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QUASI-CLASSICAL DESCRIPTION OF HEAVY ION REACTIONS

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1 Introduction

The method is suggested for calculating both elastic and inelastic scattering of light and heavy ions on nuclei and for describing simple transfer reactions. For this aim we construct nuclear distorted waves using the high-energy approximation (HEA) in quasi-classical (QC) scattering on an optical (complex) potential. The method can be applied under the conditions $kR \gg 1$ and $E \gg V$. In sec.2, the main attention is paid to peculiarities coming from the imaginary part of a potential. Thus, the so-called effective potential appears, inherent in the three-dimensional quasi-classical wave functions. In principle, it can be parametrised independently of the primary input potential, and the corresponding QC-distorted waves may be applied then to describe the nuclear collision processes in the framework of the distorted wave Born approximation (DWBA).

Note that usually the one-dimensional quasi-classical approximation (WKB-method) is used for calculations of the partial phases in considerating the light- and heavy ion collisions with nuclei. In this case the classical deflection angle turns out to be a necessary atribute of the QC-theory (see, e.g.[1]). Instead, the three-dimensional HEA is adjusted for calculating the hadron- and electron-nuclear scattering [2]. In the consideration of the latter processes the straight-line trajectories as integration paths are usually used. Here we try to join both these methods, where a simple way is developed to include a distortion of the straight line trajectories. Thus, one can avoid the traditional partial wave decompositions, for which in this case a very large number of terms is to be included. In principle, this gives us the possibility to avoid complicated numerical calculations and to obtain, in the framework of DWBA, analytical expressions for qualitative physical estimations and for a quantitative comparison with experimental data. In sections 3-5, the basic formulae are presented for elastic, inelastic scattering and transfer reactions, respectively. Sec.6 summarizes the results of comparison with experimental data and the main conclusions.

2 Quasi-classics for the Optical Model

We start with the wave equation with the complex potential $\bar{V}$ included as follows

$$\Delta\Psi + K^2\Psi = 0, \quad K^2 = k^2 - \bar{U}, \quad \bar{U} = U + i\omega = \frac{2m}{\hbar^2}\bar{V}, \quad \bar{V} = V + iW. \tag{2.1}$$

Its QC-solution is written in the form

$$\Psi = ue^{i\bar{S}}, \qquad \bar{S} = S_1 + i\bar{S}_2 \tag{2.2}$$

Substituting (2.2) into (2.1), equating the real and imaginary parts to zero separately and calculating the current change along a path, one can obtain the following equations

$$(\vec{\nabla}S_1(r))^2 = k^2 - U_e, \quad \vec{\nabla}S_1\vec{\nabla}S_2 = \frac{1}{2}\omega_e, \quad \vec{\nabla}\vec{j} = u^2[x_1 - 2\vec{\nabla}S_1\vec{\nabla}S_2]e^{-2S_2}, \tag{2.3}$$

where

$$\vec{j} = u^2\vec{\nabla}S_1 e^{-2S_2}, \tag{2.4}$$

$$U_e = U + (\vec{\nabla}S_2)^2 + x_2, \quad \omega_e = -\omega + x_1, \quad x_{1,2} = 2\frac{\vec{\nabla}u\vec{\nabla}S_{1,2}}{u} + \Delta S_{1,2}. \tag{2.5}$$

In the case of a real potential, when $W=0$, we have the current conservation law $\vec{\nabla}\vec{j} = u^2 x_1 = 0$. Thus, a simple way to decouple eqs.(2.3) is to assume the imaginary part of a potential being small as compared with its real part $|W| \ll |V|$. Then, supposing that $x_1 \simeq 0$, we obtain

$$(\vec{\nabla}S_1)^2 = k^2 - U_e, \quad \vec{\nabla}S_1\vec{\nabla}S_2 = -\frac{1}{2}\omega, \quad \vec{\nabla}\vec{j} = u^2\omega e^{-2S_2}. \tag{2.6}$$

In HEA the decomposition of S in small V_e/E can be realized from eq.(2.6):

$$S_1 = \vec{k}\vec{r} - \frac{1}{2k}\int_{\vec{r}_0}^{\vec{r}} U_e(\vec{r} - \hat{k}s)ds, \quad S_2 \simeq -\frac{1}{2k}\int_{\vec{r}_0}^{\vec{r}} \omega(\vec{r} - \hat{k}s)ds, \tag{2.7}$$

where integration runs along the trajectory of motion.

From the law of changing the current in (2.6) one can obtain [3] the amplitude u:

$$u = 1 - \frac{V_e}{4E}x, \quad x = 1 + \frac{\rho}{V_e}\frac{dV_e}{d\rho}. \tag{2.8}$$

Thus, the three-dimensional QC-wave function in a nuclear optical potential is as follows:

$$\Psi = (1 - \frac{V_e}{4E})\exp\{i\vec{k}\vec{r} - \frac{i}{\hbar v}\int_0^\infty V_e(\vec{r} - \hat{k}s)ds + \frac{1}{\hbar v}\int_0^\infty W(\vec{r} - \hat{k}s)ds\}. \tag{2.9}$$

Here an effective real potential is induced by the imaginary part of the initial potential. By the way, this fact has no serious meaning, if V_e is calculated from the scattering data in the framework of quasiclassics. Then, these effective potentials will also be appropriate in calculating other processes, if one uses the corresponding QC-distorted waves.

The HEA-calculations usually start with expressions for wave functions in a typical form (2.9), where integration is supposed to run along the straight line trajectories parallel to the momentum of a particle in the asymptotics $\vec{k}(\|o\vec{z})$. However, this latter suggestion doesn't follow automatically from the basic condition $E \gg V$ of HEA. Indeed, in this case trajectories deflect near the scattering center by angles of the order V/E. This means that in phase (2.7) the correction appears of the order $k(V/E)$ to the momentum of a particle. This additional term is comparable in magnitude with the same order terms $(\hbar v)^{-1}\int V ds$ present in (2.9) as a result of the decomposition of S in V/E. The most distorted trajectory goes through the so-called external turning point $\Re$ of the closest approach of scattered particles. At this point the trajectory is parallel to the local momentum $\vec{k}_{\Re}$ directed at the angle $\theta_c/2$ with respect to $\vec{k}$ in the asymptotics. Here $\theta_c = \max(|V(\Re)|/E)$ is the corresponding deflection angle. Then

$$\vec{k}_{\Re} = \vec{k} - \vec{q}_c/2, \quad q_c/2 \simeq k\sin(\theta_c/2), \tag{2.10}$$

where $\vec{q}_c$ represents the momentum transfer in the classical motion. Now, simplifying the problem, we select a new path of integration along the straight line parallel to the momentum $\vec{k}_{\Re}$. Then, eq.(2.9) has to be rewritten in terms of $k_{\Re}$ instead of k. It is possible to show that the effect of the trajectory distortion can be neglected in the last terms of (2.9) containing the integrals of V and W, since the total contribution of the corresponding terms appeared is of an order of $(V/E)^2$. Thus, for the integrals in (2.9) we may use as an integration path the straight line along the momenta in the asymptotics. Then, changing variables $\vec{r} = \vec{\rho} + \vec{z}$, $\quad s + z = \lambda$, where $\vec{\rho} \perp \vec{z}$, and using the Wigner time reverse relation we write distorted waves in the form:

$$\Psi^{(+)}_{\vec{k}_i} = \exp\{i(\vec{k}_i - \frac{\vec{q}_{ci}}{2})\vec{r}_i - \frac{i}{\hbar v_i}\int\limits_{-\infty}^{s_i} V(\sqrt{\rho_i^2+\lambda^2})d\lambda + \frac{1}{\hbar v_i}\int\limits_{-\infty}^{s_i} W(\sqrt{\rho_i^2+\lambda^2})d\lambda\}, \tag{2.11}$$

$$\Psi^{(-)*}_{\vec{k}_f} = \exp\{-i(\vec{k}_f + \frac{\vec{q}_{cf}}{2})\vec{r}_f - \frac{i}{\hbar v_f}\int\limits_{s_f}^{\infty} V(\sqrt{\rho_f^2+\lambda^2})d\lambda + \frac{1}{\hbar v_f}\int\limits_{s_f}^{\infty} W(\sqrt{\rho_f^2+\lambda^2})d\lambda\}, \tag{2.12}$$

One should remind that if necessary, one can include here the flux factor $(1 - V/4E)$. Note that these QC-distorted waves are inherent in the heavy ion collisions, where the classical deflection angle θ_c is really observed in experimental data as the so-called limited angle of the Coulomb deflection. In principle, other trajectories for which $\theta < \theta_c$ should be also taken into account. Moreover, the incident particle can deflect at the same classical angle θ_c corresponding to the "nuclear" trajectory of motion with the radius of the closest approach $\Re_N$ smaller than the $\Re$. *However, first, contributions of these "nuclear" trajectories seem to be negligible because* of the strong nuclear absorption in this region. Second, the probability to find a particle out of any classical trajectory is a quantum effect and in quasi-classical conditions this probability has to be exponentially small. Thus, these comparably small effects cannot be important in the classical region of motion, i.e. at $\theta < \theta_c$. The only place where they give the main effect is the region out of the limited trajectory at $\theta > \theta_c$.

3 Elastic Scattering

Now we consider the heavy ion elastic scattering at energies larger than several dozen MeV per nucleon so that the QC- and HEA-conditions are fulfilled. As the elastic scattering amplitude we use the expression obtained in [3] for large angles $\theta > (1/kR)$ and $\theta > \theta_c \simeq (|V|/E)$ which cover in practice a wide region of scattering angles

$$T^{el} = -\frac{m}{2\pi\hbar^2}\int d\vec{r}\,\Psi^{(-)*}_{\vec{k}_f} V \Psi^{(+)}_{\vec{k}_i} = -\frac{m}{2\pi\hbar^2}\int d\vec{r}\,(V_N + V_C)\exp\{i\vec{q}\vec{r} + i\Phi(\vec{r})\} \tag{3.1}$$

with

$$\Phi = \Phi^{(+)}_i + \Phi^{(-)}_f, \quad \Phi^{(\pm)} = -\frac{1}{\hbar v}\int\limits_{\mp z}^{\infty} V(\sqrt{\rho^2+\lambda^2})d\lambda. \tag{3.2}$$

Here the potentials:

$$V_N = V + iW = V_0 f_V(r) + iW_0 f_W(r), \tag{3.3}$$

$$V_C = \frac{Z_1 Z_2 e^2}{R_C} \int \frac{\rho_c(x) d\vec{x}}{|\vec{r} - \vec{x}|}, \quad \rho_c = \rho_0 f_c(r) \tag{3.4}$$

with the charge density distribution $\rho_c(r)$ and the effective momentum transfer $\vec{\tilde{q}} = \vec{q} - \vec{q_c}$, where $\vec{q} \| \vec{q_c}$, $\tilde{q} = 2k(\alpha - \alpha_c)$, $\alpha = \sin(\theta/2)$, and $\alpha_c \simeq \frac{1}{2E}[V(R_t) + V_C(R_t) + iW(R_t)]$, taken at the radius of the closest approach R_t of the external limited trajectory of motion. All the distribution functions are taken in the form of the Fermi-function

$$f_p(r) = \frac{1}{1 + \exp\frac{r - R_p}{a_p}}. \tag{3.5}$$

Thus, the scattering amplitude consists of three terms:

$$T^{el} = T^{el}_V + iT^{el}_W + T^{el}_C. \tag{3.6}$$

Substituting (3.4) into T^{el}_C we obtain the 6-dimensional integral. It can be transformed to the 3-dimensional if one expands the phase Φ in $\vec{u} = \vec{r} - \vec{x}$ and then integrates over $d\vec{u}$

$$T^{el}_C = -\frac{m}{2\pi\hbar^2} \int d\vec{r} v_c(r) \exp\{i\vec{\tilde{q}}\vec{r} + i\Phi(\vec{r})\}, \qquad v_c(r) = \frac{Z_1 Z_2 e^2 \rho_0}{q_c^2 R_C} f_c(r), \tag{3.7}$$

where $q_c \simeq \tilde{q}$, and $v_c(r)$ plays a role of the quasi-potential of scattering on a spread charge. Now each of the terms of the scattering amplitude (3.4) has the same form:

$$T^{el}_P = -\frac{m}{2\pi\hbar^2} \int d\vec{r} Y_P f_p(r) \exp\{i\tilde{\Phi}(\vec{r})\}, \qquad \tilde{\Phi}(\vec{r}) = \vec{\tilde{q}}\vec{r} + \Phi(\vec{r}), \tag{3.8}$$

where Y_P is the "strength" of the corresponding part of the whole potential.

Bearing in mind, that the high-energy scattering is mainly sensitive to the internal region of interaction $r < R$, we have made integrations in the QC-phases (3.2) with simplified potentials, substituting into V_N, instead of f_p, their expansions in the "diffuseness" a-parameter [4] and using for V_C its inside-of-R expression:

$$\tilde{V}_N = V_{(0)N} \{\Theta(R_N - r) - \frac{\pi^2}{6} a^2 \delta^{(1)}(r - R_N) - ...\}, \quad \tilde{V}_C = \frac{1}{2} V_B (3 - \frac{r^2}{R_C^2}). \tag{3.9}$$

For the same reason we decompose the result of integration of (3.2) in ρ/R and get:

$$\tilde{\Phi} = \vec{\tilde{q}}\vec{r} + 2\bar{a}_0 + \frac{\bar{a}_1}{k}(\vec{k}_i - \vec{k}_f)\vec{r} + \frac{\bar{a}_2}{k^2}[(\vec{k}_i\vec{r})^2 + (\vec{k}_f\vec{r})^2] + \frac{a_3}{k^3}[(\vec{k}_i\vec{r})^3 - (\vec{k}_f\vec{r})^3], \tag{3.10}$$

where $\bar{a}_n$ are the known functions of r and parameters of the potentials, obtained as a result of integration of (3.2). In principle, they also depend on the initial and exit channel indices (i, f).

To calculate the scalar products in (3.10), we select the coordinate system with axes $oz \| \vec{q}$ and $ox \| \vec{K} = \vec{k}_i + \vec{k}_f$. Then,

$$\vec{k}_{\binom{i}{f}}\vec{r} = kr(\pm\alpha\mu + \sqrt{1 - \alpha^2}\sqrt{1 - \mu^2}\cos\varphi), \tag{3.11}$$

where $\mu = \cos\bar{\theta}$ and $\bar{\theta}$ and $\bar{\varphi}$ are the angles of vector $\vec{r}$ in a spherical coordinate system. Inserting (3.11) into (3.10), we obtain the whole phase

$$\tilde{\Phi} = 2\bar{a}_0 + \tilde{\beta}\mu + n_1\mu^2 + c_1\mu^3 + n_2(1-\mu^2)\cos^2\bar{\varphi} + c_2\mu(1-\mu^2)\cos^2\bar{\varphi}], \tag{3.12}$$

where $\tilde{\beta}$, c and n are expressed through a_n. For example,

$$\tilde{\beta} = 2k(\alpha - \alpha_c)r + \bar{\beta}, \quad \bar{\beta} = -\frac{2}{\hbar v}[(V_0 + iW_0) + \frac{Z_1 Z_2 e^2}{2R_C}(3 - \frac{r^2}{R_C^2})]\alpha r. \tag{3.13}$$

We can see that now the integrand (3.8) contains in the exponent a typical power dependence on the variables r and μ. Keeping in mind that $d\vec{r} = -r^2 dr d\mu d\bar{\varphi}$, we first integrate in (3.8) over $d\mu$ by parts

$$I = \int_{-1}^{+1} d\mu \exp[i\tilde{\Phi}(r,\mu,\bar{\phi})] = -i\frac{\exp(i\tilde{\Phi})}{\partial\tilde{\Phi}/\partial\mu}\Big|_{-1}^{+1} + i\int d\mu \exp(i\tilde{\Phi})\frac{\partial^2\tilde{\Phi}/\partial\mu^2}{(\partial\tilde{\Phi}/\partial\mu)^2}, \tag{3.14}$$

neglecting the second term, having the smallness $(kR)^{-2}$. The result is

$$I = -i\exp(2i\bar{a}_0 + in_1)[I^{(+)} - I^{(-)}], \qquad I^{(\pm)} = \frac{\exp[\pm i(\tilde{\beta} + c_1)]}{\Delta_{(\pm)} \mp \delta_{(\pm)}\cos^2\bar{\varphi}}, \tag{3.15}$$

$$\Delta_{(\pm)} = \tilde{\beta} + 3c_1 \pm 2n_1, \quad \delta_{(\pm)} = 2(n_2 \pm c_2). \tag{3.16}$$

Then the integration over $d\bar{\varphi}$ can be performed with the help of a table integral. Thus, we can write the amplitude (3.8) in the form of a one-dimensional integral [5]:

$$T_p^{el} = \frac{im}{\hbar^2}Y_p\int_0^\infty f_p(r)\Big\{F_p^{(+)}(r) - F_p^{(-)}(r)\Big\}dr, \tag{3.17}$$

where

$$F_p^{(\pm)}(r) = \frac{r}{\bar{q}L(\pm)}e^{i(2a_0+n_1)}e^{\pm i(\tilde{q}r + c_1)}, \qquad L(\pm) = \frac{1}{\bar{\beta}}\sqrt{\Delta_{(\pm)}(\Delta_{(\pm)} \mp \delta_{(\pm)})}. \tag{3.18}$$

The root singularities in the denominator are situated far from the radius of interaction R, namely, at $r_s = \frac{A_1 E}{20}R$ with A_1, the atomic number of a projectile, and E, the energy in MeV. At these distances the $f_p(r)$ decreases very fast. , and the previously suggested approximations for calculations of phases do not work. This forces us to introduce a prescription when the integration in (3.17) should be cut off at distances less than the point r_s, that is to exclude the increase of an integrand due to the nonphysical singularities. In this way one can suggest the method of calculating the integral (3.17) by using the properties of the Fermi- functions (3.5) on the complex r-plane. On the other hand, $f_p(r)$ has simple poles in the region of the nuclear surface at $r_n^{\pm} = R \pm ix_n$, where $x_n = \pi a(2n+1)$ with $n = 0, 1, 2\ldots$. It is possible to show that the integration contour for r_n^+ should be drawn in the first quadrant of the complex plane, going along the imaginary axis and then over the circles of an infinite radius. The same contour, but in the fourth quadrant, must be used in the case of r_n^-. Thus, the result is expressed through a sum of the corresponding residues at the above-mentioned poles:

$$T_p^{el} = -\frac{im}{\hbar^2} Y_p 2\pi i a_p \sum_{n=0}^{\infty} \left\{ F^{(+)}(r_n^+) + F^{(-)}(r_n^-) \right\}. \tag{3.19}$$

In practice, for the typical nuclear parameters it is enough to take into account only a couple of poles at $r_0^{(\pm)} = R \pm i x_0$ nearest to the real axis ("two-pole approximation"), because every next pair contributes approximately an order less than the previous one. However, in many cases the L-function in the denominator of $F^{(\pm)}$ can be presented as $exp(\mp\kappa)$ with $\kappa \simeq n_2/\tilde{q}r$. Then, using the decompositions in small x_n/R of the following functions

$$\tilde{q}r = \tilde{q}R \pm i\tilde{q}x_n, \quad \phi = 2\tilde{a}_0 + n_1 = \phi(R) \pm i x_n \phi'_R, \quad y = c_1 - i\kappa = y(R) \pm i x_n y'_R, \tag{3.20}$$

one can sum up all the terms in (3.19) to obtain

$$T_p^{el} \simeq \frac{\pi a m R}{\hbar^2 \tilde{q}} Y(1 + i\frac{\pi a}{R}) e^{i\phi(R)} \left\{ \frac{e^{i(\tilde{q}R + y(R))}}{\sinh[\pi a(\tilde{q} + y'_R + \phi'_R)]} + \xi \frac{e^{-i(\tilde{q}R + y(R))}}{\sinh[\pi a(\tilde{q} + y'_R - \phi'_R)]} \right\}, \tag{3.21}$$

where $\xi \simeq (1 - i\frac{\pi a}{R})/(1 + i\frac{\pi a}{R})$, and the parameters a, R, Y have the corresponding index p.

In the case of nucleus-nucleus scattering the interaction radius R is usually larger than πa, and we have $\xi \simeq 1$. If at the same time $\phi' \ll 1$, we can bring the $sinh$ out of the wave brackets. Thus, in principle, the amplitude begins to come down with angles as an exponential function with a slope determined by the thickness parameter a of a surface of an interaction. Simultaneously it oscillates with a frequency depending mainly on the radius parameter R. However, for large ϕ' only one term of (3.21) is important, and in this case no oscillations will appear in the cross section.

4 Inelastic Scattering

For calculating the inelastic scattering of light and heavy ions with excitation of the collective nuclear states we have used DWBA with the relative-motion QC-wave functions whose phases are calculated as it is shown in sec.3. The energy change in the out-channel is neglected since usually $E_{ex} \ll 1$. The transition interaction is constructed as usual with the help of derivatives in small quadrupole and octupole additions $\delta R = R \sum \alpha_{LM} Y^*_{LM}(\hat{r})$ to the radius of a potential in the elastic channel. The result for the amplitude is the same as if one used the sudden approximation

$$T^{in} = (J_f M_F | \hat{T}_V^{el} + i\hat{T}_W^{el} + \hat{T}_C^{el} | J_i M_i), \tag{4.1}$$

where

$$\hat{T}_p^{el} = -\frac{m}{2\pi\hbar^2} \int d\vec{r} Y_p f_p(r, R + \delta R) \Psi^{(-)*} \Psi^{(+)} \tag{4.2}$$

is the operator, depending on the internal nuclear coordinates α_{LM}. Then, substituting (4.2) into (4.1) we get

$$T_p^{in} = \sum_{LM} (J_f M_f | \alpha_{LM} | J_i M_i) \, \tilde{T}_{LM}^{in}, \tag{4.3}$$

where

$$\tilde{T}^{in}_{LM} = -\frac{m}{2\pi\hbar^2}Y_p R \int dr \Psi^{(-)*}\Psi^{(+)}\frac{df}{dR}Y^*_{LM}. \tag{4.4}$$

Transforming the structure matrix element in (4.3) through the reduced one and using the definition of $B\downarrow(EL)$-transition, one can write the inelastic cross section:

$$\frac{d\sigma}{d\Omega} = \frac{(2J_f+1)}{(2J_i+1)}\frac{1}{(2L+1)}\sum_{LM}\frac{B\downarrow(EL)}{D_L^2}|\tilde{T}^{in}_{LM}|^2 \tag{4.5}$$

with

$$D_L = Z_2 e\rho_0 R_C J^c_L, \quad J^c_L = \int \frac{df_c}{dR_C} r^{L+2} dr \simeq R_C^{L+2}. \tag{4.6}$$

One can show that all the terms with $M \neq 0$ may be neglected because of the additional fast oscillations in integrands as compared with the term $M = 0$. Then, the principal difference of the inelastic amplitude from the elastic one appears in integrals over $d\mu$, because now in the upper and lower limits $\mu = \pm 1$ we have to take into account the relation

$$Y_{L0}(-\mu) = (-1)^L Y_{L0}(+\mu), \tag{4.7}$$

which changes the sign of the second term in the inelastic analog of eq.(3.15) for odd L. Indeed, now we have

$$I_L = \int_{-1}^{+1} d\mu \exp(i\tilde{\Phi})Y_{L0} \simeq -i\left(\frac{\exp(i\tilde{\Phi})}{\partial\tilde{\Phi}/\partial\mu}\bigg|_{+1} - (-)^L\frac{\exp(i\tilde{\Phi})}{\partial\tilde{\Phi}/\partial\mu}\bigg|_{-1}\right)Y_{L0}(1), \tag{4.8}$$

and consequently, using the relation $df_p/dR = -df_p/dr$, we have got, instead of (3.17), its analog equation for inelastic scattering

$$T^{in}_p = -\frac{im}{\hbar^2}Y_p Y_{L0}(1)R\int_0^\infty dr\frac{df_p}{dr}\left\{F_p^{(+)}(r) - (-1)^L F_p^{(-)}(r)\right\}. \tag{4.9}$$

This integral can be calculated in analytical form if one uses the second order poles on the complex plane of the derivative df/dr. However, we show another way. Indeed, bearing in mind that the $F^{(\pm)}$-functions rapidly oscillate with increasing r because of the exponent $\tilde{q}r \gg 1$, one can integrate in (4.9) by parts

$$\int_0^\infty \frac{df_p}{dr}F_p^{(\pm)}dr = -\int_0^\infty f_p\frac{1}{r}F_p^{(\pm)}dr - O\left(\frac{1}{(\tilde{q}r)^2}\right). \tag{4.10}$$

So, substituting (4.10) into (4.9) we get a form like (3.15) for elastic scattering with some additions in the integrand, namely, the factor $(1/r)$ and the multiplier $(-1)^L$ before the second term:

$$T^{in}_p = \frac{im}{\hbar^2}Y_p Y_{L0}(1)R\int_0^\infty dr\frac{1}{r}f_p(r)\left\{F_p^{(+)}(r) - (-1)^L F_p^{(-)}(r)\right\}. \tag{4.11}$$

Subsequent calculations are the same as in the case of elastic scattering. We give here only *an approximate expression, an analog of (3.21), when summation of all the poles runs under* certain suggestions on the QC-phase behaviour. We have

$$T_p^{in} \simeq \frac{\pi a m R}{\hbar^2 \tilde{q}} Y_p Y_{L0}(1) e^{i\phi(R)} \left\{ \frac{e^{i(\tilde{q}R + y(R))}}{\sinh[\pi a(\tilde{q} + y'_R + \phi'_R)]} + (-1)^L \frac{e^{-i(\tilde{q}R + y(R))}}{\sinh[\pi a(\tilde{q} + y'_R - \phi'_R)]} \right\}. \quad (4.12)$$

It is easy to see that if the admixture ϕ' to the distorted QC-phase is negligible, all the denominators coincide and for large arguments they give, in the amplitude, a fast decreasing function $\exp(-\pi a k \theta)$ [6]. Simultaneously, the oscillating part of the amplitude as a function of the scattering angle is the *cos-* or *sin*-function in dependence of L-even or odd, respectively. In this case the cross section will have visible oscillations which coicide for excitations of the even collective states in their phases with the elastic scattering oscillations. In the other case, when ϕ' is large, the only term in (4.12) will contribute to the cross section, so that no oscillations will exist, and the slope of the cross section will depend mainly on the thickness parameter of the interaction potential.

5 One-nucleon Transfer Reactions

For simplicity, we consider the reaction $a + A \rightarrow b + B$ with transfer of a spinless x-particle, when the corresponding cross section and the amplitude in the zero-range approximation are as follows:

$$\frac{d\sigma}{d\Omega} = \frac{m_a}{m_b} \frac{k_a}{k_b} \frac{1}{2j_a + 1} \frac{2J_B + 1}{2J_A + 1} \sum_l S_l |\tilde{T}_l^{tr}|^2, \quad (5.1)$$

$$\tilde{T}_l^{tr} = -\frac{m_b D_0}{2\pi\hbar^2} \int d\vec{r}\, \Psi_b^{(-)*}(\vec{r}) \Psi_a^{(+)}(\vec{r}) \Re_l(r) Y_{l0}^*(\hat{r}), \quad (5.2)$$

where

$$D_0 = -\frac{4\pi\hbar^2}{m_a} \sqrt{\frac{\kappa_a}{2\pi}}$$

depends on the structure of an incident particle, and $\Re_l(r)$ is the radial wave function of the x-particle in the final nucleus B. The latter has the asymptotic behaviour $exp(-\kappa_l r)/r$. We have emphasised that the main effect in heavy ion reactions comes from the region near the interaction radius. This means that the behaviour of the function $\Re_l$ at $r < R$ is of no importance, and one can select it in the form

$$\Re_l(r) = \frac{\sqrt{6 a_l}}{r} \frac{d f_s(r, R, a_l)}{dr}, \quad (5.3)$$

where

$$f_s = \frac{\sinh \frac{R}{a_l}}{\cosh \frac{R}{a_l} + \cosh \frac{r}{a_l}} \simeq \frac{1}{1 + \exp \frac{r - R}{a_l}} \quad (5.4)$$

is the symmetrised Fermi-function having the asymptotics $\exp(-a_l r)/r$ and going to a constant at $r = 0$; the function (5.3) is normalised to 1, and the "diffuseness" of the transition region is to be taken $a_l = 1/\kappa_l$, where $\kappa_l = \sqrt{2 m_x \epsilon_l / \hbar^2}$ with ϵ_l, the separation energy.

Inserting (5.3) into (5.2) we get the amplitude of the typical form inherent in HEA. Moreover, here we can use the quasi-elastic approximation because the loss of energy in the reaction is comparably small and $E_a \simeq E_b$. Thus, the QC-distorted waves are taken in the same elastic channel, and the final expression for the amplitude is as follows:

$$\tilde{T}_l^{tr} = \frac{m_b}{2\pi\hbar^2} D_0 \sqrt{6a_l}\, Y_{l0}(1) 2\pi i \int_0^\infty dr \frac{1}{r}\frac{df_l}{dr}\left\{F^{(+)}(r) - (-)^l F^{(-)}(r)\right\}. \quad (5.5)$$

The only difference here is that the residues must be taken in the second order poles displayed at the same points as in the previous calculations. So, we write

$$\tilde{T}_l^{tr} = -\frac{8\sqrt{2}\pi m_b}{\hbar^2} D_0 a_l^{3/2} Y_{l0}(1) \sum_n \left\{ \frac{d}{dr}\left[\frac{e^{i(\phi+\tilde{\varphi}+c_l)}}{\tilde{q}}\right]_{r_n^+} + (-)^l \frac{d}{dr}\left[\frac{e^{i(\phi-i\tilde{\varphi}-c_l)}}{\tilde{q}}\right]_{r_n^-} \right\}. \quad (5.6)$$

Thus, we can conclude that here we also have the general exponential decrease at angles $\theta > \theta_c$, depending on the acting thickness a_l in the region of the surface of transition [7]. Its magnitude is determined by the slope of a "tail" of a bound state function in the final nucleus B.

6 Conclusion

Calculations of differential cross sections for elastic and inelastic scattering within the two-pole approximation are presented in Figs.1a,b in comparison with the experimental data from [8]. One can see a rather good agreement in the range of scattering angles $\theta > \theta_c \simeq 2^\circ$ in coincidence with the initial assumptions of the HEA-method. For each set of colliding nuclei we got the same interaction parameters for elastic and inelastic channels excluding the absorption W_0 that appeared to be about 10% as small as that in the elastic channel. The depths of potential wells are in limits of $V_0 = 60 - 70 MeV$ and $W_0 = 5 - 6 MeV$, the $B(EL)$-transitions obtained are approximately twice those cited in [8]. The most interesting result is that the thickness parameters for inelastic channels are about two-three times as small as those for elastic channels, where we have $a_{el} = 0.55 - 0.6 fm$. This might signify that in collective excitations of nuclei not all the particle states take part in forming the transition matrix elements. Otherwise, in elastic scattering the "tail" of a potential is formed from the whole set of one-particle states.

Figs.2a,b show calculations and comparisons with data from [9]. The spectroscopic factors were taken to equal 1, and the absolute values of theoretical cross sections are presented. The thickness parameters here are $a_l = 0.4 fm$ and $a_l = 0.6 fm$ for the first and the second reactions, consequently, they characterise the form factor behaviour in the surface area of interaction. The other parameters are $V_0 = 50 MeV$, $W_0 = 31$ and $48 MeV$. We should note that we use no codes for the best fit analysis of data, but we are sure that the basic conclusions we have obtained will not change.

We can summarise that investigations of heavy ion collisions in the quantum region of scattering angles $\theta > \theta_c$, outside of the limited trajectories of motion, are very sensitive to the precise structure of a nucleus-nucleus interaction. For instance, the slope of curves with θ feels the "thickness" of the acting region in the corresponding channel. It may be used also for searching the "halo" distributions of nuclei in the radioactive beams which now become available. We hope that the HEA-method suggested can be successfully used in both the qualitative and quantitative analysis of scattering processes and direct reactions.

In conclusion I would like to thank Dr.R.Roussel-Chomaz for sending the tables on elastic and inelastic scattering data.

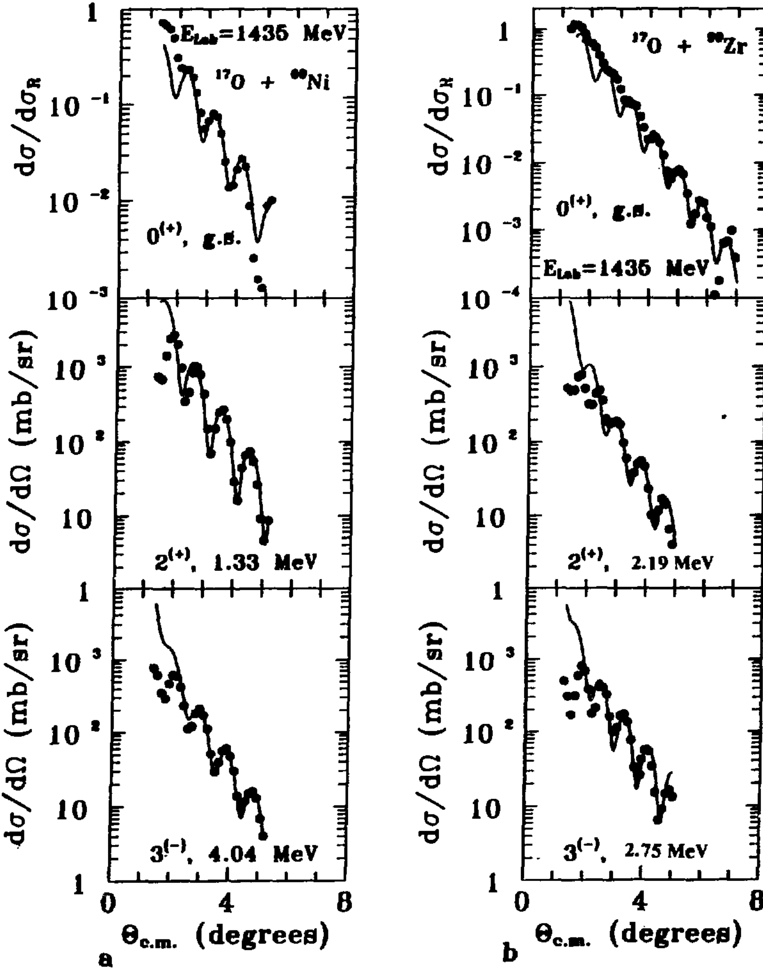
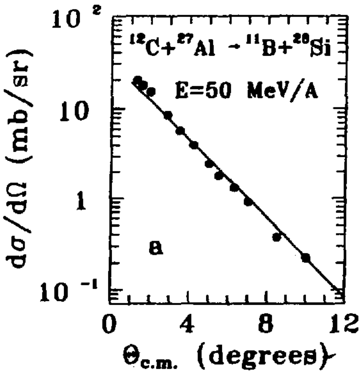


Fig.1 The heavy ion elastic and inelastic cross sections: (a) $^{17}O + ^{60}Ni$; (b) $^{17}O + ^{90}Zr$; $E_{lab} = 1435 MeV$, exp. data from [8], solid lines– theory



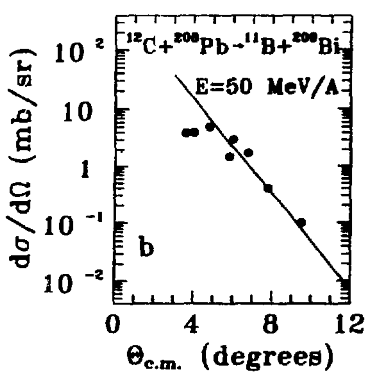


Fig.2 The transfer reaction cross sections: (a) $^{12}C+^{27}Al \Rightarrow ^{11}B+^{28}Si$; (b) $^{12}C+^{208}Pb \Rightarrow ^{11}B+^{209}Bi$; $E = 50MeV/n$, exp. data from [9], solid lines–theory

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