объединенный
институт
ядерных
исследований
дубна

E4-94-275

A.I.Vdovin, D.S.Kosov

# EXTENSION OF THE QPM TO $T \neq 0$ BASED ON THE FORMALISM OF THE THERMO FIELD DYNAMICS

Invited talk at the Fourth KINR International School on Nuclear Physics, Kiev, August 29 — September 7, 1994

1994

# 1 Introduction

The study of phenomena occurring in hot nuclear systems in the framework provided by the nuclear many body theory has attracted attention of many authors [1-6]. The problem of particular interest is collective excitations in these systems and their dependence on the temperature $T$. One of the reasons for this is the increasing amount of experimental data on the giant dipole resonances in highly excited nuclei. In studying hot nuclear systems, theorists often follow the way of extending the known models for cold nuclei to the case $T \neq 0$. It is well proved for cold heavy nuclei that the main part of the width of collective giant resonance states is due to the coupling of the RPA - states with more complex ones, like 2p-2h or four quasiparticle or two phonon states [7-9]. The same idea has been explored for giant resonances in hot nuclei in the framework of the nuclear field theory and within the theory of finite Fermi – systems extended to $T \neq 0$ (see, respectively, [2] and [3]).

We formulate here an approach based on extension of the quasiparticle - phonon nuclear model (the QPM) [8, 9] to $T \neq 0$. The QPM gives reasonable description of spreading widths of giant resonances and other resonance - like structures in spectra of heavy nuclei at the excitation energies $E_x \leq$ 20 – 25 MeV. Distinctive features of the QPM are the schematic (namely, separable) residual forces and the use of the RPA phonons as elementary blocks to describe excitations in even - even nuclei. Both ingredients simplify drastically formulas of the model as well as numerical calculations within it.

# 2 The formalism of the thermo field dynamics

To extend the QPM to $T \neq 0$, we explore the formalism of the thermo field dynamics (the TFD) [10]. The main idea behind the TFD is to define a thermal vacuum $|0(\beta)\rangle$ such that the thermal expectation value of any operator

$$\ll A \gg = \frac{1}{Tr(exp(-\beta H))} Tr[A\, exp(-\beta H)]$$

($\beta = T^{-1}$) equals the expectation value with respect to the thermal vacuum state

$$\ll A \gg = \langle 0(\beta)|A|0(\beta)\rangle$$



The extension of quantum field theory at $T = 0$ to finite temperature requires a doubling of the field degrees of freedom. In the TFD, a tilde conjugate operator $\tilde{A}$ is associated with any operator A acting in ordinary space through the tilde conjugation rules

$$(\widetilde{AB}) = \tilde{A}\tilde{B}\;;\;(c_1 A + c_2 B)^{\sim} = c_1^* \tilde{A} + c_2^* \tilde{B}\,,$$

where A and B stand for any operators and $c_1$ and $c_2$ are c-numbers. The asterisk denotes the complex conjugate. The tilde operation commutes with the hermitian conjugation operation and any tilde and non-tilde operators are assumed to commute or anticommute with each other. For any system governed by the Hamiltonian H the whole Hilbert space now is spanned by the direct product of the eigen states of H and those of the tilde Hamiltonian $\tilde{H}$ having the same eigenvalues. The time - translation operator is not the energy operator H but the thermal Hamiltonian $\mathcal{H} = H - \tilde{H}$. This means that the properties of the system excitations are obtained by the diagonalization of $\mathcal{H}$.

It is easy to see that with the doubling of Hilbert space the temperature dependent vacuum $|0(\beta)\rangle$ has to be defined as follows:

$$|0(\beta)\rangle = \frac{1}{\sqrt{Tr(exp(-\beta H))}} \sum_n exp(-\frac{\beta E_n}{2})|n\rangle \otimes |\tilde{n}\rangle$$

This state is the vacuum for the thermal quasiparticle annihilation operators:

$$\beta_{jm} = x_j a_{jm} - y_j \tilde{a}^+_{jm}$$

$$\tilde{\beta}_{jm} = x_j \tilde{a}_{jm} + y_j a^+_{jm}$$

$$\beta_{jm}|0(\beta)\rangle = \tilde{\beta}_{jm}|0(\beta)\rangle = 0$$

The transformation $\{x, y\}$ is a unitary one and due to this the standard anticommutation relations are valid for $\beta_{jm}, \beta^+_{jm}, \tilde{\beta}_{jm}, \tilde{\beta}^+_{jm}$. The coefficients $x_j, y_j$ are the thermal (Fermi in our case) occupation numbers of the states $|n\rangle$.

$$x_j = \sqrt{1 - n_j}\;,\; y_j = \sqrt{n_j}$$

$$n_j = \frac{1}{1 + exp(\beta E_j)}$$

## 3 The pairing correlations at $T \neq 0$

Now we apply [11, 12] the outlined formalism to a hot nuclear system governed by the Hamiltonian of the QPM. This Hamiltonian consists of the average fields for protons and neutrons, the monopole pp- and nn- pairing interactions and the separable multipole particle - hole interactions consisting of the isoscalar and isovector parts

$$H = H_{sp} + H_{pair} + H_{ph}$$

Within the QPM, one follows the standard way of transformation of the Hamiltonian of a system of interacting nucleons to that of interacting elementary excitation modes. The first step along this way is the Bogoliubov transformation from the creation and annihilation operators of particles and holes to the creation and annihilation operators of quasiparticles. To take into account the influence of temperature, we have to make additional canonical transformation to the operators of thermal quasiparticles that will annihilate the thermal vacuum. So the final form of the transformation from the nucleon creation and annihilation operators $a$, $a^+$, $\tilde{a}$, $\tilde{a}^+$ to thermal quasiparticles $\beta$, $\beta^+$, $\tilde{\beta}$, $\tilde{\beta}^+$, which have $|0(\beta)\rangle$ as a vacuum, is the following [5, 11, 13] :

$$\begin{pmatrix} a_{jm} \\ a^+_{\overline{jm}} \\ \tilde{a}^+_{jm} \\ \tilde{a}_{\overline{jm}} \end{pmatrix} = \begin{pmatrix} A & B \\ -B & A \end{pmatrix} \begin{pmatrix} \beta_{jm} \\ \beta^+_{\overline{jm}} \\ \tilde{\beta}^+_{jm} \\ \tilde{\beta}_{\overline{jm}} \end{pmatrix} \quad (1)$$

$$A = x_j \begin{pmatrix} u_j & v_j \\ -v_j & u_j \end{pmatrix} \qquad B = y_j \begin{pmatrix} u_j & v_j \\ -v_j & u_j \end{pmatrix}$$

To find the coefficients $u_j, v_j, x_j, y_j$, we minimize the grand thermodynamic potential $\Omega = \langle 0(\beta)|\mathcal{H}'|0(\beta)\rangle - TS$ ( $S$ is entropy of the system) for a system of nucleons governed by the Hamiltonian $H' = H_{sp} + H_{pair}$ at $T = const$. In the calculations, one should have in mind that the ground state energy $E$ equals $\langle 0(\beta)|H'|0(\beta)\rangle$ but not $\langle 0(\beta)|\mathcal{H}'|0(\beta)\rangle$ [10].

The entropy $S$ of the system is

$$S = -\sum_j (2j+1)(n_j \ln n_j + (1-n_j)\ln(1-n_j))$$

The calculation of $\Omega$ is straightforward

$$\Omega = \langle 0(\beta)|H_{av}(\tau) + H_{pair}(\tau)|0(\beta)\rangle - TS =$$



$$= \sum_j^{\tau}(2j+1)(E_j - \lambda_\tau)(u_j^2 n_j + v_j^2(1-n_j)) -$$

$$-\frac{G_\tau}{4}\left(\sum_j^{\tau}(2j+1)u_j v_j(1-2n_j)\right)^2 - TS$$

We use the following notation: $E_j$ is the single - particle energy; $\lambda_\tau$ is the chemical potential; $G_\tau$ is the constant of the pairing interaction. The index $\tau$ is an isotopic index and takes two values, $\tau = n, p$. We suppose that $\tau$ is included in the set of shell model quantum numbers $[nlj\tau]$ that we usually denote by one index j. The symbol $\sum^\tau$ means that the summation is taken only over neutron or proton single - particle states.

After variation of $\Omega$ over $u_j, v_j, n_j$ we have

$$(E_j - \lambda_\tau)u_j v_j - \frac{G_\tau}{4}(u_j^2 - v_j^2)\sum_j^{\tau}(2j+1)u_j v_j(1-2n_j) = 0$$

$$(E_j - \lambda_\tau)(u_j^2 - v_j^2) + u_j v_j G_\tau \sum_j^{\tau}(2j+1)u_j v_j(1-2n_j) +$$

$$+T(\ln n_j - \ln(1-n_j)) = 0$$

For the coefficients $u_j, v_j, n_j$ the following relations are valid:

$$u_j^2 = \frac{1}{2}\left(1 + \frac{E_j - \lambda_\tau}{\varepsilon_j}\right), \; v_j^2 = \frac{1}{2}\left(1 - \frac{E_j - \lambda_\tau}{\varepsilon_j}\right)$$

$$\varepsilon_j = \sqrt{(E_j - \lambda_\tau)^2 + \Delta_\tau^2}, \; n_j = \frac{1}{1 + exp(\beta\varepsilon_j)}$$

In its turn $\Delta_\tau, \lambda_\tau$ can be found from the equations:

$$N_\tau = \frac{1}{2}\sum_j^{\tau}(2j+1)\left(1 - \frac{E_j - \lambda_\tau}{\sqrt{(E_j - \lambda_\tau)^2 + \Delta_\tau^2}}(1-2n_j)\right)$$

$$\frac{4}{G_\tau} = \sum_j^{\tau}(2j+1)\frac{1-2n_j}{\sqrt{(E_j - \lambda_\tau)^2 + \Delta_\tau^2}}, \quad (2)$$

where $N_\tau$ is the number of nucleons of a given type.

Eqs.(2) are the well known equations for pairing at $T \neq 0$ with the Hamiltonian of Bardeen - Cooper - Shrieffer (see e.g. ref.[14]). Using the TFD formalism they were derived also in [13].

## 4 The random phase approximation at $T \neq 0$

After the transformation (1) to the thermal quasiparticles the thermal Hamiltonian of the QPM takes the form

$$\mathcal{H} = \sum_{jm} \varepsilon_{jm}(\beta^+_{jm}\beta_{jm} - \tilde{\beta}^+_{jm}\tilde{\beta}_{jm}) -$$

$$-\frac{1}{2}\sum_{\lambda\mu}\sum_{\tau,\rho=\pm1}(k_0^{(\lambda)} + \rho k_1^{(\lambda)})\{M^+_{\lambda\mu}(\tau)M_{\lambda\mu}(\rho\tau) - \tilde{M}^+_{\lambda\mu}(\tau)\tilde{M}_{\lambda\mu}(\rho\tau)\},$$

where

$$M^+_{\lambda\mu}(\tau) = \frac{(-)^{\lambda-\mu}}{\sqrt{2\lambda-1}}\sum^{\tau}_{jj'} f^{(\lambda)}_{jj'}\{[A^+_\beta(jj';\lambda\mu) +$$

$$+(-)^{\lambda-\mu}A_\beta(jj';\lambda-\mu)] + B_\beta(jj';\lambda\mu)\} \quad (2)$$

$$A^+_\beta(jj';\lambda\mu) =$$

$$= \frac{1}{2}u^{(+)}_{jj'}(\sqrt{1-n_j}\sqrt{1-n_{j'}}[\beta^+_{jm}\beta^+_{j'm'}]_{\lambda\mu} - \sqrt{n_j}\sqrt{n_{j'}}[\tilde{\beta}^+_{\overline{jm}}\tilde{\beta}^+_{\overline{j'm'}}]_{\lambda\mu}) -$$

$$-v^{(-)}_{jj'}\sqrt{1-n_j}\sqrt{n_{j'}}[\beta^+_{jm}\tilde{\beta}^+_{\overline{j'm'}}]_{\lambda\mu}$$

$$B_\beta(jj';\lambda\mu) =$$

$$= -v^{(-)}_{jj'}(\sqrt{1-n_j}\sqrt{1-n_{j'}}[\beta^+_{jm}\beta_{\overline{j'm'}}]_{\lambda\mu} + \sqrt{n_j}\sqrt{n_{j'}}[\tilde{\beta}_{jm}\tilde{\beta}^+_{\overline{j'm'}}]_{\lambda\mu}) +$$

$$+u^{(+)}_{jj'}\sqrt{1-n_j}\sqrt{n_{j'}}([\beta^+_{jm}\tilde{\beta}_{j'm'}]_{\lambda\mu} + (-)^{\lambda-\mu}[\beta_{jm}\tilde{\beta}^+_{j'm'}]_{\lambda-\mu})$$

In these formulas we use the following notation: $f^{(\lambda)}_{jj'}$ is the reduced single-particle matrix element of the multipole operator; $\kappa_0^{(\lambda)}, \kappa_1^{(\lambda)}$ are the coupling constants of the isoscalar and isovector multipole - multipole interactions, respectively; $u^{(+)}_{jj'} = u_j v_{j'} + u_{j'} v_j$ , $v^{(-)}_{jj'} = u_j u_{j'} - v_{j'} v_j$. Changing the sign of $\tau$ means changing $n \leftrightarrow p$. The square brackets $[\ ]_{\lambda\mu}$ stand for the coupling of single - particle angular momenta $j, j'$ to the sum angular momentum $\lambda$. The bar over lower indices $\overline{jm}$ denotes the time reversal state.

One can easily see from the expression for $\mathcal{H}$ that the structure of the thermal Hamiltonian in terms of the operators $\beta^+, \beta, \tilde{\beta}^+, \tilde{\beta}$ is the same as of the Hamiltonian $H$ in terms of the Bogoliubov quasiparticles $\alpha^+, \alpha$ (cf. [9]). The main difference is redefinition of vertices corresponding to terms



of the same operator structure. For example, the coefficient at the term $A^+_\beta(j_1j_2;\lambda\mu)A^+_\beta(j_3j_4;\lambda\mu)$ depends now not only on the superfluid particle-hole factor $u^{(+)}_{j_1j_2}$ as at $T = 0$ but on the particle - particle (or hole - hole) factor $v^{(-)}_{j_1j_2}$ as well. The thermal vacuum $|0(\beta)\rangle$ now formally plays the role which is similar to the role of the quasiparticle vacuum at $T = 0$. So further derivation can be done in parallel with the $T = 0$ case.

Firstly, we introduce a thermal phonon operator and redefine the thermal ground state as a phonon vacuum $|\Psi_0(\beta)\rangle$:

$$Q^+_{\lambda\mu i} = \frac{1}{2}\sum_{jj'}\Big(\psi^{\lambda i}_{jj'}[\beta^+_{jm}\beta^+_{j'm'}]_{\lambda\mu} + \tilde{\psi}^{\lambda i}_{jj'}[\tilde{\beta}^+_{\overline{jm}}\tilde{\beta}^+_{\overline{j'm'}}]_{\lambda\mu} +$$

$$+2\eta^{\lambda i}_{jj'}[\beta^+_{jm}\tilde{\beta}^+_{\overline{j'm'}}]_{\lambda\mu} + (-)^{\lambda-\mu}\phi^{\lambda i}_{jj'}[\beta_{jm}\beta_{j'm'}]_{\lambda-\mu} +$$

$$+(-)^{\lambda-\mu}\tilde{\phi}^{\lambda i}_{jj'}[\tilde{\beta}_{\overline{jm}}\tilde{\beta}_{\overline{j'm'}}]_{\lambda-\mu} + 2(-)^{\lambda-\mu}\zeta^{\lambda i}_{jj'}[\beta_{jm}\tilde{\beta}_{\overline{j'm'}}]_{\lambda-\mu}\Big)$$

$$Q_i|\Psi_0(\beta)\rangle = 0$$

Then, we suppose that the standard assumptions of the RPA are valid, namely, the number of thermal quasiparticles in the new vacuum state $|\Psi_0(\beta)\rangle$ is negligible or thermal phonon operators commute :

$$\langle\Psi_0(\beta)|\beta^+_{jm}\beta_{jm}|\Psi_0(\beta)\rangle \approx 0$$

$$\langle\Psi_0(\beta)|[Q_{\lambda\mu i}, Q^+_{\lambda'\mu' i'}]|\Psi_0(\beta)\rangle = \delta_{\lambda\lambda'}\delta_{\mu\mu'}\delta_{ii'}$$

With these assumptions it is easy to find the following constraint on the bifermionic amplitudes of the thermal phonon:

$$\frac{1}{2}\sum_{jj'}(\psi^{\lambda i}_{jj'})^2 - (\phi^{\lambda i}_{jj'})^2 + (\tilde{\psi}^{\lambda i}_{jj'})^2 - (\tilde{\phi}^{\lambda i}_{jj'})^2 + 2(\eta^{\lambda i}_{jj'})^2 - 2(\zeta^{\lambda i}_{jj'})^2 = \delta_{\lambda\lambda'}\delta_{ii'} \quad (3)$$

Making the inverse transformation from the operators $Q^+_{\lambda\mu i}, Q_{\lambda\mu i}$ to bifermionic operators $[\beta^+_1\beta^+_2]_{\lambda\mu}$ etc. we get the following expression for $\mathcal{H}$ in terms of thermal quasiparticles and phonons:

$$\mathcal{H} = \mathcal{H}_{rpa} + \mathcal{H}_{qph} = \sum_{jm}\varepsilon_j(\beta^+_{jm}\beta_{jm} - \tilde{\beta}^+_{jm}\tilde{\beta}_{jm}) -$$

$$-\frac{1}{8}\sum_{\lambda\mu i i'}\sum_{\tau,\rho=\pm1}\frac{k_0^{(\lambda)}+\rho k_1^{(\lambda)}}{2\lambda+1}D_\tau^{\lambda i}D_{\rho\tau}^{\lambda i'}\{(Q^+_{\lambda\mu i}+(-)^{\lambda-\mu}Q_{\lambda-\mu i})\times$$

$$\times(Q_{\lambda\mu i'}+(-)^{\lambda-\mu}Q^+_{\lambda-\mu i'})-$$

$$-(\tilde{Q}^+_{\lambda\mu i}+(-)^{\lambda-\mu}\tilde{Q}_{\lambda-\mu i})(\tilde{Q}_{\lambda\mu i'}+(-)^{\lambda-\mu}\tilde{Q}^+_{\lambda-\mu i'})\} \qquad (4)$$

$$-\frac{1}{4}\sum_{\lambda\mu i}\sum_{\tau\rho=\pm1}\frac{k_0^{(\lambda)}+\rho k_1^{(\lambda)}}{2\lambda+1}D_{\rho\tau}^{\lambda i'}\sum_{jj'}^{\tau}f_{jj'}^{(\lambda)}\{((-)^{\lambda-\mu}Q^+_{\lambda\mu i}+Q_{\lambda-\mu i})B_\beta(jj';\lambda-\mu)-$$

$$-((-)^{\lambda-\mu}\tilde{Q}^+_{\lambda\mu i}+\tilde{Q}_{\lambda-\mu i})\tilde{B}_\beta(jj';\lambda-\mu)+h.c.\}$$

$$D_\tau^{\lambda i}=\sum_{jj'}^{\tau}f_{jj'}^{\lambda}[u_{jj'}^{(+)}(\sqrt{1-n_j}\sqrt{1-n_{j'}}(\psi_{jj'}^{\lambda i}+\phi_{jj'}^{\lambda i})-\sqrt{n_j}\sqrt{n_{j'}}(\tilde{\psi}_{jj'}^{\lambda i}+\tilde{\phi}_{jj'}^{\lambda i}))-$$

$$-2v_{jj'}^{(-)}\sqrt{1-n_j}\sqrt{n_{j'}}(\eta_{jj'}^{\lambda i}+\zeta_{jj'}^{\lambda i})]$$

In (4) we introduce the tilde operators $\tilde{Q}_{\lambda\mu i}$ and $\tilde{B}_\beta(jj';\lambda-\mu)$, for convenience, and omit the terms like $B^+_\beta(jj';\lambda-\mu)B_\beta(jj';\lambda-\mu)$ etc. because their effect is supposed to be small in the RPA as for the analogous term at $T=0$.

The expectation value of $\mathcal{H}$ over the one-phonon state $Q^+_{\lambda\mu i}|\Psi_0(\beta)\rangle$ has the form:

$$\langle\Psi_0(\beta)|Q_{\lambda\mu i}\mathcal{H}Q^+_{\lambda\mu i}|\Psi_0(\beta)\rangle=$$

$$=\frac{1}{2}\sum_{jj'}\{(\varepsilon_j+\varepsilon_{j'})[(\psi_{jj'}^{\lambda i})^2+(\phi_{jj'}^{\lambda i})^2-(\tilde{\psi}_{jj'}^{\lambda i})^2-(\tilde{\phi}_{jj'}^{\lambda i})^2]+ \qquad (5)$$

$$+2(\varepsilon_j-\varepsilon_{j'})[(\eta_{jj'}^{\lambda i})^2+(\zeta_{jj'}^{\lambda i})^2]-\frac{1}{4}\frac{1}{2\lambda+1}\sum_{\tau,\rho=\pm1}(k_0^{(\lambda)}+\rho k_1^{(\lambda)})D_\tau^{\lambda i}D_{\rho\tau}^{\lambda i}$$

After variation of (5) at the constraint (3) over $\psi,\phi,\eta,\zeta,\tilde{\psi},\tilde{\phi}$, one gets the homogeneous system of linear equations. It can be resolved if the energy of the one-phonon state $\omega_{\lambda i}$ is the root of the following secular equation:

$$[X_\tau^{\lambda i}(\omega)+X_{-\tau}^{\lambda i}(\omega)](k_0^{(\lambda)}+k_1^{(\lambda)})-4k_0^{(\lambda)}k_1^{(\lambda)}X_\tau^{\lambda i}(\omega)X_{-\tau}^{\lambda i}(\omega)=1 \qquad (6)$$



$$X_\tau^{\lambda i}(\omega)=\frac{1}{2\lambda+1}\sum_{jj'}^{\tau}(f_{jj'}^{\lambda})^2\left(\frac{(u_{jj'}^{(+)})^2(1-n_j-n_{j'})(\varepsilon_j+\varepsilon_{j'})}{(\varepsilon_j+\varepsilon_{j'})^2-\omega^2}-\right.$$

$$\left.-\frac{(v_{jj'}^{(+)})^2(n_j-n_{j'})(\varepsilon_j-\varepsilon_{j'})}{(\varepsilon_j-\varepsilon_{j'})^2-\omega^2}\right)$$

For the bifermionic amplitudes of the one-phonon wave function one gets:

$$\psi_{jj'}^{\lambda i}=\sqrt{\frac{1}{2\mathcal{N}_\tau^{\lambda i}}}\frac{f_{jj'}^{(\lambda)}u_{jj'}^{(+)}\sqrt{1-n_j}\sqrt{1-n_{j'}}}{(\varepsilon_j+\varepsilon_{j'})-\omega_{\lambda i}}\qquad \tilde{\psi}_{jj'}^{\lambda i}=\sqrt{\frac{1}{2\mathcal{N}_\tau^{\lambda i}}}\frac{f_{jj'}^{(\lambda)}u_{jj'}^{(+)}\sqrt{n_j}\sqrt{n_{j'}}}{(\varepsilon_j+\varepsilon_{j'})+\omega_{\lambda i}}$$

$$\phi_{jj'}^{\lambda i}=\sqrt{\frac{1}{2\mathcal{N}_\tau^{\lambda i}}}\frac{f_{jj'}^{(\lambda)}u_{jj'}^{(+)}\sqrt{1-n_j}\sqrt{1-n_{j'}}}{(\varepsilon_j+\varepsilon_{j'})+\omega_{\lambda i}}\qquad \tilde{\phi}_{jj'}^{\lambda i}=\sqrt{\frac{1}{2\mathcal{N}_\tau^{\lambda i}}}\frac{f_{jj'}^{(\lambda)}u_{jj'}^{(+)}\sqrt{n_j}\sqrt{n_{j'}}}{(\varepsilon_j+\varepsilon_{j'})-\omega_{\lambda i}}$$

$$\eta_{jj'}^{\lambda i}=-\sqrt{\frac{1}{2\mathcal{N}_\tau^{\lambda i}}}\frac{f_{jj'}^{(\lambda)}v_{jj'}^{(-)}\sqrt{1-n_j}\sqrt{n_{j'}}}{(\varepsilon_j-\varepsilon_{j'})-\omega_{\lambda i}}\qquad \zeta_{jj'}^{\lambda i}=-\sqrt{\frac{1}{2\mathcal{N}_\tau^{\lambda i}}}\frac{f_{jj'}^{(\lambda)}v_{jj'}^{(-)}\sqrt{1-n_j}\sqrt{n_{j'}}}{(\varepsilon_j-\varepsilon_{j'})+\omega_{\lambda i}}$$

$$\mathcal{N}_\tau^{\lambda i}=N_\tau^{\lambda i}(\omega_{\lambda i})+\left(\frac{1-X_\tau^{\lambda i}(\omega_{\lambda i})(k_0^{(\lambda)}+k_1^{(\lambda)})}{X_{-\tau}^{\lambda i}(\omega_{\lambda i})(k_0^{(\lambda)}-k_1^{(\lambda)})}\right)^2N_{-\tau}^{\lambda i}(\omega_{\lambda i})$$

$$N_\tau^{\lambda i}(\omega)=\frac{2\lambda+1}{2}\frac{\partial}{\partial\omega}X_\tau^{\lambda i}(\omega)$$

The secular eq.(6) is the same as in [15] where it has been derived using the Green function method. This is true for the expressions for the amplitudes $\psi,\phi,\eta$ as well. The last expressions can be found, e.g., in ref.[16]. One should keep in mind that we define the phonon operator in terms of the thermal quasiparticles, thus giving rise to additional factors proportional to $n_j$ and $(1-n_j)$ in our expressions.

With the RPA values of the amplitudes $\psi,\phi,\eta,\zeta,\tilde{\psi},\tilde{\phi}$ the first two terms of $\mathcal{H}$ (5), which we denoted by $\mathcal{H}_{rpa}$, take the diagonal form in terms of the thermal phonon operators:

$$\mathcal{H}_{rpa}=\sum_{\lambda\mu i}\omega_{\lambda i}(Q^+_{\lambda\mu i}Q_{\lambda\mu i}-\tilde{Q}^+_{\lambda\mu i}\tilde{Q}_{\lambda\mu i})$$

## 5 The quasiparticle – phonon interaction at $T \neq 0$

The last term in (5) (we denote it by $\mathcal{H}_{qph}$) is the interaction term of thermal quasiparticles and thermal phonons (i.e., phonons are built of pairs of thermal quasiparticles). This term mixes the states with a different phonon number, and due to this mixing the strength of the RPA-state is fragmented over some excitation energy interval. In other words, the term $\mathcal{H}_{qph}$ produces a spreading width of a thermal one-phonon state like the case of $T = 0$ [9].

To describe the fragmentation of thermal phonons, we use the variational method with a trial wave function of the form:

$$|\Psi_\beta(JM\nu)\rangle = \Big\{\sum_i R_i(J\nu)Q^+_{JMi} + $$

$$+ \sum_{\lambda_1 i_1 \lambda_2 i_2} P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)\left[Q^+_{\lambda_1\mu_1 i_1}Q^+_{\lambda_2\mu_2 i_2}\right]_{JM}\Big\}|\Psi_0(\beta)\rangle \qquad (7)$$

So we take into account the effect of the interaction between phonons on excited states but not on the vacuum state which is supposed to be the thermal phonon vacuum $|\Psi_0(\beta)\rangle$ as before. The wave function (7) has to be normalized

$$\sum_i (R_i(J\nu))^2 + 2\sum_{\lambda_1 i_1 \lambda_2 i_2}(P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu))^2 = 1 \qquad (8)$$

To find the energy of the state (7) $\eta_{J\nu}$ and the coefficients $R, P$, we again minimize the expectation value of $\mathcal{H}$ over $|\Psi_\beta(JM\nu)\rangle$ at the constraint (8). We have

$$\langle\Psi_\nu(JM)|\mathcal{H}|\Psi_\nu(JM)\rangle = \sum_i \omega_{Ji}[R_i(J\nu)]^2 +$$

$$+2\sum_{\lambda_1 i_1 \lambda_2 i_2}(\omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2})[P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)]^2 + 2\sum_i \sum_{\lambda_1 i_1 \lambda_2 i_2} R_i(J\nu)P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji) \qquad (9)$$

where

$$U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji) = \langle\Psi_0(\beta)|Q_{JMi}\mathcal{H}_{qph}[Q^+_{\lambda_1\mu_1 i_1}Q^+_{\lambda_2\mu_2 i_2}]_{JM}|\Psi_0(\beta)\rangle =$$

$$= U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji, n) + U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji, p)$$



is the coupling matrix element of one- and two-phonon configurations. The function $U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)$ is a bilinear form of the phonon amplitudes $\psi, \phi, \eta, \tilde{\psi}, \tilde{\phi}, \zeta$, namely:

$$U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji, \tau) = -\tfrac{1}{\sqrt{2}}\sqrt{2\lambda_1+1}\sqrt{2\lambda_2+1}\times$$

$$\times\sum_{j_1 j_2 j_3}{}^{\tau}\Big[(-)^J\Gamma^{\lambda_2 i_2}_{j_1 j_2}\begin{Bmatrix}\lambda_2 & \lambda_1 & J\\ j_3 & j_2 & j_1\end{Bmatrix}\mathcal{K}^{\lambda_1 i_1 Ji}_{j_3 j_2 j_1} +$$

$$+(-)^{\lambda_1-\lambda_2}\Gamma^{\lambda_1 i_1}_{j_1 j_2}\begin{Bmatrix}\lambda_1 & \lambda_2 & J\\ j_3 & j_2 & j_1\end{Bmatrix}\mathcal{K}^{\lambda_2 i_2 Ji}_{j_3 j_2 j_1} +$$

$$+(-)^{J-\lambda_1}\Gamma^{Ji}_{j_1 j_2}\begin{Bmatrix}J & \lambda_1 & \lambda_2\\ j_3 & j_2 & j_1\end{Bmatrix}\mathcal{L}^{\lambda_1 i_1 \lambda_2 i_2}_{j_3 j_2 j_1}\Big]$$

Where

$$\mathcal{K}^{\lambda_1 i_1 Ji}_{j_3 j_2 j_1} = v^{(-)}_{j_1 j_2}\sqrt{1-n_{j_1}}\sqrt{1-n_{j_2}}(-)^{j_1+j_3+\lambda_1+J}\times$$

$$\times(\psi^{\lambda_1 i_1}_{j_1 j_3}\psi^{Ji}_{j_2 j_3} + +\phi^{\lambda_1 i_1}_{j_1 j_3}\phi^{Ji}_{j_2 j_3} + \eta^{\lambda_1 i_1}_{j_1 j_3}\eta^{Ji}_{j_2 j_3} + \zeta^{\lambda_1 i_1}_{j_1 j_3}\zeta^{Ji}_{j_2 j_3}) +$$

$$+u^{(+)}_{j_1 j_2}\sqrt{1-n_{j_1}}\sqrt{n_{j_2}}(-)^{\lambda_1+j_1+j_3}\times$$

$$\times(\psi^{\lambda_1 i_1}_{j_1 j_3}\eta^{Ji}_{j_3 j_2} + \phi^{\lambda_1 i_1}_{j_1 j_3}\zeta^{Ji}_{j_3 j_2} + \eta^{\lambda_1 i_1}_{j_1 j_3}\tilde{\psi}^{Ji}_{j_3 j_2} + \zeta^{\lambda_1 i_1}_{j_1 j_3}\tilde{\phi}^{Ji}_{j_3 j_2}) -$$

$$-u^{(+)}_{j_1 j_2}\sqrt{n_{j_1}}\sqrt{1-n_{j_2}}(-)^J(\eta^{\lambda_1 i_1}_{j_3 j_1}\psi^{Ji}_{j_2 j_3} + \zeta^{\lambda_1 i_1}_{j_3 j_1}\phi^{Ji}_{j_2 j_3} + \tilde{\psi}^{\lambda_1 i_1}_{j_3 j_1}\eta^{Ji}_{j_2 j_3} + \tilde{\phi}^{\lambda_1 i_1}_{j_3 j}\zeta^{Ji}_{j_2 j_3}) -$$

$$-v^{(-)}_{j_1 j_2}\sqrt{n_{j_1}}\sqrt{n_{j_2}}(-)^{j_2+j_3}(\eta^{\lambda_1 i_1}_{j_3 j_1}\eta^{Ji}_{j_3 j_2} + \zeta^{\lambda_1 i_1}_{j_3 j_1}\zeta^{Ji}_{j_3 j_2} + \tilde{\psi}^{\lambda_1 i_1}_{j_3 j_1}\tilde{\psi}^{Ji}_{j_3 j_2} + \tilde{\phi}^{\lambda_1 i_1}_{j_3 j_1}\tilde{\phi}^{Ji}_{j_3 j_2})$$

$$\mathcal{L}^{\lambda_1 i_1 \lambda_2 i_2}_{j_3 j_2 j_1} = v^{(-)}_{j_1 j_2}\sqrt{1-n_{j_1}}\sqrt{1-n_{j_2}}(-)^{j_1+j_3+\lambda_1+\lambda_2}\times$$

$$\times(\psi^{\lambda_1 i_1}_{j_1 j_3}\phi^{\lambda_2 i_2}_{j_2 j_3} + \phi^{\lambda_1 i_1}_{j_1 j_3}\psi^{\lambda_2 i_2}_{j_2 j_3} + \eta^{\lambda_1 i_1}_{j_1 j_3}\zeta^{\lambda_2 i_2}_{j_2 j_3} + \zeta^{\lambda_1 i_1}_{j_1 j_3}\eta^{\lambda_2 i_2}_{j_2 j_3}) +$$

$$+u^{(+)}_{j_1 j_2}\sqrt{1-n_{j_1}}\sqrt{n_{j_2}}(-)^{\lambda_1+j_1+j_2}\times$$

$$\times(\psi^{\lambda_1 i_1}_{j_1 j_3}\zeta^{\lambda_2 i_2}_{j_3 j_2} + \phi^{\lambda_1 i_1}_{j_1 j_3}\eta^{\lambda_2 i_2}_{j_3 j_2} + \eta^{\lambda_1 i_1}_{j_1 j_3}\tilde{\phi}^{\lambda_2 i_2}_{j_3 j_2} + \zeta^{\lambda_1 i_1}_{j_1 j_3}\tilde{\psi}^{\lambda_2 i_2}_{j_3 j_2}) -$$

$$-u^{(+)}_{j_1 j_2}\sqrt{n_{j_1}}\sqrt{1-n_{j_2}}(-)^{\lambda_2}(\zeta^{\lambda_1 i_1}_{j_3 j_1}\psi^{\lambda_2 i_2}_{j_2 j_1} + \eta^{\lambda_1 i_1}_{j_3 j_1}\phi^{\lambda_2 i_2}_{j_2 j_3} + \tilde{\phi}^{\lambda_1 i_1}_{j_3 j_1}\eta^{\lambda_2 i_2}_{j_2 j_3} + \tilde{\psi}^{\lambda_1 i_1}_{j_3 j_1}\zeta^{\lambda_2 i_2}_{j_2 j_3}) -$$

$$-v^{(-)}_{j_1 j_2}\sqrt{n_{j_1}}\sqrt{n_{j_2}}(-)^{j_2+j_3}(\eta^{\lambda_1 i_1}_{j_3 j_1}\zeta^{\lambda_2 i_2}_{j_3 j_2} + \zeta^{\lambda_1 i_1}_{j_3 j_1}\eta^{\lambda_2 i_2}_{j_3 j_2} + \tilde{\psi}^{\lambda_1 i_1}_{j_3 j_1}\tilde{\phi}^{\lambda_2 i_2}_{j_3 j_2} + \tilde{\phi}^{\lambda_1 i_1}_{j_3 j_1}\tilde{\psi}^{\lambda_2 i_2}_{j_3 j_2})$$

After variation of (9) at the constraint (8) one gets the homogeneous system of linear equations with the coefficients depending on energy $\eta_{J\nu}$. The system can be resolved if $\eta_{J\nu}$ is the root of the following secular equation:

$$det|(\omega_{Ji} - \eta_{J\nu})\delta_{ii'} - \frac{1}{2}\sum_{\lambda_1 i_1 \lambda_2 i_2} \frac{U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji) U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji')}{\omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2} - \eta_{J\nu}}| = 0 \qquad (10)$$

## 6 Discussion

Equation (10) for the energies of excited states built on the thermal ground (compound) state of a hot nucleus together with the expression for $U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji')$ gives us the possibility to calculate spreading of strength of a collective resonance state (a thermal RPA-state) in hot nuclei. Formally, eq.(10) has the same form as at $T = 0$ (see, e.g. [9]). The difference is that now $U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)$ depends on the temperature through the thermal occupation numbers (not only directly but through the amplitudes $\psi, \phi\, \eta$ etc. as well). Moreover, the energies of one-phonon states $\omega_{\lambda i}$ are calculated in the thermal RPA and the number of phonons of given multipolarity at finite temperature is twice as large as at $T = 0$ because of new poles $\varepsilon_j - \varepsilon_{j'}$ which appear in the thermal RPA equation (6).

As one can see from the expressions for the bifermionic amplitudes, in the leading order the non-tilde amplitudes $\psi, \phi$ are proportional to $(1 - n_j)$, the mixed amplitudes $\eta, \zeta$ are proportional to $n_j^{1/2}$ and the tilde amplitudes $\tilde{\psi}, \tilde{\phi}$ are proportional to $n_j$. So, at $T \rightarrow 0$ only the terms containing $\psi, \phi$ will survive, and, as a result, one gets for $U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)$ the same expression as in refs.[8, 9]. It means that there is a natural correspondence between the results of our approach at $T \neq 0$ and at $T = 0$.

In the above-stated consideration, we followed quite closely the way outlined in [4, 5]. The difference from, e.g. [4], is due to a specific form of a residual interaction (which is now taken to be separable for simplicity) and the phonon language when we go beyond the thermal RPA. It seems interesting to clarify in detail what the difference between the second thermal RPA approach of [4] and our way to take into account the coupling with comlex configurations by mixing one- and two- thermal phonon states is.

The present TFD approach to the damping of nuclear excitations at finite temperature differs quite noticeably from that of papers [2, 3]. In those papers, the Matsubara formalism has been used but the main point was that the thermal particle - hole and the thermal phonon excitations have been considered on equal footing, i.e., both systems were heated and both Fermi and Bose thermal occupation numbers came into play, respectively. In the TFD approach, the thermal phonons are formed of the thermal quasiparticles but the phonon system itself is not heated and, therefore, Bose thermal occupation numbers didn't appear in our consideration. Certainly, one can project the initial Hamiltonian from bifermion to boson space using some kind of boson expansion and only after that make the TFD transformation. This way has been discussed by Hatsuda [5], for example. Then, Bose thermal occupation numbers appear naturally but the structure of bosons has to be calculated at $T = 0$. This is not the case e.g. in [2] where the structure of phonons have been calculated in *the thermal RPA*. It seems to us that this is a kind of double counting.

This work was partially supported by the Russian foundation for fundamental researches under grant 94-02-05137-a

## References

[1] Bunatyan G.G., Yad.Fiz., 1977, **26**, 518 ; Sommermann H.M., Ann. Phys., 1983, **151**, 163 ; Vautherin D., Vinh Mau N., Nucl.Phys., 1984, **A422**, 140.

[2] Bortignon R.A. et al., Nucl.Phys., 1986, **A460**, 149

[3] Kamerdzhiev S.P., Preprint PhEI-1860, Obninsk, 1987.

[4] Tanabe K., Phys.Rev. C., 1988,**37**, 2802

[5] Hatsuda T., Nucl.Phys., 1989, **A492**, 187

[6] Vinh Mau N. Nucl.Phys., 1992, **A548**, 381.

[7] Bertsch G.F., Bortignon P.F., Broglia R.A. Rev. Mod. Phys., 1983, **55**, 287.

[8] Vdovin A.I., Soloviev V.G., Particles & Nuclei, 1983, **14**, 237; Voronov V.V., Soloviev V.G., Particles & Nuclei, 1983, **14**, 583

[9] Soloviev V.G. "Theory of atomic nuclei: quasiparticles and phonons" (Institute of Physics Publishing, Bristol and Philadelphia, 1992)

[10] Umezawa H., Matsumoto H., Tachiki M., Thermo field dynamics and condensed states (North-Holland, Amsterdam, 1982)

[11] Vdovin A.I., Kosov D.S., JINR, Preprint E4-93-417, Dubna, 1993.

[12] Kosov D.S., Vdovin A.I., JINR, Preprint E4-94-106, Dubna, 1994.

[13] Civitarese O. et al., Z.Phys., 1993,**A344**, 243

[14] Ignatyuk A.V. "Statisticheskie svoistva vozbuzhdennykh sostoyaniy yader" (Moscow, Energoatomizdat, 1983)

[15] Ignatyuk A.V., Yad.Fiz., 1975, **21**, 20

[16] Dussel G.G. et al., Phys.Rev. C, 1992, **46**, 558

Received by Publishing Department
on July 21, 1994.



Вдовин А.И., Косов Д.С. E4-94-275

Обобщение КФМ на конечные температуры
методами термополевой динамики

Предложен способ обобщения квазичастично-фононной модели ядра (КФМ) для описания возбуждений нагретых ядер с помощью методов термополевой динамики. После формального удвоения одночастичных степеней свободы ядра введением так называемых одночастичных тильдованных состояний последовательно выполнены два канонических преобразования — обычное и тепловое преобразования Боголюбова. Их коэффициенты найдены из условия минимума свободной энергии системы частиц только со спаривательным взаимодействием при $T \neq 0$. Затем введены тепловые фононные возбуждения, представляющие собой линейные суперпозиции прямых и обратных амплитуд, образованных парами операторов тепловых квазичастиц, и с помощью вариационного принципа получены уравнения теплового приближения случайной фазы. Получено выражение для теплового гамильтониана КФМ в терминах тепловых квазичастиц и тепловых фононов, которое содержит взаимодействие этих двух типов возбуждений. Выведены уравнения для состояний, представляющих собой суперпозицию одно- и двухфононных компонент, и получено выражение для матричного элемента взаимодействия.

Работа выполнена в Лаборатории теоретической физики им.Н.Н.Боголюбова, ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 1994

Vdovin A.I., Kosov D.S. E4-94-275

Extension of the QPM to $T \neq 0$ Based on the Formalism
of the Thermo Field Dynamics

The way of extending the quasiparticle — phonon nuclear model (the QPM) to finite temperature is presented. It is based on a formalism of the thermo field dynamics (the TFD). After formal doubling of the single-particle degrees of freedom by introducing the so called tilde-states, the usual and thermal Bogoliubov transformations are made. The coefficients of the transformations are determined by minimizing the free energy potential of a hot nucleus in the thermal vacuum state taking into account only single — particle and pairing terms of the QPM Hamiltonian. Then the thermal phonon operator is introduced as a linear superposition of forwardgoing- and backwardgoing bi-thermal-quasiparticle amplitudes, and with the variational principle the thermal RPA equations are derived. The expression for the thermal QPM Hamiltonian in terms of thermal quasiparticles and thermal phonons is given which contains the interaction of these two types of excitation modes. The equation for the states taking into account the mixing of one- and two-thermal phonon components and the expression of the corresponding coupling matrix element are derived.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 1994