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DYNAMICAL INSTABILITY AND SOLITON CONCEPT

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I. MOTIVATION

Nonlinear dynamic phenomena in different complex systems are currently a topic of considerable interest in modern physics. It is mainly caused by great progress in a development of the methods to solve exactly nonlinear partial differential equations. A hole class of these equations admits solutions in a form of so-called solitary waves - solitons (For a last decade history of a soliton concept in nonrelativistic nuclear physics see review [1]).

An existence of solitary waves is determined by two essential factors, namely, nonlinearity and dispersion. Both the factors, which are responsible for the stability of a wave, are connected in their turn with two different types of instability. A localized pulse will tend to spread out due to dispersion terms of the equations of motion. The nonlinearity which is responsible for the formation of solitary waves, on the other hand, automatically leads to their destruction, if it is alone. Both instabitities may compensate each other and lead to stable solutions (solitons).

Let us look from these points of view at multifragmentation phenomena. The formation and breakup of a highly excited and compressed nuclear system is the most striking process observed in intermediate-energy heavy ion reactions. How does such a system expand and finally disassemble when passing through a regime of dynamical instabilities? What is the mechanism of the clustering (stable light and intermediate mass fragment production)? Quite a variety of models have been developed to discuss this question (see, for instance, the recent review [2] and Proceedings [3]). The dynamical clusterization in the presence of instabilities is the focus of attention of the intermediate energy heavy ions physics [4], [5]. Ten years ago multifragmentation has been associated with the onset of the spinodal instability [6]. This instability is associated with the transit of a homogeneous fluid across a domain of negative pressure, which leads to its breaking up into droplets of denser liquid embedded in a lower density vapor. Since the spinodal instability can occur in an infinite system, it can be called the bulk or volume instability. On the other hand, it physically means that pressure depends on density, that is just a nonlinearity in terms of density [7].

Recently [8], it has been pointed out that a new kind of instability (sheet instability) may play an important role in multifragmentation. This new instability can be assigned to the class of surface instabilities of the Rayleigh-Taylor kind [9]. System escapes from the high surface energy of the intermediate complex by breaking up into a number of spherical fragments with less overall surface. At the same time, it physically means the existence of the gradient terms of the equations of motion, i.e the dispersion [7].

The spinodal instability and the Rayleigh-Taylor instability may compensate each other and lead to stable quasi-soliton type objects. In the next Sec. we present this physical picture using a simple analytical model proposed to describe the time evolution of compressed nuclear systems.

II. BASIC EQUATIONS

A complete quantum - mechanical description of a colliding nuclear systems is not yet constructed. The Schrõdinger equation is linear in nuclear many-body wave function, but we are not able to solve it. The reduced description in a truncated space of collective degrees of freedom inevitably leads to nonlinear problem.

Both classical (hydrodynamics [10]) and semiclassical (time dependent Hartree-Fock (TDHF), temperature dependent Hartree - Fock, the dynamical Thomas - Fermi theory, the Boltzmann - Nordheim -Vlasov (BNV [11]) or Botzmann - Ueling - Uhlenbeck (BUU) [12]) equations are used now to simulate numerically a breakup phase in heavy ion collisions. All of them are based on a supposition that at the later stage, when hard collisions are nearly over, the created excitation evolves according to the mean-field description.

At the same time, an analysis of stability of nonlinear dynamical systems and an analysis of nonlinear evolution of initial complex states is a traditional goal of Soliton Theory. The inverse methods to integrate nonlinear evolution equations are often more effective than a direct numerical integration. Let us demonstrate this statement for a very simple case [13].

The type of systems under our consideration are uncharged slabs of nuclear matter. The slabs are finite in the z coordinate and infinite and homogeneous in two transverse directions.

The basic equations for the static mean-field description of the slabs are the following

$$\psi_{k_\perp n}(\boldsymbol{x}) = \frac{1}{\sqrt{\Omega}}\psi_n(z)exp(i\boldsymbol{k}_\perp\vec{r}),$$

$$\epsilon_{k_\perp n} = \frac{\hbar^2 k_\perp^2}{2m} + \epsilon_n,$$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dz^2}\psi_n(z) + U(z)\psi_n(z) = \epsilon_n\psi_n(z),$$

$$\rho(\boldsymbol{x}) \Longrightarrow \rho(z) = \sum_{n=1}^{N_0} a_n\psi_n^2(z), \tag{1}$$

$$A \Longrightarrow \mathcal{A} = (6A\rho_N^2/\pi)^{1/3} = \sum_{n=1}^{N_0} a_n,$$

$$a_n = \frac{2m}{\pi\hbar^2}(\epsilon_F - \epsilon_n),$$

$$\frac{E}{A} \Longrightarrow \frac{\hbar^2}{2m\mathcal{A}}\Big(\sum_{n=1}^{N_0} a_n\int_{-\infty}^{\infty}(\frac{d\psi_n}{dz})^2dz + \frac{\pi}{2}\sum_{n=1}^{N_0} a_n^2\Big) + \frac{1}{\mathcal{A}}\int_{-\infty}^{\infty}\mathcal{E}[\rho(z)]dz,$$

where $\boldsymbol{r} \equiv (x,y), \boldsymbol{k}_\perp \equiv (k_x,k_y)$, and Ω is the transverse normalization area. N_0 is the number of occupied bound orbitals.

The dynamical description will be done in the framework of the inverse mean field method. One can found the details of this approach in [14], [15]. We concentrate here only on essentials.

The evolution of a system is given by the famous hydrodynamical Korteweg-de Vries equation (KdV) for the mean-field potential $U(z,t)$

$$\sum_{n=1}^{N}\frac{\partial U}{\partial(S_n t)} = 6U\frac{\partial U}{\partial z} - \frac{\hbar^2}{2m}\frac{\partial^3 U}{\partial z^3}. \tag{2}$$

where S_n are constants which are determined by the initial conditions.

General solution of KdV Eq. (2) can be derived in principle via direct methods numerically. This way is to assign a functional of interaction $\mathcal{E}$ (as usual an effective density dependent Skyrme force), a total number of particles (or a thickness of a slab $\mathcal{A}$) and to solve Hartree-Fock equations to derive a spectrum of the single particle states ϵ_n and wave functions $\psi_n(z,0)$, the density profile $\rho(z,0)$ and the one-body potential $U(z,0)$ for the initial compressed nucleus. Then, one calculate an evolution of the one-body potential with the help of Eq. (2).

However, there is an inverse method to solve KdV Eq. (2). The main advantage of this way is to reduce the solution of the nonlinear KdV Eq. (2) to the solution of the linear Schrödinger - type equation

$$-\frac{\hbar^2}{2m}\frac{d^2}{dz^2}\psi_n(z,t) + U(z,t)\psi_n(z,t) = e_n\psi_n(z,t), \tag{3}$$

and linear integral Gelfand - Levitan - Marchenko equation to derive the function $K(x,y)$

$$K(x,y) + B(x+y) + \int_x^{\infty} B(y+z)K(x,z)dz = 0. \tag{4}$$

The kernel B is determined by the reflection coefficients $R(k)(e_k = \hbar^2k^2/2m)$. and by the N bound state eigenvalues $e_n = -\hbar^2\kappa_n^2/2m$. N is the total number of the bound orbitals.

$$B(z) = \sum_{n=1}^{N} C_n^2(\kappa_n) + \frac{1}{\pi}\int_{-\infty}^{\infty} R(k)exp(ikz)dk.$$

The coefficients C_n are uniquely specified by the boundary conditions

$$C_n(\kappa_n) = \lim_{z\to\infty}\psi_n(z)exp(\kappa_n z),$$

and the wanted single particle potential is given by

$$U(z,t) = -\frac{\hbar^2}{m}\frac{\partial}{\partial z}K(z,z).$$

The time t is included in Eqs. (3,4) only as a parameter, so it has been omitted in the above formulas.

The general solution, $U(z,t)$, should naturally contain both, contributions due to the continuum of the spectrum and to its discrete part. There is no way to obtain the general solution $U(z,t)$ in a closed form. Eqs. (3,4) have to be solved only numerically.

However in the case of reflectless ($R(k) = 0$) symmetrical ($U(-z,0) = U(z,0)$) potentials one can derive the following basic relations

$$U(z,t) = -\frac{\hbar^2}{m}\frac{\partial^2}{\partial z^2}ln(det\|M\|) = -\frac{2\hbar^2}{m}\sum_{n=1}^{N}\kappa_n\psi_n^2(z,t),$$

$$\psi_n(z,t) = \sum_{n=1}^{N} (M^{-1})_{nl} \lambda_l(z,t),$$

$$\lambda_n(z,t) = C_n(\kappa_n) exp(-\kappa_n z + 2\hbar^2 \kappa_n^3 S_n t/m)), \tag{5}$$

$$M_{nl}(z,t) = \delta_{nl} + \frac{\lambda_n(z,t)\lambda_l(z,t)}{\kappa_n + \kappa_l},$$

$$C_n(\kappa_n) = \Big(2\kappa_n \prod_{l \neq n} \frac{\kappa_n + \kappa_l}{\kappa_n - \kappa_l}\Big)^{1/2}.$$

So, the wave functions, potential and the density profile are completely defined by the bound state eigenvalues. The first step is to solve the Schrödinger Eq. (3) for the initial potential $U(z,0)$, which is suitable to simulate compressed nuclear system or to simulate this state with the help of spectrum. Then one calculates the evolution of $\rho(z,t)$ and $U(z,t)$ with the help of Eqs. (5).

III. THREE-LEVEL SYSTEM

Although there is definitely some progress in the application of the inverse methods to nuclear physics they are not yet too popular. As illustration of these methods, we consider a one-dimensional three-level system in detail. A three-level system may be useful for modelling the evolution of light nuclei, for instance, of oxygen [16].

Let us present the main formulas to calculate wave functions of a three-level system $\kappa_3 > \kappa_2 > \kappa_1$

$$\psi_1(z,t) = \Big(2\kappa_1 \big(\tfrac{\kappa_2+\kappa_1}{\kappa_2-\kappa_1}\big)\big(\tfrac{\kappa_3+\kappa_1}{\kappa_3-\kappa_1}\big)\Big)^{1/2} D^{-1}(z,t)\big(ch(\xi_2+\xi_3) - \big(\tfrac{\kappa_3+\kappa_2}{\kappa_3-\kappa_2}\big) ch(\xi_3-\xi_2)\big),$$

$$\psi_2(z,t) = \Big(2\kappa_2 \big(\tfrac{\kappa_2+\kappa_1}{\kappa_2-\kappa_1}\big)\big(\tfrac{\kappa_3+\kappa_2}{\kappa_3-\kappa_2}\big)\Big)^{1/2} D^{-1}(z,t)\big(sh(\xi_3+\xi_1) - \big(\tfrac{\kappa_3+\kappa_1}{\kappa_3-\kappa_1}\big) ch(\xi_3-\xi_1)\big),$$

$$\psi_3(z,t) = \Big(2\kappa_3 \big(\tfrac{\kappa_3+\kappa_2}{\kappa_3-\kappa_2}\big)\big(\tfrac{\kappa_3+\kappa_1}{\kappa_3-\kappa_1}\big)\Big)^{1/2} D^{-1}(z,t)\big(ch(\xi_1+\xi_2) - \big(\tfrac{\kappa_2+\kappa_1}{\kappa_2-\kappa_1}\big) ch(\xi_2-\xi_1)\big),$$

$$D(z,t) = ch(\xi_1+\xi_2+\xi_3) + \big(\frac{\kappa_2+\kappa_1}{\kappa_2-\kappa_1}\big)\big(\frac{\kappa_3+\kappa_1}{\kappa_3-\kappa_1}\big) ch(\xi_3+\xi_2-\xi_1)$$
$$+ \big(\frac{\kappa_2+\kappa_1}{\kappa_2-\kappa_1}\big)\big(\frac{\kappa_3+\kappa_2}{\kappa_3-\kappa_2}\big) ch(\xi_3+\xi_1-\xi_2) + \big(\frac{\kappa_3+\kappa_1}{\kappa_3-\kappa_1}\big)\big(\frac{\kappa_3+\kappa_2}{\kappa_3-\kappa_2}\big) ch(\xi_2+\xi_1-\xi_3),$$

$$\xi_n(z,t) \equiv \kappa_n z - 2\hbar^2\kappa_n^3 S_n t/m \qquad n = 1,2,3.$$

The asymptotic behavior of the wave functions and the mean - field potential have the following form (ξ_n is fixed)

$$\lim_{t\to\infty}\psi_k = \delta_{kn}\sqrt{\kappa_n}sech(\xi_n - \xi_n^0),$$

$$\lim_{t\to\infty}U(z,t) = -\frac{\hbar^2\kappa_n^2}{m}sech^2(\xi_n - \xi_n^0),$$

$$\xi_n^0 = \frac{1}{2}ln\Big(\prod_{k=1}^{n-1}(\frac{\kappa_n - \kappa_k}{\kappa_n + \kappa_k})^2\prod_{m\neq n}^{n}|\frac{\kappa_n + \kappa_m}{\kappa_n - \kappa_m}|\Big).$$

So, for large z and t the time - dependent one - body potential and the corresponding density distributions are represented by a set of stable solitary waves. The energy spectrum of an initially compressed system completely determines widths, velocities and the phase shifts of the solitons.

The number of waves is equal to the number of occupied bound orbitals. Thickness ('number' of particles) of an n - wave is equal to a_n.

Reflecting terms ($R(k) \neq 0$) of $U(z,t)$ cause ripples (oscillating waves of a small amplitude) in addition to the solitons in the final state.

The initially compressed system expands so that for large times one can observe separate density solitons and ripples ('emissions'). This picture is in accordance with the TDHF simulation of the time evolution of a compressed O^{16} nucleus [16]. The disassembly shows collective flow and clusterization. It is important to note that the clusterization was not observed in the absence of the self-consistent mean-field potential, i.e this confirmes our supposition that the nonlinearity is very important for the clustering.

It is necessary to note that the present model is too primitive in order to describe a real breakup process. However this model can be used to illustrate an inverse mean-field method scheme, a nonlinear principle of superposition and the idea that nonlinearity and dispersion terms of the evolution equation can lead to clusterization in the final channel.

Certainly there are a lot of open questions relative to the presented approach. The most crucial concerns the generalization to 3+1 dimensional model with a finite temperature. One possible way to do it keeping symplicity of the approach would be to use an information theory [17], [18]. Such investigations are in progress.

IV. SUMMARY

The problem of dynamical instability and clusterization (stable fragments formation) in a breakup of excited nuclear systems are considered from the points of view of the soliton concept. It is noted that the volume (spinodal) instability can be associated with nonlinear terms, and the surface (Rayleigh-Taylor type) instability, with the dispersion terms in the evolution equations. The both instabilities above may compensate each other and lead to stable solutions (solitons).

The simple analytical model is presented to describe the time evolution of the cold compressed nuclear systems in the framework of the inverse mean-field method. It is demonstrated that the nonlinearity and dispersion terms of the evolution equations can lead to clusterization in the final channel.

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REFERENCES

[1] Kartavenko V.G., Particles and Nuclei. **24** (1993) 1469.

[2] Moretto L.G., Wozniak G.J., Ann. Rev. Nucl. & Part. Sci. **43** (1993) 379.

[3] Proceeding of the International Workshop on Dynamical Fluctuations and Correlations in Nuclear Collisions. Aussois. France. March 16-60. 1992; Nucl. Phys., **545** (1992).

[4] Knoll J., Strack B., Phys. Lett. **112B** (1984) 45.

[5] Burgio G.F., Ph. Chomaz, Randrup J., Phys. Rev. Lett. **69** (1992) 885

[6] Siemens, P.J., Nature, **305** (1983) 410.

[7] Kartavenko V.G., Clusters'93, 2nd Int. Conf., (Santorini, Greece). 1993, p.63.

[8] Moretto L.G., Tso K., Colonna N., Wozniak G.J. Phys. Rev. Lett., **69** (1992) 1184.

[9] Lord Rayleigh, Scientific Papers, Article 58, p.361, New York: Dover 1964.

[10] Stöcker H. et al., Prog. Part. Nucl. Phys., **4** (1980) 113.

[11] Stöcker H., Greiner W., Phys. Rep. **137** (1986) 277.

[12] Stöcker H, Greiner W., Phys. Rep. **160** (1988) 189.

[13] Kartavenko V., III Int. Conf. on Nucl.-Nucl. Collis. (San-Malo, France), 1988, p.142.

[14] Hefter E.F., Journ. de Phys., **45** (1984) C6:67.

[15] Hefter E.F., Kartavenko V.G., JINR Rapid. Comm., **3** (1987) 29.

[16] Dhar A., Das Gupta S., Phys. Rev., **C30** (1984) 1545.

[17] Gridnev K.A., Clusters'93, 2nd Int. Conf., (Santorini, Greece), 1993, p.51.

[18] Shannon C.E., Weaver W., The Mathematical Theory of Communication, University of Illinois Press, Urbana, Illinois, 1979.

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