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LANDAU QUASIENERGY SPECTRUM DESTRUCTION FOR AN ELECTRON IN BOTH A STATIC MAGNETIC FIELD AND A RESONANT ELECTROMAGNETIC WAVE

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I. Introduction

Consider the free nonrelativistic electron in a static magnetic field H, taking electron spin into account. A plane-polarized wave propagates along a magnetic field, its frequency Ω is close to the electron rotation frequency $\omega_L = eH/mc$:

$$|\Omega - \omega_L| \ << \ \omega_L \ , \tag{1}$$

where $-e$ is the electron charge, m is the electron mass and c is the velocity of light. Let the electron be located in a domain with dimension $L << \lambda$, where $\lambda = 2\pi c/\Omega$ is the length of the light wave. So we can describe the electron-wave interaction using a dipole approximation for the kinetic part of the Hamiltonian. We include the wave magnetic field in the spin part of the Hamiltonian because the corresponding interaction frequency is close to the resonant one, so spin components have to be mixed. In section II of the present paper we include and transform the corresponding Schrödinger equation.

As the electron Hamiltonian is time-dependent and time-periodic, one can include quasienergies and steady states [1] (quasienergy states [2, 3]) to describe electron behaviour. In section III we construct these values according to the resonant approximation of (1). It is shown that if $\Omega \neq \omega_L$, the electron quasienergy spectrum is discrete and equivalent to the Landau energy spectrum in a static magnetic field. Steady state components corresponding to rotation have a finite norm. When $\Omega = \omega_L$, the electron quasienergy spectrum changes into a continuous one and the steady state rotation components become un-normalizable. Destruction of the Landau spectrum in the resonant case corresponds to electron transversal motion transmutation (from finite to infinite) in the resonant wave.

II. Schrödinger Equation Transform

Let j be $1,2,3$; α, β be ± 1; x_j — the coordinates, t — time, ψ_α — the electron wave function, $\sigma^{(j)}_{\alpha\beta}$ — the Pauli matrixes and $\hbar$ — Planck's constant.

Let the static magnetic field to be directed along the x_3 axis and the wave electric field oscillate along the x_2 axis with amplitude E. We can then describe the electron by the following Schrödinger equation:

$$i\hbar\frac{\partial}{\partial t}\psi_\alpha = \frac{1}{2m}\left\{\left(-i\hbar\frac{\partial}{\partial x_1} - \frac{e}{c}Hx_2\right)^2 + \left(-i\hbar\frac{\partial}{\partial x_2} + \frac{cE}{\Omega}\cos(\Omega t)\right)^2 - \right.$$
$$\left. -\hbar^2\frac{\partial^2}{\partial x_3^2}\right\}\psi_\alpha + \frac{e\hbar}{2mc}\sum_\beta\left\{-E\sin(\Omega t)\sigma^{(1)}_{\alpha\beta} + H\sigma^{(3)}_{\alpha\beta}\right\}\psi_\beta \ . \tag{2}$$

Next, we search for solutions of (2) in the form:

$$\psi_\alpha(x_j,t) = \exp\left\{\frac{i}{\hbar}\left[p_1x_1 + p_3x_3 - \frac{p_3^2}{2m}t - \frac{eE\xi}{\Omega}\cos(\Omega t)\right]\right\}\gamma_\alpha(\xi,t) \ , \tag{3}$$

where p_1, p_2 are real numbers and $\xi = x_2 - (p_1c)/(eH)$.

Two functions, $\gamma_1(\xi,t)$ and $\gamma_{-1}(\xi,t)$, satisfy the following set of equations:

$$i\hbar\frac{\partial\gamma_1}{\partial t} = \left\{\hat{H}_{osc} + \frac{\hbar\omega_L}{2} + eE\xi\sin(\Omega t)\right\}\gamma_1 - \frac{\epsilon\hbar\omega_L}{2}\sin(\Omega t)\gamma_{-1}$$
$$i\hbar\frac{\partial\gamma_{-1}}{\partial t} = \left\{\hat{H}_{osc} - \frac{\hbar\omega_L}{2} + eE\xi\sin(\Omega t)\right\}\gamma_{-1} - \frac{\epsilon\hbar\omega_L}{2}\sin(\Omega t)\gamma_1 \ , \tag{4}$$

where $\epsilon = E/H$ and

$$\hat{H}_{osc} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial\xi^2} + \frac{m\omega_L^2}{2}\xi^2 \ . \tag{5}$$

In the frame of the resonant approximation (1) we can rewrite (4) as

$$i\hbar\frac{\partial\gamma_1}{\partial t} = \left\{\hat{H}_{osc} + \frac{\hbar\omega_L}{2} + eE\xi\sin(\Omega t)\right\}\gamma_1 - \frac{i\epsilon\hbar\omega_L}{4}\exp(-i\Omega t)\gamma_{-1}$$
$$i\hbar\frac{\partial\gamma_{-1}}{\partial t} = \left\{\hat{H}_{osc} - \frac{\hbar\omega_L}{2} + eE\xi\sin(\Omega t)\right\}\gamma_{-1} + \frac{i\epsilon\hbar\omega_L}{4}\exp(i\Omega t)\gamma_1 \ . \tag{6}$$

Then, we search for solutions of (6) in the form:

$$\gamma_{-1}(\xi,t) = i\beta e^{i\Omega t}\gamma_1(\xi,t) \ , \tag{7}$$

where β is a constant. It is easy to show that (7) can be the solution of (6) if

$$\beta = \beta_{1,2} = \left\{2(\Omega - \omega_L) \pm \sqrt{4(\Omega-\omega_L)^2 + \epsilon^2\omega_L^2}\right\}/(\epsilon\omega_L) \ . \tag{8}$$

Let us present $\gamma_1(\xi,t)$ in following form for the two cases corresponding to $\beta = \beta_1$, $\beta = \beta_2$:

$$\gamma_1(\xi,t) = \exp\left\{-it\left[\frac{\Omega}{2} \pm \frac{1}{4}\sqrt{4(\Omega-\omega_L)^2 + \epsilon^2\omega_L^2}\right]\right\}\Phi(\xi,t) \ . \tag{9}$$

Then $\Phi(\xi,t)$ satisfies the following equation in both cases:

$$i\hbar\frac{\partial}{\partial t}\Phi(\xi,t) = \left\{\hat{H}_{osc} + eE\xi\sin(\Omega t)\right\}\Phi(\xi,t) \ . \tag{10}$$

Equation (10) is the well-known equation for a harmonic oscillator acted upon by a time-dependent oscillating force. Thus, we have shown that the electron, located in both a static magnetic field and an electromagnetic wave with the frequency close to the resonant one, described by the Schrödinger equation for a harmonic oscillator.

III. Quasienergies and Steady States Construction

Quasienergies and steady states (quasienergy states) for the quantum systems described by (10) and for some more general class of Hamiltonians were exactly constructed in [4]. Below we construct them for our special case (10) and give a draft proof for the conclusions used. The results are especially clear if we use the resonant approximation (1) and introduce as standard operators:

$$a = -i\sqrt{\hbar/2m\omega_L}\frac{\partial}{\partial\xi} - i\xi\sqrt{m\omega_L/2\hbar}\ ; a^+ = -i\sqrt{\hbar/2m\omega_L}\frac{\partial}{\partial\xi} + i\xi\sqrt{m\omega_L/2\hbar}\ . \quad (11)$$

Then we can rewrite (10) as approximately:

$$i\frac{\partial}{\partial t}|\Phi(t)>=\left\{\omega_L\left(a^+a+\frac{1}{2}\right)+\alpha\left(ae^{it\Omega}+a^+e^{-it\Omega}\right)\right\}|\Phi(t)>, \quad (12)$$

where $|\Phi(t)>$ is the corresponding ket-vector and $\alpha = eE/\sqrt{8m\hbar\omega_L}$.

Now let us use complex representation [5]. If z is a complex number and $|z>$ is a corresponding coherent state, then

$$\Phi(z,t) = \exp\left\{|z|^2/2\right\} < \bar{z}|\Phi(t)> \quad (13)$$

is an analitical function of z in the whole complex plane at any t (the overline means a complex conjugation). We can rewrite (12) for $\Phi(z,t)$:

$$i\frac{\partial}{\partial t}\Phi(z,t) = \left\{\omega_L\left(z\frac{\partial}{\partial z}+\frac{1}{2}\right)+\alpha\left(e^{it\Omega}\frac{\partial}{\partial z}+e^{-it\Omega}z\right)\right\}\Phi(z,t)\ . \quad (14)$$

It was shown in [4] that equation (14) has resonant solutions at $\omega_L = q\Omega$, where q is an integer. Here we consider resonant solutions for $q = 1$ only because the restriction (1) is fulfilled.

If $\Omega \neq \omega_L$, each solution of equation (14) can be represented as a linear combination of following solutions:

$$\Phi_n(z,t) = e^{-itE_n/\hbar}\phi_n(z,t)\ ;\ \phi_n(z,t) = \phi_n(z,t+2\pi/\Omega)\ ;\ n = 0,\ 1,\ 2\ldots\ , \quad (15)$$

where quasienergies E_n and steady states (quasienergy states) $\phi_n(z,t)$ are given by:

$$E_n = \hbar\omega_L\left(n+\frac{1}{2}\right)+\frac{\hbar\alpha^2}{\Omega-\omega_L}$$

$$\phi_n(z,t) = \frac{1}{\sqrt{n!}}\left[ze^{-it\Omega}-\frac{\alpha}{\Omega-\omega_L}\right]^n\exp\left\{itn\Omega+\frac{ze^{-it\Omega}\alpha}{\Omega-\omega_L}-\frac{|\alpha|^2}{2(\Omega-\omega_L)^2}\right\}. \quad (16)$$

It is easy to show [4] that the vectors $\phi_n(z,t)$, $n = 0, 1, 2\ldots$, form the complete orthogonal basis in the corresponding Hilbert space and their norms are equal to

one. It shows that we have found all the solutions of equation (2) (in resonant approximation). The quasienergy spectrum (16) is equivalent to the Landau energy spectrum for a free electron in a static magnetic field.

As quasienergies are only defined modulo $q\hbar\Omega$, where q is an integer, we can use an equivalent representation for quasienergies and steady states (quasienergy states) instead of (16) [1, 2, 3]:

$$\tilde{E}_n = \hbar(\omega_L-\Omega)n + \hbar\omega_L/2 + \hbar\alpha^2/(\Omega-\omega_L)\ ;\ \tilde{\phi}_n(z,t) = e^{-itn\Omega}\phi_n(z,t)\ . \quad (17)$$

This form shows that when the wave frequency Ω approaches to the electron rotation frequency ω_L, the quasienergy spectrum almost degenerates.

In the resonant case, when $\Omega = \omega_L$, each solution of equation (14) can be represented as a linear combination of following solutions:

$$\Phi_E(z,t) = e^{-itE/\hbar}\phi_E(z,t)\ ;\ \phi_E(z,t) = \phi_E(z,t+2\pi/\Omega)\ ;\ -\infty < E < \infty\ ; \quad (18)$$

where E is the quasienergy, and the steady states (quasienergy states) $\phi_E(z,t)$ are given by [4]:

$$\phi_E(z,t) = (2\pi\hbar^2\alpha^2)^{-1/4}\exp\left\{-\left(\frac{E-\hbar\Omega/2}{2\hbar\alpha}\right)^2+\frac{ze^{-it\Omega}(E-\hbar\Omega/2)}{\hbar\alpha}-\frac{z^2e^{-2it\Omega}}{2}\right\}. \quad (19)$$

It is easy to show [4] that the steady states (19) satisfy the following relations at any t:

$$\frac{1}{\pi}\int dzd\bar{z}e^{-|z|^2}\bar{\phi}_E(z,t)\phi_{E'}(z,t) = \delta(E-E')$$

$$\int_{-\infty}^{\infty} dE\bar{\phi}_E(z,t)\phi_E(z',t) = e^{\bar{z}z'}\ , \quad (20)$$

where δ is the Dirac delta-function. These relations mean that the corresponding ket-vectors satisfy the orthogonality and completeness relations [6]:

$$<\phi_E|\phi_{E'}>=\delta(E-E') \quad ; \quad \int_{-\infty}^{\infty} dE|\phi_E><\phi_E| = I\ , \quad (21)$$

where I is the unit operator in the corresponding Hilbert space. The relations (21) show that we have found, for the resonant case, all solutions of equation (2) (in resonant approximation).

We can see that the Landau spectrum (16) is destroyed in the resonant case, because the quasienergy spectrum is a continuous one: $-\infty < E < \infty$. As follows from (21), resonant steady states (quasienergy states) are un-normalizable. The spectrum and steady states reconstruction correspond to electron transversal motion transmutation (from finite to infinite) in the resonant wave.

IV. Conclusion

The behaviour of the electron located in both a static magnetic field and an electromagnetic wave with the frequency close to the resonant one is described in terms of quasienergies and steady states (quasienergy states).

It is shown that the well-known electron transversal motion transmutation (from finite to infinite) in the resonant wave leads to the descrete quasienergy spectrum destruction and to the forming of a new steady states basis, consisting of un-normalizabled vectors. The corresponding quasienergy spectrum is continuous.

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