV$ [5] and (d) $^{3}He +^{208} Pb \Rightarrow {}^{4}He +^{207} Pb$ at $E = 47.5MeV$ [6]. The corresponding calculations have been performed with the following parametrs: (a) $V_0 = 50MeV$, $W_0 = 38MeV$, $a_I = 0.6fm$, $r_0 = 1.2fm$; (b) $V_0 = 50MeV$, $W_0 = 19$, $a_I = 0.4fm$, $r_0 = 1.2fm$; (c) $V_0 = 50MeV$, $W_0 = 15MeV$, $a_I = 0.5fm$, $r_0 = 1.2fm$; (d) $V_0 = 50MeV$, $W_0 = 3MeV$, $a_I = 0.5fm$, $r_0 = 1.25fm$. In all the cases, we have taken the spectroscopic factors equal to 1. The values D_0 and α_c were calculated according to formulae in the text.One can mention that for explanation of experimental data at various bombarding energies from $50MeV$ to $600MeV$ the main effect comes from changing the depth of the imaginary part of the potential W_0 from $3MeV$ to $38MeV$ and thickness parameter a_I changing in the limits of $0.5 \div 0.6fm$. Figs. 1 and 2 show the agreement of our calculations with experimental data presented both in absolute values and in the form of angular disributions. At higher energies, the reaction is characterized by a simple exponential angular distribution. At the energy decrease the diffraction-like picture in angular distribution appears according to equationi (21). From Fig.3 we see that the results of calculations are very sensitive to the choice of the parameter α_c, and a small deviation of the trajectory radius parameter r_{0I} leads to a significant change of the differential cross section in its absolute value. Thus,when θ_c changes with respect to $Re\theta_c = 0.027$ (solid line) and $Re\theta_c = 0.018$ (dashed line) the cross section is changed approximatly one third order of its value.

One can see that the DWBA calculations with the quasi-classical distorted waves obtained in the framework of HEA give good agreement with experimental data at energies begining from $10MeV$ per nucleon and higher. The absolute values of theoretical cross sections presented are shown without any renormalization factors, which means that the theory has rather good prediction possibilities. We can summarise that investigations of heavy ion-collisions, e.g. simple transfer reactions, in the quantum region of scattering angles $\theta > \theta_c$, outside the

Fig.1 Angular distributions for the stripping reactions (a) $^{12}C + ^{208}Pb \Rightarrow ^{11}B + ^{209}Bi$, $E = 50 MeV/n$; (b) $^{12}C + ^{27}Al \Rightarrow ^{11}B + ^{28}Si$, $E = 50 MeV/n$. Solid lines are the theoretical calculations, squared points are the experimental data from [4]



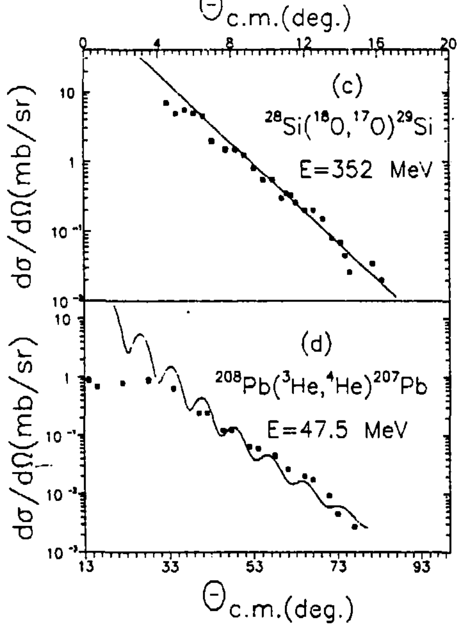


Fig.2 Angular distributions for the proton stripping in (c) $^{18}O + ^{28}Si \Rightarrow ^{17}O + ^{29}Si$, $E = 352 MeV$; and for the neutron pick up reaction (d) $^{3}He + ^{208}Pb \Rightarrow ^{4}He + ^{207}Pb$, $E = 47.5 MeV$. Solid lines are the theoretical calculation, squared points are the experimental data from [5,6].

limited trajectories of motion, are very sensitive to the precise structure of a nucleus-nucleus interaction. For instance, the slope of curves with θ feels the "thickness" of the acting region in the channel (Fig.3). It may be used also for searching the "halo" distributions of nuclei in the radioactive beams which now become available. We hope that the HEA-method suggested can be successfully used in both the qualitative and quantitative analysis of direct reactions.

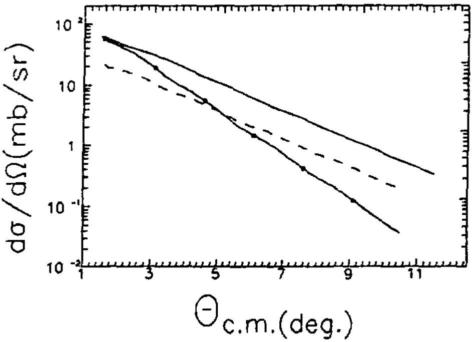


Fig.3 Influences of the classical angle θ_c introduced on the absolute value of the cross section and of the thickness parameter a_t on the form of the angular distribution. The solid line is the calculation for the reaction (b) with R_t for $r_{0t} = 0.17 fm$, the dashed line corresponds to the same reaction but for $r_{0t} = 1.2 fm$. The solid line with stars is the calculation for the latter case but at a larger thickness parameter $a_t = 0.6 fm$.

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