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NUCLEAR DISTORTED WAVES IN HIGH-ENERGY APPROXIMATION

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1 Introduction

Here we construct nuclear distorted waves using the high-energy approximation (HEA) in quasi-classical (QC) scattering on the optical (complex) potential. The method can be applied under the conditions $kR \gg 1$ and $E \gg V$. In sec.2, the main attention is paid to peculiarities coming from the imaginary part of a potential. Thus, the so-called effective potential appears, inherent in the three-dimensional quasi-classical wave functions. In principle, it can be parametrised independently of the primary input potential, and the corresponding QC-distorted waves may be applied then to describe the nuclear collision processes in the framework of the distorted wave Born approximation (DWBA). Note that usually the one-dimensional quasi-classical approximation is used for calculations of the partial phases in considerating the light- and heavy ion collisions with nuclei. In this case the classical deflection angle turns out to be a necessary atribute of the QC-theory (see, e.g.[1]). Instead, the three-dimensional HEA is adjusted for calculating the hadron- and electron-nuclear scattering [2-5]. In the consideration of the latter processes the straight-line trajectories as integration paths are usually used. In Sec.3 we try to join both these methods, where a simple way is developed to include a distortion of the straight line trajectories. In sec.4 we obtain an explicit form for the three-dimensional distorted waves when both the Coulomb and nuclear potentials are included, and the method is developed for calculating the typical matrix elements, appearing in considerations of various reactions in the framework of DWBA. Thus, we avoid the traditional partial wave decompositions, for which in this case a very large number of terms is to be included. This gives us the possibility to obtain, in the framework of DWBA, analytical expressions for qualitative physical estimations and for a quantitative comparison with experimental data.

2 Quasi-classics for the Optical Model

We start with the wave equation

$$\Delta\Psi + K^2\Psi = 0, \qquad K^2 = k^2 - \bar{U} \tag{2.1}$$

with the complex potential $\bar{U}$ included as follows

$$\bar{U} = U + i\omega = \frac{2m}{\hbar^2}\bar{V}, \qquad \bar{V} = V + iW. \tag{2.2}$$

The solution of eq.(2.1) is written in the form

$$\Psi = ue^{iS}, \qquad \tilde{S} = S_1 + iS_2. \tag{2.3}$$

Substituting (2.3) into (2.1), equating to zero separately the real and imaginary parts of eq.(2.1), and calculating the current change along a path, one can obtain the following equations

$$\vec{\nabla} S_1(r) = \tilde{\kappa}_1^2, \tag{2.4}$$

$$\vec{\nabla} S_1 \vec{\nabla} S_2 = \frac{1}{2}\tilde{\kappa}_2^2, \tag{2.5}$$

$$\vec{\nabla} \vec{j} = u^2[x_1 - 2\vec{\nabla} S_1 \vec{\nabla} S_2]e^{-2S_2}, \tag{2.6}$$

where

$$\vec{j} = u^2 \vec{\nabla} S_1 e^{-2S_2}, \tag{2.7}$$

$$\tilde{\kappa}_1^2 = k^2 - U_e, \qquad \tilde{\kappa}_2^2 = \omega_e, \tag{2.8}$$

$$U_e = U + (\vec{\nabla} S_2)^2 + x_2, \qquad \omega_e = -\omega + x_1, \tag{2.9}$$

$$x_{1,2} = 2\frac{\vec{\nabla} u \vec{\nabla} S_{1,2}}{u} + \Delta S_{1,2}. \tag{2.10}$$

In the case of a real potential, when W=0, we have $\tilde{S} = S_1 = S$, S_2=0, and then we obtain the known expressions [6] for the wave function, the action function and the current conservation law:

$$\Psi = ue^{iS}, \tag{2.11}$$

$$(\vec{\nabla} S)^2 = k^2 - U, \tag{2.12}$$

$$\vec{\nabla} \vec{j} = u^2 x_1 = 0, \tag{2.13}$$

$$\vec{j} = u^2 \vec{\nabla} S. \tag{2.14}$$

Note that eq.(2.13) is a result of equality to zero of the imaginary part of the wave equation in the case of the real potential. This means that the current is conserved if an absorption along the path is absent.

A simple way to decouple eqs.(2.4) and (2.5) is to assume the imaginary part of a potential being small as compared with its real part $|W| \ll |V|$. Then, we can suppose that $x_1 \simeq 0$ (see eq.(2.13)), and instead of eqs.(2.4)-(2.6) we obtain the following set of equations:

$$(\vec{\nabla} S_1)^2 = k^2 - U_e, \tag{2.15}$$

$$\vec{\nabla} S_1 \vec{\nabla} S_2 = -\frac{1}{2}\omega, \tag{2.16}$$

$$\vec{\nabla} \vec{j} = u^2 \omega e^{-2S_2}. \tag{2.17}$$

In the case of basic conditions of HEA the decomposition of S in small V_e/E can be realized from eq.(2.15). Thus,

$$S_1 \simeq S_0 + \int_{\vec{r}_0}^{\vec{r}} [k - \frac{1}{2k} U_e(\vec{r} - \hat{k}s)]ds = \vec{k}\vec{r} - \frac{1}{2k}\int_{\vec{r}_0}^{\vec{r}} U_e(\vec{r} - \hat{k}s)ds, \tag{2.18}$$

where $\hat{k} = \vec{k}/k$ and $S_0 = \vec{k}\vec{r}_0$ is determined in the asymptotics so that to compensate the contribution of the constant $-\vec{k}\vec{r}_0(r_0 \to \infty)$, and integration runs along the trajectory of motion. Substituting the solution (2.18) into (2.16), one gets:

$$S_2 \simeq -\frac{1}{2k}\int_{\vec{r}_0}^{\vec{r}} .\omega(\vec{r} - \hat{k}s)ds \tag{2.19}$$

Now let us obtain the amplitude u of the wave function (2.3). For this aim we use the law (2.17) of changing the current:

$$\vec{\nabla}u^2\vec{K}e^{-2S_2} = \omega u^2 e - 2S_2, \tag{2.20}$$

where $\vec{K} = \vec{S}_1$ is the local momentum at the point $\vec{r}$. We consider the change of a flux moving from $-\infty$ to the center of scattering. In our case of HEA we can select for the path of motion a straight line along the oz-axis with an impact parameter ρ. Then we use the Gauss-Ostrogradsky theorem for a transformation of (2.20):

$$\int_{(S)} u^2(\vec{K}\vec{n})e^{-2S_2}ds = \int_{(V)} u^2\omega e^{-2S_2}dV, \tag{2.21}$$

where integration in the left- and the right-hand side of eq.(2.21) is done over the surface and volume of a tube of the flux, respectively, and $\vec{n}$ is the normal vector to the surface of the tube. One can note that everywhere along the straight line we have $\vec{\rho} \perp \vec{K}$. Therefore a contribution to the left integral occurs only from the left D_0 and the right D sides of the current tube, where $\vec{K}\vec{n} = \pm 1$. In the right integral of (2.21) we write the volume element using the average surfaces of the sides D_0 and D, that is $dV = \langle D\rangle dz = [(D_0 + D)/2]dz$. Thus, we get:

$$-u_0^2 e^{-2S_2(-\infty)}D_0 k + u^2 D e^{-2S_2}K = \int_{-\infty}^{z} u^2\omega e^{-2S_2}\frac{D_0 + D}{2}dz. \tag{2.22}$$

The relation of D_0/D can be obtained by using the momentum conservation law

$$\rho_0 k = \rho K \simeq \rho k(1 - \frac{V_e}{2E}), \tag{2.23}$$

from which it follows that

$$d\rho_0 = (1 - \frac{V_e}{2E}x)d\rho, \qquad x = 1 + \frac{\rho}{V_e}\frac{\partial V_e}{\partial \rho}, \tag{2.24}$$

$$D_0/D = \rho_0 d\rho_0 d\varphi/\rho d\rho d\varphi \simeq 1 - \frac{V_e}{2E}(1 + x), \tag{2.25}$$

$$\frac{D_0 + D}{2} = D[1 - \frac{V_e}{4E}(1 + x)] = Dl. \tag{2.26}$$

Denoting $Y = u^2 D e^{-2S_2}K$, we rewrite (2.22) in the form

$$-Y(-\infty) + Y = \int_{-\infty}^{z} \omega\frac{l}{K}Y dz. \tag{2.27}$$

Solving equation with the boundary conditions $u_0^2 = 1$, $e^{-2S_0(-\infty)} = 1$ and taking account of $l/K \simeq 1/k$, we have

$$Y(z) = Y(-\infty)\exp(\frac{1}{k}\int_{-\infty}^{z}\omega dz), \tag{2.28}$$

from which there follows the final result:

$$u = 1 - \frac{V_c}{4E}x. \tag{2.29}$$

Thus, gathering together eqs.(2.3),(2.18),(2.19) and (2.29), we can write the three-dimensional quasi-classical wave function in the case of scattering in the field of a nuclear optical potential as follows:

$$\Psi = (1 - \frac{V}{4E})\exp\{i\vec{k}\vec{r} - \frac{i}{2k}\int_0^\infty U(\vec{r} - \hat{k}s)ds + \frac{1}{2k}\int_0^\infty \omega(\vec{r} - \hat{k}s)ds\}, \tag{2.30}$$

Now one can write down the *in* and *out* three-dimensional quasi-classical wave functions in the high-energy approximation, using the time reverse relation:

$$\Psi^{(-)*}(\vec{r}, \vec{k}) = \Psi^{(+)}(\vec{r}, -\vec{k}). \tag{2.31}$$

This permits us to write distorted waves in the form

$$\Psi^{(+)}_{\vec{k}_i} = \exp\big[i\vec{k}_i\vec{r}_i - \frac{i}{\hbar v_i}\int_{-\infty}^{0} V(\vec{r} + \hat{k}s)ds + \frac{1}{\hbar v_i}\int_{-\infty}^{0} W(\vec{r} + \hat{k}s)ds\big], \tag{2.32}$$

$$\Psi^{(-)*}_{\vec{k}_f} = \exp\big[-i\vec{k}_f\vec{r}_f - \frac{i}{\hbar v_f}\int_0^{\infty} V(\vec{r} + \hat{k}s)ds + \frac{1}{\hbar v_f}\int_0^{\infty} W(\vec{r} + \hat{k}s)ds\big], \tag{2.33}$$

We omit here the index c in the real part of a potential, having in mind that this potential is selected specially for quasi-classical calculations. Then, we omit also the preexponential functions because of their small contribution in the case of nuclear collisions, for which we intend to apply this method. It is interesting to note that according to (2.31), the $\Psi^{(-)}$-function itself has an opposite sign of the imaginary part of the complex potential, as compared with the one in $\Psi^{(+)}$, which means the increase in the flux when going back to the scattering center.

Usually, these expressions for $\Psi^{(\pm)}$ are obtained using the prescription, when in quasi-classical wave functions for the real potential the latter is changed to the complex one, and the amplitude pre-exponential factor is taken to be unity. However, we see that, first, the form of eq.(2.30) is right if the absorption term of a potential W is small as compared with its real part V_c. Then, in (2.30) one should use an effective real part of the potential, indused by the imaginary part of the initial potential. In the general case the action function has to be obtained by solving the coupled channel equations (2.4), (2.5) and the dependence of S on V and W will be non-linear (see, e.g. [7]).Thus, we conclude that the potentials usually used in quasi-classical scattering on nuclei should not coincide with those in the initial wave equation. By the way, this fact has no serious meaning, if the V_c-potentials is calculated directly from the

scattering data in the framework of quasi-classics. Then, these effective potentials will also be appropriate in calculating other processes, if one uses the corresponding quasi-classical distorted waves. Second, the pre-exponential factor in (2.30) makes small alternations in the final results and can be omitted in nuclear scattering, where we have to use the phenomenological effective potentials. But in those cases, where exact real potentials can be introduced, e.g. the Coulomb potentials of nuclei, this factor is necessary to take into account. In [8] this factor was introduced in a phenomenological way for discribing the electron scattering on nuclei, where it makes correction to form factors sometimes about 30% of their value.

3 Distorted Trajectory Approximation

The HEA-calculations usually start with expressions for wave functions in typical forms (2.32), (2.33), where integration is supposed to run along the straight line trajectories, parallel to the momentum of a particle in asymptotics $\vec{k}(\|\vec{oz})$. However this latter suggestion doesn't follow automatically from the basic condition $E \gg V$ of HEA. Indeed, one can learn from mechanics that in this case trajectories deflect near the scattering center by angles of the order V/E. This means that in the phase (2.18) the correction appears of the order $k(V/E)$ to the momentum of a particle near the most important point of contribution $\vec{r}$. This additional term is comparable in magnitude with the same order terms $(\hbar v)^{-1}\int V ds$, which present in (2.32), (2.33) as a result of an expansion of S in V/E in (2.18). The most distorted trajectory goes through the so-called external turning point $\Re$ of the closest approach of scattered particles. At this point the trajectory is parallel to the local momentum $\vec{k}_{\Re}$, directed at the angle $\theta_c/2$ with respect to $\vec{k}$ in the asymptotics. Here $\theta_c \simeq |V|/E$ is the corresponding deflection angle in asymptotics at the end of motion. Then we can write

$$\vec{k}_{\Re} = \vec{k} - \vec{q}_c/2, \tag{3.1}$$

where $\vec{q}_c$ represents the momentum transfer in the classical motion. The quantity $k_{\Re}$ is obtained from the conservation law of angular momentum:

$$\rho k \simeq \Re k_{\Re}, \tag{3.2}$$

where $\rho \simeq \Re\cos(\theta_c/2)$ is the impact parameter. So, we have

$$k_{\Re} = k\cos(\theta_c/2), \quad q_c/2 \simeq k\sin(\theta_c/2). \tag{3.3}$$

Now, simplifying the problem, we select a new path of integration along the straight line parallel to the momentum $\vec{k}_{\Re}$. Then, eq.(2.18) has to be rewritten in the following form:

$$S_1 = S_0 + \vec{k}_{\Re}\vec{r} - \vec{k}\vec{r}_0 - \frac{1}{\hbar v}\int_{\vec{r}_0}^{\vec{r}} V(\vec{r} - \hat{k}_{\Re}s)ds. \tag{3.4}$$

Now we show that the effect of the trajectory distortion can be neglected in the last term of (3.4) containing the integral of V. For this aim we take into account that the main contribution to the integral comes from the upper limit at the point of the closest approach $\vec{r} \sim \vec{\Re}$. Also, in

the case of small classical angles we can use the small parameter $q_c/k \simeq V/E \ll 1$ in further expansions:

$$\widehat{k}_{\Re} \simeq \frac{\vec{k}}{\sqrt{k^2+q_c^2}} - \frac{\vec{q}_c/2}{\sqrt{k^2+q_c^2}} \simeq \widehat{k} - \frac{q_c}{2k}\widehat{q}_c, \tag{3.5}$$

$$V(\vec{r}-\widehat{k}_{\Re}s) \simeq V(\vec{R}) + \frac{q_c}{2k}\vec{\nabla}_R V\widehat{q}_c s = V(\vec{R}) + \frac{q_c}{2k}\frac{dV}{dR}\widehat{R}\widehat{q}_c s, \tag{3.6}$$

where we denote $\vec{R} = \vec{r} - \widehat{k}s$, $\widehat{R} = \vec{R}/R$, $\widehat{q}_c = \vec{q}_c/q_c$. Note that near the point $\vec{r} \sim \vec{\Re}$ we have $s/R \ll 1$ and $\vec{r} \perp \widehat{k}$, which permits us to write $\widehat{R}\widehat{q}_c \simeq 1$ and $R^2 \simeq r^2 + s^2$. Then, substituting $dV/dR = (dV/ds)(R/s)$ into (3.6), we obtain from eq.(3.4) in the region of the closest-approach point $r \sim \Re$

$$\int_{\vec{r}_0}^{\vec{r}} V(\vec{r}-\widehat{k}_{\Re})ds \simeq \int_{\vec{r}_0}^{\vec{r}} V(\vec{r}-\widehat{k}s)ds + \frac{q_c}{2k}rV(r). \tag{3.7}$$

It is clear that the last term here can be neglected, since $q_c \simeq k(V/E)$ and its total contribution is of an order of $(V/E)^2$.

Now, using eqs.(2.3),(2.10),(3.4) and (3.7), we can write the *in* and *out* QC-wave functions in the high-energy approximation with the trajectory distortion included. For this aim we direct the $0z$-axis parallel to these momenta, and change variables $\vec{r} = \vec{\rho} + \vec{z}$, $s = z - \lambda$, where $\vec{\rho} \perp \vec{z}$. Thus, we can write

$$\Psi^{(+)}_{\vec{k}_i} = \exp[iS_0^i + i(\vec{k}_i - \frac{\vec{q}_{ci}}{2})\vec{r}_i - \frac{i}{\hbar v_i}\int_{-\infty}^{z_i} V(\sqrt{\rho_i^2+\lambda^2})d\lambda + \frac{1}{\hbar v_i}\int_{-\infty}^{z_i} W(\sqrt{\rho_i^2+\lambda^2})d\lambda], \tag{3.8}$$

$$\Psi^{(-)*}_{\vec{k}_f} = \exp[iS_0^f - i(\vec{k}_f + \frac{\vec{q}_{cf}}{2})\vec{r}_f - \frac{i}{\hbar v_f}\int_{z_f}^{\infty} V(\sqrt{\rho_f^2+\lambda^2})d\lambda + \frac{1}{\hbar v_f}\int_{z_f}^{\infty} W(\sqrt{\rho_f^2+\lambda^2})d\lambda]. \tag{3.9}$$

For a Coulomb field these integrals will give the logarithmic contribution $-\eta \ln 2kz_0$ at large z_0, where $\eta = Z_1Z_2e^2/\hbar v$. Therefore it is convenient to select the constant

$$S_0 = \eta \ln 2kz_0 \quad at \quad z_0 \to \infty, \tag{3.10}$$

so that to have the usual plane waves in the asymptotics. One should remind that if necessary one can include in (3.8),(3.9) the flux factor $(1 - V/4E)$, as it was done in (2.30). Note that QC-distorted waves in the form (3.8),(3.9) are inherent in the heavy ion collisions, where the classical deflection angle θ_c is really observed in experimental data as the so-called limited angle of a Coulomb deflection. In principle, other trajectories for which $\theta < \theta_c$ should be also taken into account. Moreover, the incident particle can deflect at the same classical angle θ_c, corresponding to the "nuclear" trajectory of motion with the radius of the closest approach $\Re_N$, smaller than the $\Re$, inherent in a limited Coulomb trajectory. However, it is clear, first, that contributions of these "nuclear" trajectories seem to be negligible because of the strong nuclear absorption in this region. Second, the probability to find a particle out of any classical trajectory is a quantum effect and in quasi-classical conditions this probability has to be exponentially small. Thus, usually these comparably small effects cannot be important in the classical region of motion at $\theta < \theta_c$. The only place where they give the main effect is the region out of the limited trajectory, inherent in the given potential of scattering, i.e. at angles $\theta > \theta_c$.

4 Transition Matrix Elements in Quasi-Elastic Approximation

Based on the QC-distorted waves (3.8),(3.9) we show how one can calculate typical matrix elements of the nuclear scattering processes and direct reactions having the following form:

$$t = \int d\vec{r}\,\Psi^{(-)*}_{\vec{k}_f}\hat{O}(\vec{r})\Psi^{(+)}_{\vec{k}_i} = \int d\vec{r}\,\hat{O}(\vec{r})\exp\{i\tilde{\Phi}(\vec{r})\}, \tag{4.1}$$

$$\tilde{\Phi} = \vec{q}_{ef}\vec{r} + \Phi(\vec{r}), \quad \Phi = \Phi^{(+)}_i + \Phi^{(-)}_f, \quad \Phi^{(\pm)} = \Phi^{(\pm)}_N + \Phi^{(\pm)}_C. \tag{4.2}$$

Here the nuclear and the Coulomb parts of the whole phase are:

$$\Phi^{(\pm)}_N = -\frac{1}{\hbar v}\int_{\mp z}^{\infty} V_N(\sqrt{\rho^2+\lambda^2})d\lambda, \quad \Phi^{(\pm)}_C = \eta\ln 2kz_0 - \frac{1}{\hbar v}\int_{\mp z}^{\infty} V_C(\sqrt{\rho^2+\lambda^2})d\lambda, \tag{4.3}$$

where $kz_0 \to \infty$, the effective momentum transfer $\vec{q}_{ef} = \vec{q} - \vec{q}_c$ with $\vec{q}\|\vec{q}_c$, $q_{ef} = 2k(\alpha - \alpha_c)$, $\alpha = \sin(\theta/2)$, and $\alpha_c \simeq V(R_t)/2E$, taken at the radius R_t of the limited trajectory of motion. In HEA it is natural to use the quasi-elastic approximation working well when the energy loss is very small, as compared with the collision energy, so one can take $E_i \simeq E_f$ and $k_i \simeq k_f$.

We write the transition operator in the general form:

$$\hat{O}(\vec{r}) = P(r)f^{(l)}(r)Y_{L0}(\hat{r}) \tag{4.4}$$

with the typical Fermi-like functions usually used in many practical calculations

$$f(r) = (1+\exp\frac{r-R}{a})^{-1}, \quad f^{(l)}(r) = \frac{d^l f}{dR^l}, \quad l = 1,2,3\ldots, \tag{4.5}$$

where R and a correspond to the radius and the "surface thickness" of an interaction. For example, in the case of elastic scattering we have [3,9] $L = 0$, $l = 0$, $P(r) = 1$; for inelastic scattering with the collective state excitations there are various combinations of $L = 0,2,3,4$, $l = 1,2$, $P(r) = 1, 1/r$; for direct one nucleon transfer reactions one may use $L \geq 1$, $l = 1$, $P(r) = 1/r$. Thus, we consider the typical form:

$$t = \int d\vec{r}\,P(r)f^{(l)}(r)\exp\{i\tilde{\Phi}(\vec{r})\}Y_{L0}(\hat{r}), \tag{4.6}$$

and first calculate the QC-phases (4.3). For this aim we take into account that the high-energy scattering is mainly sensitive to the internal and surface region of interaction, the fact, which permits us to use simplified potentials in (4.3). Namely, instead of the nuclear Woods-Saxon potential we take its expansion [10] in the diffuseness parameter a, and for the Coulomb potential we use only its inside-of-R expression:

$$V_N \Rightarrow \tilde{V}_N = V_0\{\Theta(R-r) - \frac{\pi^2}{6}a^2\delta^{(1)}(r-R) - \ldots\}, \tag{4.7}$$

$$V_C \Rightarrow \tilde{V}_C = \frac{1}{2}V_B(3 - \frac{r^2}{R_C^2}), \cdot\ V_B = \frac{Z_1Z_2e^2}{R_C}. \tag{4.8}$$

Substituting (4.7) into (4.3) and expanding, in ρ/R_V, the result of integration of $\tilde{V}_N$ we get:

$$\Phi_V^{(\pm)} = -B^V\left\{\sqrt{R_V^2-\rho^2} \pm z\Theta(\sqrt{R_V^2-\rho^2}-z) + \frac{\pi^2}{6}\frac{a_v^2\rho^2}{(R_V^2-\rho^2)^{3/2}}\left[1+\Theta(z-\sqrt{R_V^2-\rho^2})\right]\right\} \simeq$$

$$\simeq -B^V\left[R_V \pm z - \frac{\rho^2}{2dR_V}(1+3.29\frac{a_v^2}{R_V^2}) - 2.46R_V\frac{a_v^2}{R_V^2}(\frac{\rho}{R_V})^4\right], \qquad B^V = \frac{V_0}{\hbar v}. \tag{4.9}$$

Neglecting terms with $(a/R_V)^2$ which are usually small in absolute values, we obtain:

$$\Phi_V^{(\pm)} = -B^V\left[R - \frac{\rho^2}{2dR_V} \pm z\right]. \tag{4.10}$$

Here the large dimensionless parameter $d(\gg 1)$ is introduced to fit the mentioned expansion $1-\rho^2/2dR_V^2$ to the initial expression $\sqrt{1-\rho^2/R_V^2}$ in the most important region near the radius interaction.

As to the Coulomb phase we follow the prescription of ref.[4], supposing

$$\eta \ln 2kz_0 = \frac{1}{\hbar v}\int_0^\infty V_C(\lambda)d\lambda, \quad at \quad kz_0 \to \infty \tag{4.11}$$

with V_C, the exact Coulomb field of the spread nuclear charge. Indeed, in the lower limit integral (4.11) has to be zero (since V_C at small r looks like (4.8)), and at large λ we have the true logarithmic behaviour. Then, substituting (4.11) into (4.3), we obtain:

$$\Phi_C^{(\pm)} = \frac{1}{\hbar v}\int_0^\infty V_C(\lambda)d\lambda - \frac{1}{\hbar v}\int_{\mp z}^\infty V_C(\sqrt{\rho^2+\lambda^2})d\lambda = -\frac{1}{\hbar v}\int_{\mp z}^0 V_C(\sqrt{\rho^2+\lambda^2})d\lambda + \delta\Phi_C^{(\pm)}, \tag{4.12}$$

where

$$\delta\Phi_C^{(\pm)} = \frac{1}{\hbar v}\int_0^\infty \left[V_C(\lambda) - V_C(\sqrt{\rho^2+\lambda^2})\right]d\lambda. \tag{4.13}$$

To calculate (4.13), we expand $V_C(\sqrt{\rho^2+\lambda^2})$ in powers of small ρ,

$$V_C(\sqrt{\rho^2+\lambda^2}) = V_C(\lambda) + \frac{\rho^2}{2\lambda}\frac{\partial V_C}{\partial\lambda} + \ldots. \tag{4.14}$$

and use the estimation from [4]:

$$\delta\Phi_C = \frac{\rho^2}{2\hbar v}\int_0^\infty \frac{1}{\lambda}\frac{\partial V_C}{\partial\lambda}d\lambda \simeq \eta\frac{\rho^2}{8}\left\langle\frac{1}{r^2}\right\rangle. \tag{4.15}$$

Bearing in mind the smallness of ρ/R, we neglect this term in the first stage. Thus,

$$\Phi_C^{(\pm)} = -\frac{1}{\hbar v}\int_{\mp z}^0 \tilde{V}_C(\sqrt{\rho^2+\lambda^2})d\lambda = \mp B^C(3-\frac{\rho^2}{R_C^2})z \pm B^C\frac{z^3}{3R_C^2}, \quad B^C = \frac{V_B}{\hbar v}. \tag{4.16}$$

Then inserting relations $z = (\vec{k}\vec{r})/k$ and $\rho^2 = r^2 - (\vec{k}\vec{r})^2/k^2$ in (4.10) and (4.16) we obtain the whole phase in (4.2):

$$\tilde{\Phi} = \vec{q}_{ef}\vec{r} + 2a_0 + \frac{a_1}{k}(\vec{k}_i - \vec{k}_f)\vec{r} + \frac{a_2}{k^2}[(\vec{k}_i\vec{r})^2 + (\vec{k}_f\vec{r})^2] + \frac{a_3}{k^3}[(\vec{k}_i\vec{r})^3 - (\vec{k}_f\vec{r})^3], \tag{4.17}$$

where a_n are the known functions of r and parameters of potentials:

$$a_0 = -B^V R_V + \frac{B^V r^2}{2dR_V}, \quad a_1 = -(B^V + 3B^C) + \frac{B^C r^2}{R_C^2}, \quad a_2 = -\frac{B^V}{2dR_V^2}, \quad a_3 = -\frac{2B^C}{3R_C^2}. \tag{4.18}$$

Selecting axes $oz \| \vec{q}$ and $ox \| \vec{K} = \vec{k}_i + \vec{k}_f$, we obtain the scalar products in (4.17)

$$\vec{k}_{(f)}^{(i)}\vec{r} = kr(\pm\alpha\mu + \sqrt{1-\alpha^2}\sqrt{1-\mu^2}\cos\bar{\varphi}), \tag{4.19}$$

where $\mu = \cos\bar{\theta}$, and $\bar{\theta}$, $\bar{\varphi}$ are the angles of $\vec{r}$ in a spherical coordinate system. So,

$$\tilde{\Phi} = 2a_0 + \tilde{\beta}\mu + n_1\mu^2 + c_1\mu^3 + n_2(1-\mu^2)\cos^2\bar{\varphi} + c_2\mu(1-\mu^2)\cos^2\bar{\varphi} \tag{4.20}$$

with $\tilde{\beta}$, c and n expressed through a_n, giving the forms:

$$\begin{aligned} \tilde{\beta} &= \tilde{q}r = q_{ef}r + 2k_\delta\alpha r, \quad k_\delta = -[B^V + B^C(3 - \frac{r^2}{R_C^2})], \\ n_1 &= -\frac{B^V}{dR_V}(\alpha r)^2, \qquad n_2 = -\frac{B^V}{dR_V}(1-\alpha^2)r^2, \\ c_1 &= -\frac{4B^C}{3R_C^2}(\alpha r)^3, \qquad c_2 = -\frac{4B^C}{R_C^2}(1-\alpha^2)\alpha r^3. \end{aligned} \tag{4.21}$$

Now one can see that integrand (4.6) contains, in the exponent, a typical power dependence on the variables r and μ with a large argument $q_{ef}r$. Keeping in mind that $d\vec{r} = -r^2 dr d\mu d\bar{\varphi}$, we integrate in (4.6) over $d\mu$ by parts, retaining only the lower order term in $1/q_{ef}R$:

$$I = \int_{-1}^{+1} d\mu \exp[i\tilde{\Phi}(r,\mu,\bar{\phi})]Y_{L0}(\mu) \simeq -i\frac{\exp(i\tilde{\Phi})}{\partial\tilde{\Phi}/\partial\mu}Y_{L0}(\mu)\Big|_{-1}^{+1} =$$

$$= -i\exp(2i\tilde{a}_0 + in_1)[I^{(+)}(r,\bar{\varphi}) - (-1)^L I^{(-)}(r,\bar{\varphi})]Y_{L0}(1), \tag{4.22}$$

where

$$I^{(\pm)} = \frac{\exp[\pm i(\tilde{\beta} + c_1)]}{\Delta_{(\pm)} \mp \delta_{(\pm)}\cos^2\bar{\varphi}}, \quad \Delta_{(\pm)} = \tilde{\beta} + 3c_1 \pm 2n_1, \quad \delta_{(\pm)} = 2(n_2 \pm c_2). \tag{4.23}$$

Then, the integration over $d\bar{\varphi}$ can be performed with the help of a table integral:

$$\int_0^{2\pi} \frac{dx}{a^2 + (b^2 - a^2)\cos^2 x} = \frac{2\pi}{ab}. \tag{4.24}$$

Thus, eq.(4.6) is transformed to the one-dimensional integral:

$$t = \int_0^\infty dr P(r) f^{(l)}(r) \Big\{ F^{(+)}(r) - (-1)^L F^{(-)}(r) \Big\} Y_{L0}(1), \tag{4.25}$$

where

$$F^{(\pm)}(r) = \frac{r}{\tilde{q}\tilde{L}(\pm)} e^{i(2a_0+n_1)} e^{\pm i(\tilde{q}r+c_1)}, \qquad \tilde{L}(\pm) = \frac{l}{\tilde{\beta}} \sqrt{\Delta_{(\pm)}(\Delta_{(\pm)} \mp \delta_{(\pm)})}. \tag{4.26}$$

The root singularities in the denominator are situated far from the radius of interaction R, namely, at $r_s = \frac{A_1 E}{20} R$ with A_1, the atomic number of a projectile, E, the energy in MeV, and R in fm. However, at this distances the Fermi-like $f^{(l)}(r)$-functions in the integrand decrease very fast. This forces us to introduce a prescription when the integration in (4.25) should be cut off at distances less than r_s. For this aim we use the properties of the functions $f^{(l)}(r)$ on the complex r-plane, where they have poles near the radius of nuclear interaction at $r_n^\pm = R \pm i x_n$ with $x_n = \pi a(2n+1)$ and $n = 0, 1, 2 \ldots$.

For example we consider in some details the case, corresponding to elastic scattering, when $L = 0$, l=0, $P(r) = 1$. (For an exact amplitude of scattering one needs to take $P(r) = -m/2\pi\hbar^2 Y_{00}$). In this case we have the simple poles of $f(r)$ in $r_n^\pm$, the integration contour for r_n^+ should be selected in the first quadrant of the complex plane, going along the imaginary axis and then over the circles of an infinite radius. The same contour, but in the fourth quadrant, must be used in the case of r_n^-. Thus, the result is expressed through a sum of the corresponding residues at the above-mentioned poles:

$$t = -2\pi i a Y_{00}(1) \sum_{n=0}^{\infty} \{ F^{(+)}(r_n^+) + F^{(-)}(r_n^-) \}. \tag{4.27}$$

In practice, for the typical nuclear parameters it is usually enough to take into account only a couple of poles at $r_0^\pm = R \pm i x_0$ nearest to the real axis ("two-pole approximation"), since, in principle, every next pair of poles contributes approximately an order smaller than the previous one. In many cases the $\tilde{L}$-function in the denominator of $F^{(\pm)}$ can be presented as $\exp(\mp\kappa)$ with $\kappa \simeq n_2/(\tilde{\beta}r)$, because other terms in $\tilde{L}$ usually occurr to be negligible. Then, expanding the functions in small (x_n/R)

$$\tilde{q}r = \tilde{q}R \pm i\tilde{q}x_n, \quad \phi = 2a_0 + n_1 = \phi(R) \pm i x_n \phi'_R, \quad y = c_1 - i\kappa = y(R) \pm i x_n y'_R, \tag{4.28}$$

and using the relations

$$\sum_{n=0}^{\infty} \exp[-(2n+1)x] = \frac{1}{2\sinh x}; \quad \sum_{n=0}^{\infty} (2n+1)\exp[-(2n+1)x] = \frac{\coth x}{2\sinh x}, \tag{4.29}$$

we sum up all the terms in (4.20) to obtain

$$t \simeq -\frac{i\pi a R}{\tilde{q}} Y_{00}(1)(1 + i\frac{\pi a}{R}) e^{i\phi(R)} \left\{ \frac{e^{i(\tilde{q}R + y(R))}}{\sinh[\pi a(\tilde{q} + y'_R + \phi'_R)]} + \xi \frac{e^{-i(\tilde{q}R + y(R))}}{\sinh[\pi a(\tilde{q} + y'_R - \phi'_R)]} \right\}, \tag{4.30}$$

where

$$\xi \simeq (1 - i\frac{\pi a}{R})/(1 + i\frac{\pi a}{R}). \tag{4.31}$$

In a qu ilitative consideration one can put $\xi \simeq 1$, since the interaction radius R is usually larger than πa . If at the same time $(\phi'/\pi a) \ll 1$, we can bring the $sinh(\pi a q_{ef})$ out of the wave brackets and obtain at large $\pi a q_{ef}$:

$$t \sim e^{-\pi a k(\alpha - \alpha_c)} \cos(\tilde{q}R + y'_R), \tag{4.32}$$

where $\alpha = \sin(\theta/2)$. We see that the elastic amplitude t is decreasing with the angle of scattering as an exponential function [11] with a slope determined by the thickness parameter a of a surface of an interaction. Simultaneously it oscillates with a frequency depending mainly on the radius parameter R. This is a typical behaviour for the so-called Fraunhofer diffraction scattering on nuclei. However, for large $(\phi'/\pi a)$ only one term of (4.30) gives contribution, and in this case no oscillations will appear. So,

$$|t|^2 \sim e^{-2\pi a k(\alpha - \alpha_c)}. \tag{4.33}$$

Under some conditions this kind behaviour is observed in the heavy ion elastic scattering at angles θ larger than θ_c, the limited deflection angle.

In other cases, when $f^{(l)}(r)$ takes place in the integrand, the pole method can also be applied to obtain simple analytical expressions for the corresponding matrix elements.

5 Conclusion

It is clear that the consideration of high-energy processes in the language of nuclear potentials needs adequate methods of their description. The method developed here takes account of some specific features of nuclear collision processes including the heavy-ion reactions, as compared with those suggested for scattering on nuclei of fast electrons [4,5] and protons [12]. The results obtained can be summerised as follows:

1. The three-dimensional quasi-classics is adopted for constructing nuclear distorted waves using the high-energy approximation, when the conditions $kR \gg 1$, $E \gg |V|$, $\theta > \theta_c \simeq (1/kR)$, $(|V|/E)$ are fulfilled.

2. The complex nuclear potential is transformed, in the corresponding eikonal-like QC-distorted waves, into an effective potential which may be selected independently of the initial one for the description of experimental data. Then, based on the idea of the distorted wave Born approximation, the obtained distorted waves may be used for further applications in other direct channels, e.g., for inelastic scattering and nucleon transfer reactions.

3. An important role is established of a deflection of the trajectory of motion from the straight lines, usually used as paths of integration, and the prescription of introducing the deflection angle θ_c is suggested. This effect is most significant in heavy-ion collisions, and can change the corresponding cross sections in orders of value.

4. A method is developed for calculations of the matrix elements for typical direct processes including the calculations of QC-phases and integrations over the angular and r-variables in analytic form. This gives a possibility both for the qualitative analysis of main physical features of processes and the quantitative fit to experimental data.

5. The method can be applied for scattering angles $\theta > \theta_c$, forbidden in the classical mechanics. Therefore, in this region all the cross sections fall down with angles as an exponential function having a slope determined by the "diffuseness" parameter of the surface of an interaction potential.

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