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E4-94-158

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SOME ASPECTS OF THE OZI-RULE VIOLATION IN THE REACTION $\bar{p}p \to \phi\pi^0$

Submitted to «Zeitschrift für Physik»

1994

1 Introduction

The recent experimental data on the $\bar{p}p$ and $\bar{p}n$ annihilation at rest obtained by the ASTERIX, CRYSTAL BARREL and OBELIX groups[1, 2, 3] at LEAR, have shown that the branching ratios of the reactions $\bar{p}p \to \phi\pi^0$, $\bar{p}n \to \phi\pi^-$ and $\bar{p}p \to \phi\gamma$ are much bigger than expected from naive OZI-rule estimations. Indeed, let θ be the $\phi - \omega$ mixing angle such that the ω and ϕ states are constructed from the u, d and s quarks as follows:

$$\omega = \frac{1}{\sqrt{6}}(\sqrt{2}cos\theta + sin\theta)(u\bar{u} + d\bar{d}) + \frac{1}{\sqrt{3}}(cos\theta - \sqrt{2}sin\theta)s\bar{s},$$
$$\phi = \frac{1}{\sqrt{6}}(cos\theta - \sqrt{2}sin\theta)(u\bar{u} + d\bar{d}) - \frac{1}{\sqrt{3}}(\sqrt{2}cos\theta + sin\theta)s\bar{s} \qquad (1)$$

Then if θ takes the values $(36 \div 39)^0$ (see, for example, ref. [4]), the ϕ/ω production ratio takes the values $|(cos\theta - \sqrt{2}sin\theta)/(\sqrt{2}cos\theta + sin\theta)|^2 = (0.2 \div 4.2) \cdot 10^{-3}$ while in practice [1, 2, 3]

$$Br(\bar{p}p \to \phi\pi^0)/Br(\bar{p}p \to \omega\pi^0) = 0.14 \pm 0.04 \qquad (2)$$

$$Br(\bar{p}n \to \phi\pi^-)/Br(\bar{p}n \to \omega\pi^-) = 0.16 \pm 0.04 \quad (3)$$

$$Br(\bar{p}p \to \phi\gamma)/Br(\bar{p}p \to \omega\gamma) = 0.33 \pm 0.15 \quad (4)$$

As follows from the isotopic invariance, the reactions $\bar{p}p \to \phi\pi^0$ and $\bar{p}n \to \phi\pi^-$ can be easily related to each other (see, for example, refs. [5, 6]) and therefore any result on the first reaction can be immediately applied to the second one. As shown in ref. [7], the OZI-rule violation in the reaction $\bar{p}p \to \phi\gamma$ can be explained in the framework of the vector dominance model, and, as argued in refs. [6, 7], the OZI-rule violation in the reaction $\bar{p}p \to \phi\pi^0$ can be explained by the rescattering mechanism with K^*K and $\rho\rho$ mesons in the intermediate state.

In the present paper the possibility that the OZI-rule violation in the reaction $\bar{p}p \to \phi\pi^0$ can be explained by the rescattering mechanism is considered in detail. In Secs. 2 - 6 we consider two rescattering processes $\bar{p}p \to (\bar{K}^*K + K^*\bar{K}) \to K\bar{K}\pi \to \phi\pi^0$ and $\bar{p}p \to \rho^+\rho^- \to (\rho^+\pi^- + \rho^-\pi^+)\pi^0 \to \phi\pi^0$ and show that the model in which the $(\bar{K}^*K)$, $(K^*\bar{K})$ and $\rho^+\rho^-$ mesons are put on their mass shells (Model A) qualitatively explains the observed value of the branching ratio when the $\bar{p}p$ system at rest annihilates from the S state of the hydrogen like atom. In the framework of this model we consider in Secs. 7 and 8 the annihilation $\bar{p}p \to \phi\pi^0$ from the P state of the hydrogen like atom and the reaction $\bar{p}p \to f_2'\pi^0$. The results of the paper are discussed in Sec. 9.

2 The problem of calculating the process $\bar{p}p \to \phi\pi^0$ with K^*K intermediate states

Let us first consider the contribution of K^*K intermediate states. There exist four diagrams shown in Fig.1 and, as easily follows from the isotopic invariance, the contributions of these diagrams in the channel with the isospin $I = 1$ and spin $S = 1$ are equal to each other. To calculate these contributions we have to know the amplitudes of the reactions $\bar{p}p \to K^{*+}K^-$, $K^{*+} \to \pi^0 K^+$ and $K^+K^- \to \phi$ entering into the diagram a) of Fig. 1. We use p_1 and p_2 to denote the 4-momenta of the initial proton and antiproton respectively, k_1 and k_2 to denote the 4-momenta of the final π^0 and ϕ mesons respectively, k_1', k_2' and k_3' to denote the 4-momenta of the K^{*+}, K^- and K^+ mesons respectively, and e and e' to denote the polarization vectors of the ϕ and K^{*+}

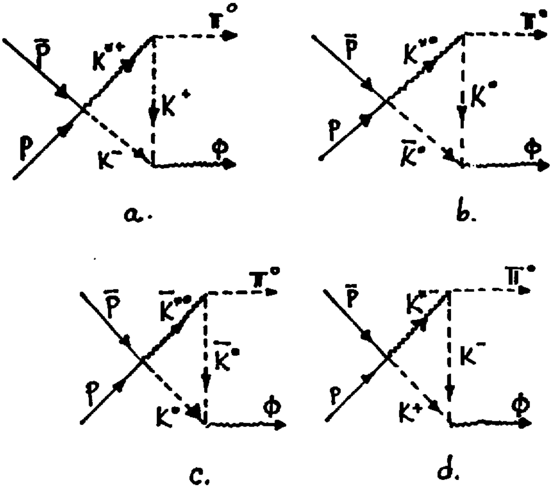


Fig.1

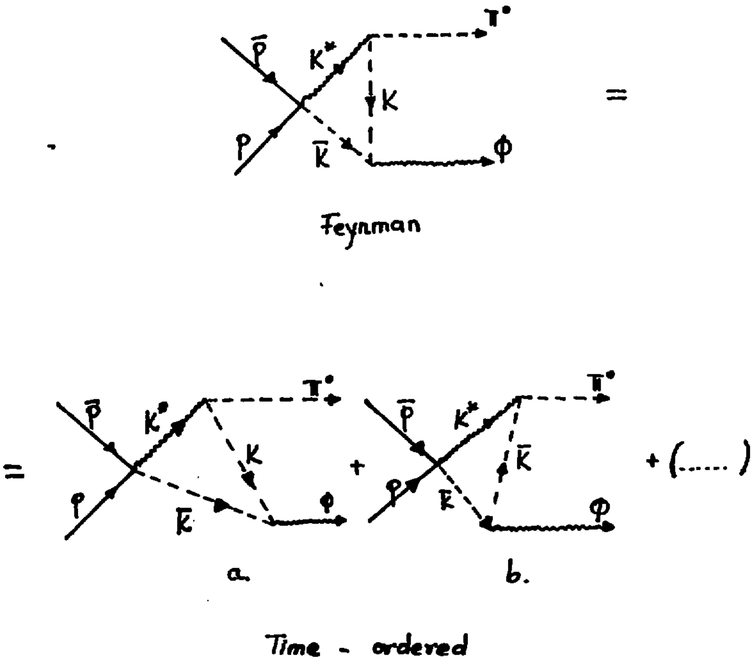


Fig.2

mesons respectively. The initial proton is described by the Dirac byspinor $u(p_1)$ and the initial antiproton is described by the Dirac byspinor with the negative energy $v(p_2)$. We also use m, m_π, m_K, m_* and m_ϕ to denote the proton mass and the masses of the corresponding mesons.

Consider the amplitude $\bar{p}p \to K^{*+}K^-$. If all particles are on-shell, the only amplitude in the channel with $I = S = 1$ which survives when the momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ are small is the following

$$M^{(11)}_{\bar{p}p\to K^{*+}K^-} = f^{(11)}_{\bar{p}p\to K^{*+}K^-}[\bar{v}(p_2)\gamma^\mu u(p_1)]e_{\mu\nu\rho\sigma}e'^{*\nu}k_1'^{\rho}k_2'^{\sigma} \qquad (5)$$

where $f_{\bar{p}p\to K^{*+}K^-}$ is some constant, $\mu,\nu,\rho,\sigma = 0,1,2,3$. $e_{\mu\nu\rho\sigma}$ is the absolutely antisymmetric tensor ($e_{0123} = -1$), γ^μ is the Dirac γ-matrix, and a sum over repeated indices is assumed. The total cross-section corresponding to the amplitude (5) can be calculated in a standard way and the result is

$$\sigma^{(11)}_{\bar{p}p\to K^{*+}K^-} = |f^{(11)}_{\bar{p}p\to K^{*+}K^-}|^2\frac{(3m^2+2p^2)k'^3}{12\pi p} \qquad (6)$$

where $\mathbf{p}$ is the proton momentum in the c.m.frame of the $\bar{p}p$ system, $p = |\mathbf{p}|$ and k' is the magnitude of the c.m.frame momentum for the $K^{*+}K^-$ system.

By analogy with Eqs. (5) and (6), the amplitude of the reaction $\bar{p}p \to \phi\pi^0$ has the form

$$M_{\bar{p}p\to\phi\pi^0} = f_{\bar{p}p\to\phi\pi^0}[\bar{v}(p_2)\gamma^\mu u(p_1)]e_{\mu\nu\rho\sigma}e^{*\nu}k_1^{\rho}k_2^{\sigma} \qquad (7)$$

where $f_{\bar{p}p\to\phi\pi^0}$ is some constant, and the total cross-section corresponding to the amplitude (7) has the form

$$\sigma_{\bar{p}p\to\phi\pi^0} = |f_{\bar{p}p\to\phi\pi^0}|^2\frac{(3m^2+2p^2)k^3}{12\pi p}. \qquad (8)$$

where k is the magnitude of the c.m.frame momentum for the $\phi\pi^0$ system.

The amplitude of the reaction $K^{*+} \to \pi^0K^+$ has the form

$$M_{K^{*+}\to\pi^0K^+} = f_{K^{*+}\to\pi^0K^+}(k_1 - k_3')_\mu e'^\mu \qquad (9)$$

and a standard calculation shows that the width of the decay is equal to

$$\Gamma_{K^{*+}\to\pi^0K^+} = \frac{|f_{K^{*+}\to\pi^0K^+}|^2k_{\pi K}^3}{6\pi m_*^2} \qquad (10)$$

where $k_{\pi K}$ is the magnitude of the 3-momentum in the c.m.frame of the πK system. If Γ_* is the total width of K^{*+} then it is easy to show that $\Gamma_* = 3\Gamma_{K^{*+}\to\pi^0K^+}$.

By analogy with Eqs. (9) and (10), the amplitude of the reaction $K^+K^- \to \phi$ is given by

$$M_{K^+K^-\to\phi} = f_{K^+K^-\to\phi}(k'_{2\mu} - k'_{3\mu})\epsilon^{\mu*}. \tag{11}$$

and the width of the decay $\phi \to K^+K^-$ is equal to

$$\Gamma_{\phi\to K^+K^-} = \frac{|f_{K^+K^-\to\phi}|^2 k^3_{KK}}{6\pi m^2_\phi} \tag{12}$$

where k_{KK} is the magnitude in the c.m.frame of the KK system. Since ϕ decays into KK in 87% cases it is easy to show that $2\Gamma_{\phi\to K^+K^-} = 0.87\Gamma_\phi$ where Γ_ϕ is the total width of ϕ.

Taking into account Eqs. (5), (9), (11) and the fact that all the four diagrams in Fig. 1 give equal contributions, we can write for the amplitude of the reaction $pp \to \phi\pi^0$

$$M_{pp\to\phi\pi^0} = 8i[v(p_2)\gamma^\mu u(p_1)]\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\lambda}k_1^\nu \cdot$$
$$\int f^{(11)}_{pp\to K^{*+}K^-} f_{K^{*+}\to\pi^0K^+} f_{K^+K^-\to\phi} k_1'^\rho k_2'^\sigma (k_2'^\lambda - k_3'^\lambda)\cdot$$
$$\frac{\delta^{(4)}(k_1' - k_1 - k_3')\delta^{(4)}(k_2 - k_2' - k_3')d^4k_1'd^4k_2'd^4k_3'}{(2\pi)^4[k_1'^2 - (m_* - i\Gamma_*/2)^2](k_2'^2 - m_K^2 + i0)(k_3'^2 - m_K^2 + i0)} \tag{13}$$

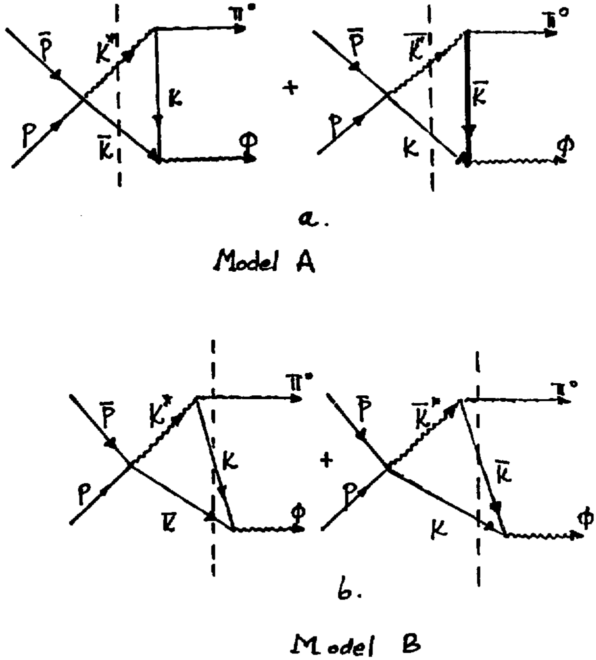
Let us note that the term with $k_1'^\nu k_1'^\beta$ in the propagator $\Pi^{\nu\beta} = (k_1'^\nu k_1'^\beta/m_*^2 - g_{\nu\beta})$ of the K^* meson ($g_{\nu\beta}$ is the metric tensor in Minkowski space) does not contribute to the amplitude (13) since $\epsilon_{\mu\nu\rho\sigma}k_1'^\nu k_1'^\rho = 0$ and for the same reason $k_1^\nu - k_3'^\nu$ can be replaced by $2k_1^\nu$. We have also taken into account that the K^* meson is the Breit-Wigner resonance and therefore the propagator of the K^* meson depends on the complex mass $(m_* - i\Gamma_*/2)$.

In the general case the quantities $f_{pp\to K^{*+}K^-}$, $f_{K^{*+}\to\pi^0K^+}$ and $f_{K^+K^-\to\phi}$ entering into Eq. (13) differ from the corresponding quantities in Eqs. (5), (9) and (11) since the K^{*+}, K^- and K^+ mesons are off shell. One might assume that the dependence of these quantities on the off shell form factor is not strong and neglect this dependence. However the integral in Eq. (13) strongly diverges in this case. Therefore we should either introduce the form factors "by hands" or try to estimate the amplitude (13) with the help of additional assumptions.

It is important to note that the covariant Feynman approach does not fully agree with our physical intuition that the process $pp \to \phi\pi^0$ can be described as $pp \to (K^*\bar{K} + \bar{K}^*K) \to K\bar{K}\pi \to \phi\pi^0$. As a rule, one Feynman diagram contains the contribution of a few diagrams of the "old fashioned" time ordered perturbation theory. In particular, the three vertices in the Feynman diagram in Fig. 1 are not necessarily time ordered as we assume. For example, the Feynman diagram in Fig. 2 contains the contributions of the diagrams a) and b) of the time ordered perturbation theory. The diagram a) indeed describes the process $pp \to \phi\pi^0$ as $pp \to (K^*\bar{K} + \bar{K}^*K) \to K\bar{K}\pi^0 \to \phi\pi^0$ while the diagram b) describes the unphysical process $pp \to K^*\bar{K} \to K^*\bar{K}\phi \to \phi\pi^0$ since the virtual K meson in this diagram decays into K and ϕ and then the interaction between K^* and $\bar{K}$ leads to the production of π^0.

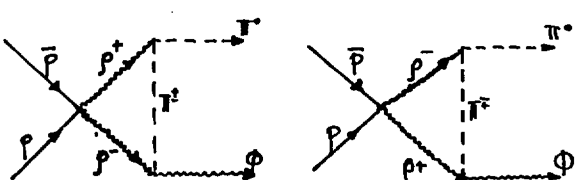
The difficulties with the interpretation of Feynman diagrams and with the divergence in Eq. (13) can be partly overcome if we assume that the main contribution to the integral in Eq. (13) is given by the residues in the poles of the propagators of some intermediate particles. According to our interpretation of the process $pp \to \phi\pi^0$ we choose two possibilities which we call Model A and Model B. In Model A we drop Γ_* in Eq. (13) and replace $[(k_1'^2 - m_*^2 + i0)(k_2'^2 - m_K^2 + i0)]^{-1}$ by $(-2i\pi)^2\theta(k_1'^0)\theta(k_2'^0)\delta(k_1'^2 - m_*^2)\delta(k_2'^2 - m_K^2)/2$. Analogously, in Model B we replace $[(k_2'^2 - m_K^2 + i0)(k_3'^2 - m_K^2 + i0)]^{-1}$ by $(-2i\pi)^2\theta(k_2'^0)\theta(k_3'^0)\delta(k_2'^2 - m_K^2)\delta(k_3'^2 - m_K^2)/2$. Schematically Model A can be described by Fig. 3a), i.e. K^* and $\bar{K}$ in the diagram of Fig. 2a) are on-mass shell. Analogously, Model B can be described by Fig. 3b), i.e. K and $\bar{K}$ mesons in the diagram of Fig. 2a) are on-mass shell.

From the theoretical point of view Model B seems more substantiated than Model A. Indeed, as shown in refs. [8, 9], the on-shell approximation is connected with the unitarity relation for the S-matrix but this relation must be formulated only in terms of stable particles. In particular, $K\bar{K}\pi^0$ is an admissible intermediate state while $K^*\bar{K}$ is not. In addition, the vertices $K^{*+} \to \pi^0K^+$ and $K^+K^- \to \phi$ entering into the amplitude $K^*\bar{K} \to \phi\pi^0$ in Model A are not necessarily time ordered and therefore this amplitude contains the contribution of not only the process $K^*\bar{K} \to K\bar{K}\pi^0 \to \phi\pi^0$ but also the contribution of the unphysical process $K^*\bar{K} \to K^*\bar{K}\phi \to \phi\pi^0$. However, as shown in refs. [6, 7], the numerical results in Model A are



$\bar{p}$
p
K^*
$\bar{K}$
K
π^0
Φ
+
a.
Model A
b.
Model B

Fig.3



$\bar{p}$
p
ρ^+
ρ^-
$\pi^{\pm}$
π^0
Φ

Fig.4

in qualitative agreement with the experimental data. For this reason we investigate below the consequences of both Model A and Model B.

3 The problem of calculating the process $\bar{p}p \to \phi\pi^0$ with $\rho^+\rho^-$ intermediate states

As shown in ref. [7], the $\rho^+\rho^-$ intermediate states may essentially contribute to the process $\bar{p}p \to \phi\pi^0$. There exist two diagrams describing the process $\bar{p}p \to \phi\pi^0$ via $\rho^+\rho^-$: $\bar{p}p \to \rho^+\rho^- \to \pi^+\pi^0\rho^- \to \phi\pi^0$ and $\bar{p}p \to \rho^+\rho^- \to \rho^+\pi^-\pi^0 \to \phi\pi^0$ (see Fig. 4) and the contributions of these diagrams are equal to each other if $I = S = 1$. To find these contributions we need the expressions defining the amplitudes $\bar{p}p \to \rho^+\rho^-$, $\rho^+ \to \pi^+\pi^0$ and $\rho^-\pi^+ \to \phi$.

When $I = S = 1$, the only amplitude which survives in the limit when $\mathbf{p}_1$ and $\mathbf{p}_2$ are small is the following.

$$M^{(11)}_{\bar{p}p\to\rho^+\rho^-} = f^{(11)}_{\bar{p}p\to\rho^+\rho^-}[\bar{v}(p_2)\gamma^\mu u(p_1)][e'^*_{1\mu}(Pe'^*_2) - e'^*_{2\mu}(Pe'^*_1)] \qquad (11)$$

where e'_i $(i = 1, 2)$ are the polarization 4-vectors of the ρ^+ and ρ^- mesons respectively and $P = p_1 + p_2$. We take into account that the C parity of the $\rho^+\rho^-$ system should be equal to -1. A standard calculation shows that the total cross-section $\sigma^{(11)}_{\bar{p}p\to\rho^+\rho^-}$ has the form

$$\sigma^{(11)}_{pp\to\rho^+\rho^-} = |f^{(11)}_{\bar{p}p\to\rho^+\rho^-}|^2 \frac{(3m^2 + 2p^2)(E_\rho^2 + m_\rho^2)k'^3}{6\pi p m_\rho^4} \qquad (15)$$

where now k' is the magnitude of the c.m.frame momentum in the $\rho^+\rho^-$ system, m_ρ is the mass of the ρ meson and $E_\rho = (m_\rho^2 + k'^2)^{1/2}$.

The amplitude $\rho^+ \to \pi^+\pi^0$ and the decay width of the ρ meson can be written by analogy with Eqs. (9) and (10):

$$M_{\rho^+\to\pi^+\pi^0} = f_{\rho^+\to\pi^+\pi^0}(k_1 - k'_3)_\mu e_1'^\mu, \quad \Gamma_{\rho^+\to\pi^+\pi^0} = \frac{|f_{\rho^+\to\pi^+\pi^0}|^2 k_{\pi\pi}^3}{6\pi m_\rho^2} \qquad (16)$$

where k_1 and k'_3 are the 4-momenta of π^0 and π^+ respectively and $k_{\pi\pi}$ is the magnitude of the 3-momentum in the c.m.frame of the $\pi\pi$ system.

The amplitude $\pi^+\rho^- \to \phi$ has the form

$$M_{\pi^+\rho^-\to\phi} = f_{\pi^+\rho^-\to\phi}\epsilon_{\alpha\beta\gamma\delta}e^{\alpha *}e_2'^{\beta}k_2^{\gamma}k_2'^{\delta} \qquad (17)$$

where k_2' is the 4-momentum of ρ^-. A direct calculation shows that the decay width $\Gamma_{\phi\to\pi^+\rho^-}$ is equal to

$$\Gamma_{\phi\to\pi^+\rho^-} = \frac{|f_{\phi\to\pi^+\rho^-}|^2 k_{\pi\rho}^3}{12\pi} \qquad (18)$$

where $k_{\pi\rho}$ is the magnitude of the c.m.frame momentum in the $\pi\rho$ system. Since ϕ decays into $\pi\rho$ in 12% cases it is obvious that $\Gamma_{\phi\to\pi^+\rho^-} = 0.12\Gamma_\phi/3$.

As follows from Eqs. (14), (16) and (17), the amplitude $pp \to \phi\pi^0$ corresponding to the Feynman diagrams in Fig.4 can be written in the form

$$M_{pp\to\phi\pi^0} = 2i[v(p_2)\gamma^\mu u(p_1)]\epsilon_{\alpha\beta\gamma\delta}e^{\alpha *}k_2^{\gamma}P_\nu \int f_{pp\to\rho^+\rho^-} f_{\rho^+\to\pi^+\pi^0}\cdot$$

$$f_{\pi^+\rho^-\to\phi}k_2'^{\delta}(k_1-k_3')^{\rho}[(\frac{k'_{1\mu}k'_{1\rho}}{m_\rho^2}-g_{\mu\rho})g_{\nu\beta} - (\frac{k'_{1\nu}k'_{1\rho}}{m_\rho^2}-g_{\nu\rho})g_{\mu\beta}]\cdot$$

$$\frac{\delta^{(4)}(k_1'-k_1-k_3')\delta^{(4)}(k_2-k_2'-k_3')d^4k_1'd^4k_2'd^4k_3'}{(2\pi)^4[k_1'^2-(m_\rho-i\Gamma_\rho/2)^2][k_2'^2-(m_\rho-i\Gamma_\rho/2)^2](k_3'^2-m_\pi^2+i0)} \qquad (19)$$

As in Eq. (13), the integral in Eq. (19) diverges if no form factors are introduced into the vertices $pp \to \rho^+\rho^-$, $\rho^+ \to \pi^+\pi^0$ and $\rho^-\pi^+ \to \phi$. By analogy with Sec. 2 we use the on-shell approximation where the intermediate states are either $\rho^+\rho^-$ or $\rho\pi\pi$. We again call the corresponding models as Model A and Model B respectively. These models correspond to the cuts of the Feynman diagrams as shown in Fig.5.

4 The contribution of K^*K intermediate states in Model A

As follows from the prescription described in Sec. 2, Eq. (13) in Model A reads

$$M_{pp\to\phi\pi^0} = -8i[v(p_2)\gamma^\mu u(p_1)]\epsilon_{\mu\nu\rho\sigma}e^{*\lambda}k_1^\nu k_2^\rho f^{(11)}_{pp\to K^{*+}K^-}\cdot$$

$$\int f_{K^{*+}\to\pi^0K^+} f_{K^+K^-\to\phi} k_2'^{\sigma} k_2'^{\lambda} \theta(k_1'^0)\theta(k_2'^0)\delta(k_1'^2 - m_*^2)\delta(k_2'^2 - m_K^2)\cdot$$

$$\frac{\delta^{(4)}(k_1 + k_2 - k_1' - k_2')d^4k_1'd^4k_2'}{(2\pi)^2[(k_1' - k_1)^2 - m_K^2 + i0]} \tag{20}$$

where we have taken into account that $(k_{2\lambda}\epsilon^{\lambda}) = 0$. The quantity $f^{(11)}_{pp\to K^{*+}K^-}$ in this expression is the same as in Eq. (5) since K^* and K are on-mass shell.

It is convenient to consider Eq. (20) in the c.m.frame of the pp system which, at the same time, is the c.m.frame of the K^*K and $\phi\pi^0$ systems. The vector P in this frame of reference has the components $P^0 = \sqrt{s}$, $\mathbf{P} = 0$ and therefore Eq. (20) can be written in the form

$$M_{pp\to\phi\pi^0} = \frac{-i}{4\pi^2 k} f^{(11)}_{pp\to K^{*+}K^-}[\bar v(p_2)\gamma^i u(p_1)]\epsilon_{ikl}k^k \cdot$$

$$\int do' f_{K^{*+}\to\pi^0K^+} f_{K^+K^-\to\phi}\frac{k'^l(E_K\epsilon^{0*} + k'^m\epsilon^{m*})}{a - x} \tag{21}$$

where $a = (2E_*E_\pi + m_K^2 - m_*^2 - m_\pi^2)/2kk'$, $E_\pi = (m_\pi^2 + k^2)^{1/2}$, $E_* = (m_*^2 + k'^2)^{1/2}$, $E_K = (m_K^2 + k'^2)^{1/2}$, $k = |\mathbf{k}|$, $k' = |\mathbf{k}'|$, $\mathbf{k} \equiv \mathbf{k}_1$, $\mathbf{k}' \equiv \mathbf{k}_1'$, $\mathbf{n} = \mathbf{k}/k$, $\mathbf{n}' = \mathbf{k}'/k'$, $x = \mathbf{n}\mathbf{n}'$, do' is the element of the solid angle corresponding to the unit vector $\mathbf{n}'$ and a sum over repeated indices $i, k, l, m = 1, 2, 3$ is assumed.

Let us consider the integrals

$$I^l = \int f(x,s)k'^l do', \quad I^{lm} = \int f(x,s)k'^l k'^m do' \tag{22}$$

where $f(x,s)$ is an arbitrary function of x and s. It is easy to show that

$$I^l = 2\pi\frac{k'}{k}k^l\int_{-1}^{1} f(x,s)dx,$$
$$I^{lm} = \pi(k')^2\int_{-1}^{1} f(x,s)[(1 - x^2)\delta^{lm} + (3x^2 - 1)\frac{k^l k^m}{k^2}]dx \tag{23}$$

where δ^{lm} is the Cronecker symbol. Then as follows from Eqs. (7), (21-23)

$$f_{pp\to\phi\pi^0} = \frac{i(k')^2}{4\pi k\sqrt{s}} f^{(11)}_{pp\to K^{*+}K^-}\int_{-1}^{1} f_{K^{*+}\to\pi^0K^+}(k_3'^2) f_{K^+K^-\to\phi}(k_3'^2)\frac{1 - x^2}{a - x}dx \tag{24}$$

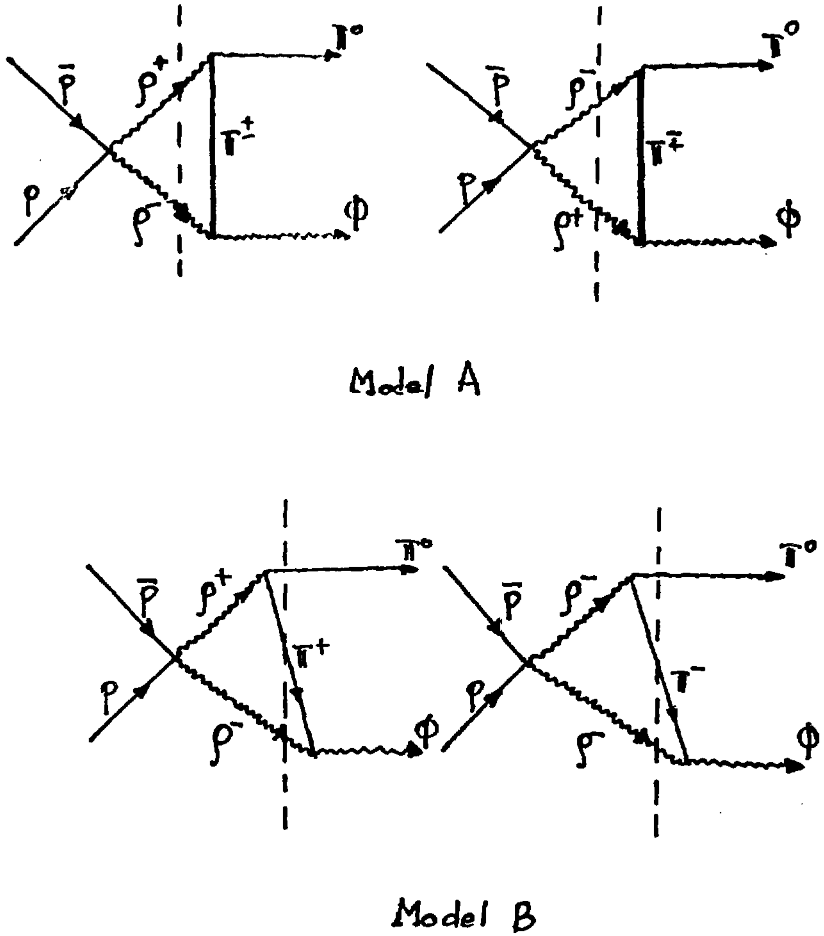


Fig.5

We explicitly note that $f_{K^{*+}\to\pi^0 K^+}$ and $f_{K^+K^-\to\phi}$ depend on the off-shell form factor for the K meson with the 4-momentum k_3'. The importance of taking into account this form factor has been pointed out in refs. [10, 7]. Following these references we write

$$f_{K^{*+}\to\pi^0 K^+}(k_3'^2) = f_{K^{*+}\to\pi^0 K^+}\frac{\Lambda - m_K^2}{\Lambda - k_3'^2}; \quad f_{K^+K^-\to\phi}(k_3'^2) = f_{K^+K^-\to\phi}\frac{\Lambda - m_K^2}{\Lambda - k_3'^2} \tag{25}$$

where now the quantities $f_{K^{*+}\to\pi^0 K^+}$ and $f_{K^+K^-\to\phi}$ are the same as in Eqs. (7) and (11). Then we get from Eq. (24) the final result

$$f_{\bar{p}p\to\phi\pi^0} = \frac{i(k')^2}{4\pi k\sqrt{s}} f^{(11)}_{\bar{p}p\to K^{*+}K^-} f_{K^{*+}\to\pi^0 K^+} f_{K^+K^-\to\phi} \cdot$$

$$\int_{-1}^{1} \frac{1-x^2}{a-x}\left[\frac{\Lambda - m_K^2}{\Lambda + 2E_* E_\pi - m_*^2 - m_\pi^2 - 2kk'x}\right]^2 dx \tag{26}$$

As follows from Eqs. (6), (8), (10), (12) and (26)

$$R \equiv \frac{\sigma_{\bar{p}p\to\phi\pi^0}}{\sigma^{11}_{\bar{p}p\to K^{*+}K^-}} = 0.87 \cdot \frac{3}{8}\frac{kk'\Gamma_*\Gamma_\phi m_*^2 m_\phi^2}{s(k_{\pi K}k_{KK})^3} \cdot$$

$$\left|\int_{-1}^{1} \frac{1-x^2}{a-x}\left[\frac{\Lambda - m_K^2}{\Lambda + 2E_* E_\pi - m_*^2 - m_\pi^2 - 2kk'x}\right]^2 dx\right|^2 \tag{27}$$

Since for the amplitudes $\bar{p}p \to K^{*+}K^-$ and $\bar{p}p \to \phi\pi^0$ we assume the structure defined by Eqs. (5) and (7), Eq. (27) can be valid only if the value of p is rather small. In Fig.6 we show the dependence of R on the laboratory momentum p_{lab} in the range $(0 \div 0.4) GeV/c$ what corresponds to the values of p in the range $(0 \div 0.2) GeV/c$. Following refs. [10, 7] we choose for Λ the values of $1.2 GeV^2$, $2 GeV^2$ and $\Lambda = \infty$ what means the absence of the off-shell form factors. We see that Model A predicts that R practically does not depend on p_{lab} in the range $0 - 0.4 GeV/c$.

In refs. [1, 2] the branching ratio of the reaction $\bar{p}p \to \phi\pi^0$ has been measured not for the annihilation in flight but for the annihilation at rest from the S state of the hydrogen-like $\bar{p}p$ atom. When $p \to 0$ only the contribution of the S wave survives in Eq. (27). Assuming that the $\bar{p}p$ system in the hydrogen-like atom is unpolarized and taking for the branching ratio $BR(\bar{p}p \to K^{*+}K^-)^{(11)}$ its experimental value $5.85 \cdot 10^{-4}$ [11] we get for the

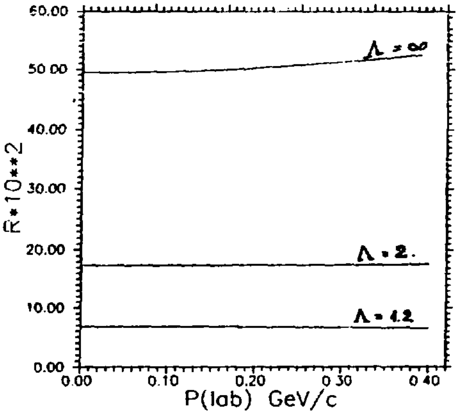

60.00
50.00
40.00
30.00
20.00
10.00
0.00
R*10**2
0.00
0.10
0.20
0.30
0.40
P(lab) GeV/c
Λ = ∞
Λ = 2.
Λ = 4.2

Fig.6

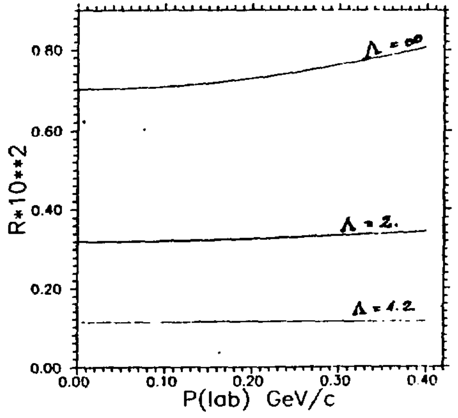

0.80
0.60
0.40
0.20
0.00
R*10**2
0.00
0.10
0.20
0.30
0.40
P(lab) GeV/c
Λ = ∞
Λ = 2.
Λ = 4.2

Fig.7

branching ratio $BR(pp \to \phi\pi^0)$ the values $2.9 \cdot 10^{-4}$, $0.99 \cdot 10^{-4}$ and $0.1 \cdot 10^{-4}$ for $\Lambda = \infty$, $\Lambda = 2GeV^2$ and $\Lambda = 1.2GeV^2$ respectively. According to ref. [2] $BR(pp \to \phi\pi^0) = (7.8 \pm 0.4) \cdot 10^{-4}$. We conclude that if the off-shell form factor for the K meson does not strongly depend on k'_3 then the contribution of K^*K intermediate states in Model A is in qualitative agreement with experimental data.

5 The contribution of $\rho^+\rho^-$ intermediate states in Model A

The calculation of this contribution is analogous to the calculation in the preceding section. Using Eqs. (14), (16), (17), (22) and (23) we get

$$f_{pp\to\phi\pi^0} = \frac{i(k')^3}{8\pi m_\rho^2\sqrt{s}} f^{(11)}_{pp\to\rho^+\rho^-} f_{\rho^+\to\pi^+\pi^0} f_{\pi^+\rho^-\to\phi} F(s) \tag{28}$$

where

$$F(s) = \int_{-1}^{1} [(1-x^2)(E_\rho E_\pi - kk'x) + 2E_\rho(\frac{E_\rho kx}{k'} - E_\pi) -$$

$$-2xE_\phi(E_\rho x - E_\pi \frac{k'}{k})][\frac{\Lambda - m_\pi^2}{\Lambda + 2E_\rho E_\pi - 2kk'x - m_\rho^2 - m_\pi^2}]^2 \cdot$$

$$\cdot \frac{dx}{2E_\rho E_\pi - 2kk'x - m_\rho^2 - i0} \tag{29}$$

In contrast with the K^*K case, now the kinematical conditions are such that all the three intermediate particles can be on-mass shell in contradiction with the Peierls theorem [12]. In turn, this theorem follows from the fundamental fact that the S-matrix can be formulated only in terms of stable particles. However such a situation is only a formal difficulty which takes place because we drop Γ_ρ in the propagators of the ρ^+ and ρ^- mesons and treat these mesons as stable particles.

As follows from Eqs. (15), (16), (18) and (28)

$$R_1 = \frac{\sigma_{pp\to\phi\pi^0}}{\sigma^{(11)}_{pp\to\rho^+\rho^-}} = 0.12\frac{3}{4}(\frac{kk'}{k_{\pi\rho}k_{\pi\pi}})^3 \frac{\Gamma_\rho\Gamma_\phi m_\rho^2}{s(s+4m_\rho^2)}|F(s)|^2 \tag{30}$$

In Fig.7 the result for R_1 as a function of p_{lab} is shown for the cases $\Lambda = 1.2GeV^2$, $\Lambda = 2GeV^2$ and $\Lambda = \infty$. We again see that the dependence of R_1 on p_{lab} is weak. If $p_{lab} = 0$ then $R_1 = 1.13\ 10^{-3}$, $R_1 = 3.2\ 10^{-3}$ and $R_1 = 7.01\ 10^{-3}$ for these three cases respectively. The experimental value of $BR(pp \to \rho^+\rho^-)^{(11)}$ at rest is unknown, but the theoretical model developed in ref. [13] predicts the value of $23.6 \cdot 10^{-3}$. Then the contribution of $\rho^+\rho^-$ intermediate states to $BR(\bar{p}p \to \phi\pi^0)$ at rest is $1.6\ 10^{-4}$ if $\Lambda = \infty$. Therefore, as first noted in ref. [7], Model A predicts a rather substantial contribution of $\rho^+\rho^-$ intermediate states to the branching ratio of the reaction $\bar{p}p \to \phi\pi^0$.

6 The contribution of $K\bar{K}\pi^0$ and $\rho\pi\pi^0$ intermediate states in Model B

As follows from the prescription described in Sec. 2, Eq. (13) in Model B reads

$$M_{pp\to\phi\pi^0} = 4i f^{(11)}_{\bar{p}p\to K^{*+}K^-} f_{K^{*+}\to\pi^0 K^+} f_{K^+K^-\to\phi}[v(p_2)\gamma^\mu u(p_1)]e_{\mu\nu\rho\sigma}e^{*\lambda}k_1^\nu \cdot$$

$$\int \frac{k_3'^\rho k_2'^\sigma (k_2'^\lambda - k_3'^\lambda)\delta^{(4)}(k_2 - k_2' - k_3')d^3\mathbf{k}_2' d^3\mathbf{k}_3'}{16\pi^2\omega_K(\mathbf{k}_2')\omega_K(\mathbf{k}_3')[(k_1 + k_3')^2 - (m_* - i\Gamma_*/2)^2]} \tag{31}$$

where $\omega_K(\mathbf{k}) = (m_K^2 + \mathbf{k}'^2)^{1/2}$, we take into account that the constants $f_{K^{*+}\to\pi^0 K^+}$ and $f_{K^+K^-\to\phi}$ are the same as in Eqs. (9) and (11), and no form factor is introduced into the vertex $\bar{p}p \to K^*\bar{K}$.

It is obvious that

$$e_{\mu\nu\rho\sigma}k_3'^\rho k_2'^\sigma = e_{\mu\nu\rho\sigma}(k_2'^\rho + k_3'^\rho)(k_2'^\sigma - k_3'^\sigma)/2$$

and therefore Eq. (31) can be written in the form

$$M_{pp\to\phi\pi^0} = 2i f^{(11)}_{\bar{p}p\to K^{*+}K^-} f_{K^{*+}\to\pi^0 K^+} f_{K^+K^-\to\phi}[\bar{v}(p_2)\gamma^\mu u(p_1)]e_{\mu\nu\rho\sigma}e^{*\lambda}k_2^\rho k_1^\sigma I_{\sigma\lambda} \tag{32}$$

where $I_{\sigma\lambda}$ is the relativistic symmetric tensor

$$I_{\sigma\lambda} = \int \frac{(k_2'^\sigma - k_3'^\sigma)(k_2'^\lambda - k_3'^\lambda)\delta^{(4)}(k_2 - k_2' - k_3')d^3\mathbf{k}_2' d^3\mathbf{k}_3'}{16\pi^2\omega_K(\mathbf{k}_2')\omega_K(\mathbf{k}_3')[(k_1 + k_3')^2 - (m_* - i\Gamma_*/2)^2]} \tag{33}$$

This tensor depends only on k_1 and k_2 and therefore the general form of $I_{\sigma\lambda}$ is

$$I_{\sigma\lambda} = c_1 g_{\sigma\lambda} + c_2 k_{1\sigma} k_{1\lambda} + c_3 k_{2\sigma} k_{2\lambda} + c_4 (k_{1\sigma} k_{2\lambda} + k_{2\sigma} k_{1\lambda}) \qquad (34)$$

It is obvious that only $c_1 g_{\sigma\lambda}$ contributes to Eq. (32). The simplest way of calculating c_1 is to consider Eq. (33) in the reference frame where the final ϕ meson is at rest. The magnitude of the pion momentum in this reference frame is $q = (\sqrt{s}k)/m_\phi$ and, as follows from Eqs. (33) and (34):

$$\frac{k_{K\bar{K}}}{4\pi^2 m_\phi} \int \frac{do' k'^i k'^l}{m_\pi^2 + m_K^2 + m_\phi (m_\pi^2 + q^2)^{1/2} + 2q k_{K\bar{K}} x - (m_* - i\Gamma_*/2)^2} =$$

$$-c_1 \delta_{il} + c_2 q_i q_l \qquad (35)$$

where $\mathbf{q}$ is the pion momentum, $\mathbf{k}'$ is the momentum of the K meson, $x = \mathbf{q}\mathbf{k}'/q k_{K\bar{K}}$ and we integrate over the solid angle corresponding to the unit vector $\mathbf{n} = \mathbf{k}'/k_{K\bar{K}}$. Then the quantity c_1 can be easily calculated by analogy with the calculation of the quantity c_1 in Sec.4 and, the final result for $f_{pp\to\phi\pi^0}$ is:

$$f_{\bar{p}p\to\phi\pi^0} = -i f^{(11)}_{\bar{p}p\to K^{*+}K^-} f_{K^{*+}\to K^+\pi^0} f_{K^+K^-\to\phi} \frac{(k_{K\bar{K}})^2}{4\pi\sqrt{s}k} [2b + (1-b^2) ln(\frac{b+1}{b-1})] \qquad (36)$$

where $b = [m_\pi^2 + m_K^2 + m_\phi(m_\pi^2 + q^2) - (m_* - i\Gamma/2)^2]/2q k_{K\bar{K}}$ and we have taken into account that:

$$\int_{-1}^{1} \frac{(1-x^2)dx}{b-x} = 2b + (1-b^2) ln(\frac{b+1}{b-1}) \qquad (37)$$

By analogy with the derivation of Eq. (27) we now get:

$$\frac{\sigma_{\bar{p}p\to\phi\pi^0}}{\sigma^{(11)}_{\bar{p}p\to K^{*+}K^-}} = 0.87 \frac{3}{8} \frac{k k_{K\bar{K}} \Gamma_* \Gamma_\phi m_*^2 m_\phi^2}{s k_{\pi K}^3 k'^3} |2b + (1-b^2) ln(\frac{b+1}{b-1})|^2 \qquad (38)$$

A simple numerical calculation shows that, if $s = 4m^2$ then $\sigma_{pp\to\phi\pi^0} \approx 10^{-4} \cdot \sigma^{(11)}_{\bar{p}p\to K^{*+}K^-}$. Therefore the contribution of $K\bar{K}\pi^0$ intermediate states in Model B is negligible.

Let us now consider the contribution of $(\rho^+\pi^- + \rho^-\pi^+)\pi^0$ intermediate states in Model B. In this model Eq. (19) reads:

$$f_{pp\to\phi\pi^0}[v(p_2)\gamma^\mu u(p_1)]\epsilon_{\mu\nu\rho\sigma}\epsilon^{\nu *}k_1^\rho k_2^\sigma =$$

$$-if^{(11)}_{pp\to\rho^+\rho^-}f_{\rho^+\to\pi^+\pi^0}f_{\pi^+\rho^-\to\phi}[v(p_2)\gamma^\mu u(p_1)]\epsilon_{\alpha\beta\gamma\delta}\epsilon^{\alpha *}k_2^\gamma$$

$$\int\frac{(2\pi)^4\delta^{(4)}(k_2-k_2'-k_3)d^3\mathbf{k_2'}d^3\mathbf{k_3'}}{[2(2\pi)^3]^2\omega_\rho(\mathbf{k_2'})\omega_\pi(\mathbf{k_3'})[(k_1+k_3')^2-(m_\rho-i\Gamma_\rho/2)^2]}$$

$$k_2'^\delta[(k_1-k_3')_\mu P_\beta - g_{\mu\beta}(P,k_1-k_3')] \qquad (39)$$

where $\omega_\rho(\mathbf{k}') = (m_\rho^2+\mathbf{k}'^2)^{1/2}$, $\omega_\pi(\mathbf{k}') = m_\pi^2+\mathbf{k}'^2)^{1/2}$.

It is obvious that

$$\int\frac{(2\pi)^4\delta^{(4)}(k_2-k_2'-k_3)d^3\mathbf{k_2'}d^3\mathbf{k_3'}}{[2(2\pi)^3]^2\omega_\rho(\mathbf{k_2'})\omega_\pi(\mathbf{k_3'})[(k_1+k_3')^2-(m_\rho-i\Gamma_\rho/2)^2]}k_2'^\delta k'^\mu =$$

$$c_1g^{\mu\delta}+c_2k_1^\mu k_1^\delta+c_3k_2^\mu k_2^\delta+c_4k_1^\mu k_2^\delta+c_5k_2^\mu k_1^\delta \qquad (40)$$

where the c_i $(i=1,\ldots,5)$ are some relativistically invariant quantities. As follows from Eq. (39), we have to calculate only c_1, c_2 and c_5. It is convenient to calculate these quantities in the reference frame where the final ϕ meson is at rest and use Eqs. (23). The final result is the following (compare with Eq. (30))

$$\frac{\sigma_{pp\to\phi\pi^0}}{\sigma^{(11)}_{pp\to\rho^+\rho^-}} = 0.12\frac{3}{16}\frac{kk_{\pi\rho}}{k'^3k^3_{\pi\pi}}\frac{\Gamma_\rho\Gamma_\phi m_\rho^6}{s(E_\rho^2+m_\rho^2)}|F_1(s)|^2 \qquad (41)$$

where , as in Eq. (30), k' is the magnitude of the c.m. frame momentum in the $\rho^+\rho^-$ system and

$$F_1(s) = \int_{-1}^{1}\frac{dx}{2m_\pi^2+2\omega_\pi(k_{\pi\rho})+2qk_{\pi\rho}x-(m_\rho-i\Gamma_\rho/2)^2}$$

$$\{\frac{1}{2}(s-m_\phi^2)[x-\frac{k_{\pi\rho}}{2q}(1-3x^2)]-\frac{1}{2}(s+m_\phi^2)$$

$$[\frac{\omega_\pi(k_{\pi\rho})x}{m_\phi}-\frac{\omega_\pi(q)k_{\pi\rho}}{2m_\phi q}(1-3x^2)]-k_{\pi\rho}q(1-x^2)\} \qquad (42)$$

A simple numerical calculation shows that if $s = 4m^2$, then Eq. (11) can be written as

$$\sigma_{pp \to \phi\pi^0} = 3.13 \cdot 10^{-5}\, \sigma^{(11)}_{pp \to \rho^+\rho^-} \tag{13}$$

Therefore, if we again assume that $\sigma^{(11)}_{pp \to \rho^+\rho^-} = 23.6 \cdot 10^{-3}$ [13] then, the $(\rho^+\pi^- + \rho^-\pi^+)\pi^0$ intermediate states in Model B don't play an important role.

7 The relation between the branching ratios of the reactions $\bar{p}p \to \phi\pi^o$ and $\bar{p}p \to K^*\bar{K}$ in the annihilation from the P state of the hydrogen like $\bar{p}p$ atom

In contrast with the annihilation $pp \to \phi\pi^0$ from the S state of the hydrogen like pp atom, the branching ratio of this annihilation from the P state is small and the reaction $\bar{p}p \to \phi\pi^o$ from the P state was not observed as yet. The data on the annihilation $pp \to K^*K$ from the P state are also much more scarce that for the annihilation from the S state , but experiments which are under way are expected to give a more detailed information on the pp annihilation from the P state. In view of the above discussion it is interesting to investigate what is the prediction of Model A for the ratio of the rates of the reactions $pp \to \phi\pi^o$ and $pp \to K^*K$ in the annihilation from P state. More exactly, since the annihilation $pp \to \phi\pi^o$ from the P state can go only in the channel with $I = 1$, $S = 0$, Model A makes it possible to give predictions on the quantity $Br(pp \to \phi\pi^0)/Br(K^{*+}K^-)^{(10)}$.

To describe the relativistically invariant amplitude for the annihilation $pp \to \phi\pi^0$ from the P state we have to construct the relativistic wave function describing the pp system not in the case when the antiproton and proton have definite momenta, but when they have the definite quantum numbers $L = 1$, $S = 0$. However since we need only the ratio of the quantities $BR(\bar{p}p \to \phi\pi^)$ and $Br(pp \to K^{*+}K^-)^{(10)}$, the following procedure can be used. We again describe the antiproton and proton by the Dirac byspinors and write such relativistically invariant amplitudes $pp \to \phi\pi^0$ and $pp \to K^{*+}K^-$ which are of order $|\mathbf{p}|/m$ when $|\mathbf{p}| \to 0$. Therefore, when $|\mathbf{p}| \to 0$ the leading contribution to the corresponding cross-sections are given by the P states and these cross-sections are also of order $|\mathbf{p}|/m$. However the ratio $\sigma_{\bar{p}p\to\phi\pi^0}/\sigma^{(10)}_{\bar{p}p\to K^{*+}K^-}$ when $|\mathbf{p}|\to 0$ becomes just the ratio of the quantities $BR(\bar{p}p\to\phi\pi^0)$ and $BR(\bar{p}p\to K^{*+}K^-)$ in the annihilation from the P state of the hydrogen like $\bar{p}p$ atom if we assume that p and p in this state are unpolarized.

The general form of the amplitude $\bar{p}p\to\phi\pi^0$ with the needed properties is the following

$$M_{pp\to\phi\pi^0} = [\bar{v}(p_2)\gamma^5 u(p_1)][F_1'(p_1-p_2,e^*) + \frac{F_2'}{m_\phi^2}(p_1-p_2,k_1-k_2)(k_1-k_2,e^*)] \tag{44}$$

where F_1' and F_2' become constants when $|\mathbf{p}|\to 0$. In contrast with the annihilation from the S state the amplitude given by Eq. (44) is defined by two unknown constants since the final $\phi\pi^0$ system has the orbital angular momentum either $L=0$ or $L=2$.

It is convenient to consider the amplitude (44) in the c.m.frame. Then we can write

$$M_{\bar{p}p\to\phi\pi^0} = [\bar{v}(p_2)\gamma^5 u(p_1)][F_1(\mathbf{pe}^*) + \frac{F_2}{m_\phi^2}(\mathbf{pk})(\mathbf{ke}^*)] \tag{45}$$

where F_1 and F_2 are the linear combinations of F_1' and F_2'. Analogously we can write

$$M^{(10)}_{pp\to K^{*+}K^-} = [\bar{v}(p_2)\gamma^5 u(p_1)][f_1(\mathbf{pe}^*) + \frac{f_2}{m_*^2}(\mathbf{pk}')(\mathbf{k}'\mathbf{e}'^*)] \tag{46}$$

where f_1 and f_2 are another constants. As easily follows from Eqs. (45) and (46)

$$R_2 = \frac{Br(\bar{p}p\to\phi\pi^0)_{L=1}}{Br(\bar{p}p\to K^{*+}K^-)^{(10)}_{L=1}} =$$

$$\frac{k[|F_1|^2(1+\frac{k^2}{3m_\phi^2}) + \frac{k^2}{3m_\phi^2}(1+\frac{k^2}{m_\phi^2})(F_1F_2^* + F_1^*F_2 + \frac{k^2}{m_\phi^2}|F_2|^2])}{k'[|f_1|^2(1+\frac{k'^2}{3m_*^2}) + \frac{k'^2}{3m_*^2}(1+\frac{k'^2}{m_*^2})(f_1f_2^* + f_1^*f_2 + \frac{k'^2}{m_*^2}|f_2|^2])} \tag{47}$$

By analogy with the derivation in Sec. 4 we obtain that in Model A

$$M_{\bar{p}p\to\phi\pi^0} = \frac{-ik'}{2\pi^2\sqrt{s}}[\bar{v}(p_2)\gamma^5 u(p_1)]f_{K^{*+}\to\pi^0K^+}f_{K^+K^-\to\phi\mathbf{P}}$$

$$\int \frac{do'(k_{2\lambda}e^{\lambda_*})}{(k_1'-k_1)^2-m_K^2}[f_1(\frac{\mathbf{k}'(k_1k_1')}{m_*^2}-\mathbf{k})+\frac{f_2}{m_*^2}\mathbf{k}'(\frac{(k')^2(k_1k_1')}{m_*^2}-\mathbf{k}\mathbf{k}')](48)$$

Since the relation between the reactions $\bar{p}p \to \phi\pi^0$ and $\bar{p}p \to K^{*+}K^-$ in the annihilation from the S state can be qualitatively explained assuming that the off shell form factors in the vertices $K^{*+} \to \pi^0 K^+$ and $K^+K^- \to \phi$ don't considerably diminish the amplitude $\bar{p}p \to \phi\pi^0$, we don't take into account the contribution of these form factors.

Using Eq. (23) we can derive the relation between the quantities F_i and f_i $(1 = 1,2)$, and the final result is the following

$$F_i = \frac{ik'}{\pi\sqrt{s}} f_{K^{*+}\to\pi^0K^+} f_{K^+K^-\to\phi} \sum_{l=1}^{2} A_{il} f_l \tag{49}$$

where

$$\begin{aligned}
A_{11} &= \frac{k'}{4km_*^2}\int_{-1}^{1}\frac{(1-x^2)(E_*E_\pi-kk'x)dx}{a-x}\\
A_{12} &= \frac{k'^2}{4km_*^2}\int_{-1}^{1}[\frac{k'(E_*E_\pi-kk'x)}{m_*^2}-kx]\frac{(1-x^2)dx}{a-x}\\
A_{21} &= \frac{m_\phi^2}{2kk'}\int_{-1}^{1}\{\frac{(E_*E_\pi-kk'x)}{m_*^2}[-\frac{E_Kk'x}{E_\phi k}+\frac{k'^2(3x^2-1)}{2k^2}]+\\
&\quad \frac{E_K}{E_\phi}-\frac{k'x}{k}\}\frac{dx}{a-x}\\
A_{22} &= \frac{m_\phi^2k'}{2m_*^2k^2}\int_{-1}^{1}[\frac{k'(E_*E_\pi-kk'x)}{m_*^2}-kx]\\
&\quad [-\frac{E_Kx}{E_\phi}+\frac{k'(3x^2-1)}{2k}]\frac{dx}{a-x}
\end{aligned} \tag{50}$$

As follows from simple numerical calculations and Eqs. (10), (12), (47), (49) and (50)

$$R_2 = \frac{0.77+0.36yz+0.044y^2}{1.16+0.46yz+0.11y^2} \tag{51}$$

where $y = |f_2/f_1|$ and z is the cosine of the relative phase of the quantities f_1 and f_2. If $f_2 = 0$ then $R_2 = 0.66$ and if $f_1 = 0$ then $R_2 = 0.40$. However in the general case the quantity R_2 can take the values from $R_{min} = 0.02$ when $y = 1.2$, $z = -1$ to $R_{max} = 0.67$ when $y = 0.7$, $z = 1$. In addition, if we take into account a possible contribution of the off shell form factors then we can conclude that the quantities $Br(pp \to \phi\pi^0)_{L=1}$ and $Br(pp \to K^{*+}K^-)^{(10)}_{L=1}$ are probably of the same order of magnitude but, at the same time one cannot exclude the possibility that the first quantity is much smaller that the second.

8 The problem of the OZI-rule violation in the reaction $\bar{p}p \to f_2'\pi^0$

The situation with the $f_2 - f_2'$ mixing is analogous to that with the $\omega - \phi$ mixing, but the mixing angle isn't so close to the ideal one: according to ref. [4], $cos\theta = 0.78$. Therefore, as follows from the OZI-rule and Eq. (1), the ratio $BR(pp \to f_2'\pi^0)/BR(pp \to f_2\pi^0)$ should be approximately equal to 0.01. The experimental data on the branching ratio for the annihilation $pp \to f_2\pi^0$ at rest are $(3.4 \pm 0.5) \cdot 10^{-2}$, $(2.1 \pm 0.7) \cdot 10^{-2}$ and $(2.0 \pm 0.6)10^{-2}$ in the cases of the ${}^1S_0$, ${}^3P_1$ and ${}^3P_2$ states respectively [14]. Therefore the quantity $BR(pp \to f_2'\pi^0)$ is expected to be of order 10^{-4} in the cases of ${}^1S_0$ and ${}^3P_2$ states and of order 10^{-3} in the case of ${}^3P_1$ state. This makes it is necessary to estimate the role of the rescattering contribution in the reaction $pp \to f_2'\pi^0$.

The major decay mode of the f_2' meson is KK as well as for the ϕ meson. Therefore, in view of the above discussion it is reasonable to estimate the role of $(K^*K + K^*)$ intermediate states in Model A. We shall consider only the S-wave annihilation, and we shall see that even the upper bound for the rescattering contribution is much less than the value expected from the OZI-rule.

The only relativistically invariant amplitude of the process $pp \to K^{*+}K^-$ which survives when $p \to 0$ and $K^{*+}K^-$ system is in the state with $I = 1$, $S = 0$ is the following

$$M^{(10)}_{pp \to K^{*+}K^-} = f^{(10)}_{K^{*+}K^-}[v(p_2)\gamma^5 u(p_1)](e^{*} P) \qquad (52)$$

where $f^{(10)}_{K^{*+}K^-}$ is the some constant. Then the corresponding cross-section is equal to

$$\sigma^{(10)}_{pp\to K^{*+}K^-} = \frac{|f^{(10)}_{K^{*+}K^-}|^2 s k'^3}{32\pi m_*^2 p} \tag{53}$$

We also need the amplitude of the reaction $K^+K^- \to f_2'$. It has the form

$$M_{K^+K^-\to f'} = f_{K^+K^-\to f'}(k_3' - k_2')_\mu (k_3' - k_2')_\nu \epsilon^{*\mu\nu} \tag{54}$$

where $\epsilon^{\mu\nu}$ is the polarization tensor of the final f_2' meson. The corresponding decay width is equal to

$$\Gamma_{f'\to K^+K^-} = \frac{4|f_{K^+K^-\to f'}|^2 k_{KK}^5}{15\pi m_{f'}^2} \tag{55}$$

where k_{KK} is now the magnitude of the momentum of the K^+ and K^- mesons in the reference frame where the f_2' meson is at rest. Since the decay of the f_2' meson into KK occurs in 72% cases, then the total width of the f_2' meson is equal to $\Gamma_{f_2'} = 2\Gamma_{K^+K^-\to f_2'}/0.72$.

As follows from Eqs. (9), (52) and (54), if the form factors are dropped, then the amplitude of the reaction $pp \to f_2'\pi^0$ in Model A is equal to

$$M_{pp\to f_2'\pi^0} = 16 f^{(10)}_{K^{*+}K^-} f_{K^{*+}\to\pi^0 K^+} f_{K^+K^-\to f_2'} [\bar{v}(p_2)\gamma^5 u(p_1)] \epsilon^{*\mu\nu} I_{\mu\nu} \tag{56}$$

where

$$I_{\mu\nu} = \int \frac{(2\pi)^4 \delta^{(4)}(k_1 + k_2 - k_1' - k_2') d^3\mathbf{k}_1' d^3\mathbf{k}_2'}{(2(2\pi)^3)^2 \omega_*(\mathbf{k}_1')\omega_K(\mathbf{k}_2')[(k_1' - k_1)^2 - m_K^2 + i0]}$$

$$[\frac{(Pk_1')(k_1 k_1')}{m_*^2} - (Pk_1)] k_{2\mu}' k_{2\nu}', \tag{57}$$

$\omega_*(\mathbf{k}') = (m_*^2 + \mathbf{k}'^2)^{1/2}$ and k_2 is the 4-momentum of the final f_2' meson.

The quantity $I_{\mu\nu}$ is the relativistic symmetric tensor which depends only on k_1 and k_2, and since $P = k_1 + k_2$ we can write

$$I_{\mu\nu} = c_1 P_\mu P_\nu + c_2 g_{\mu\nu} + c_3(P_\mu k_{2\nu} + P_\nu k_{2\mu} + c_4 k_{2\mu} k_{2\nu} \tag{58}$$

where c_i $(i = 1, \ldots 4)$ are some constants which may depend only on s. Since $\epsilon^{\mu\nu} g_{\mu\nu} = \epsilon^{\mu\nu} k_{2\mu} = \epsilon^{\mu\nu} k_{2\nu} = 0$, only the term with c_1 contributes to Eq. (56).

Therefore it is sufficient to find only c_1. For this purpose we note that the tensor

$$X_{\mu\nu} = \frac{(Pk_2)^2 k_{2\mu} k_{2\nu}}{m_{f'}^4} - \frac{(Pk_2)}{m_{f'}^2}(k_{2\mu}P_\nu + k_{2\nu}P_\mu) + P_\mu P_\nu - \frac{1}{3}(\frac{k_{2\mu}k_{2\nu}}{m_{f'}^2} - g_{\mu\nu})$$

$$[\frac{(Pk_2)^2}{m_{f'}^2} - P^2] \tag{59}$$

has the property

$$X^{\mu\nu}g_{\mu\nu} = X^{\mu\nu}k_{2\mu} = X^{\mu\nu}k_{2\nu} = 0 \tag{60}$$

Therefore, as follows from Eqs. (58) and (60)

$$c_1 = \frac{I_{\mu\nu}X^{\mu\nu}}{P_\mu P_\nu X^{\mu\nu}} \tag{61}$$

and, as follows from Eq. (56)

$$M_{pp\to f_2'\pi^0} = 4 f^{(10)}_{K^{*+}K^-} f_{K^{*+}\to\pi^0 K^+} f_{K^+K^-\to f_2'} [\bar{v}(p_2)\gamma^5 u(p_1)] c_1 e^{\mu\nu *} P_\mu P_\nu \tag{62}$$

The explicit expression for c_1 can be easily obtained in the c.m.frame of the $\pi^0 f_2'$ system (by analogy with Sec. 4). In this frame of reference

$$\frac{(2\pi)^4\delta^{(4)}(k_1 + k_2 - k_1' - k_2')}{[2(2\pi)^3]^2\omega_*(\mathbf{k}_1')\omega_K(\mathbf{k}_2')} = \frac{k'd\sigma'}{16\pi^2\sqrt{s}} \tag{63}$$

where $d\sigma'$ has the same sense as in Sec. 4.

Taking into account Eqs. (10), (57), (59), (61), (65-67), the final result can be written in the form

$$\frac{\sigma_{\bar{p}p\to f_2'\pi^0}}{\sigma^{(10)}_{\bar{p}p\to K^{*+}K^-}} = 0.72\frac{45}{2}\frac{kk'\Gamma_{f_2'}\Gamma_*}{sk^3_{\pi K}k^5_{K\bar{K}}m^2_{f'}}$$

$$|\int_{-1}^{1}\frac{k'E_\pi - E_* kx}{m_\pi^2 + m_*^2 - 2E_\pi E_* + 2kk'x - m_K^2 + i0}$$

$$\{(E_K k - E_{f'}k'x)^2 - \frac{1}{3}[(E_{f'}E_K - kk'x)^2 - m_K^2 m_{f'}^2]\}dx|^2 \tag{64}$$

A simple numerical calculation gives for $s = 4m^2$: $BR(\bar{p}p \to f_2'\pi^0) = (2.66\cdot 10^{-2} BR(\bar{p} \to K^{*+}K^-)^{(10)}$. According to ref. [11], $BR(\bar{p}p \to K^{*+}K^-) =$

$(1.7 \pm 0.7) \cdot 10^{-4}$. Therefore even the upper bound of the quantity $BR(\bar{p}p \rightarrow f_2'\pi^0)$ is of order 10^{-6}.

We have also calculated the contribution of the $\rho\pi$ channel to the reaction $\bar{p}p \rightarrow f_2'\pi^0$. The corresponding amplitude has the same spin structure as the amplitude describing the $(K^*\bar{K} + \bar{K}^*K)$ contribution. A simple numerical calculation gives $BR(\bar{p}p \rightarrow f_2'\pi^0) = (4.08 \cdot 10^{-4}) BR(\bar{p}p \rightarrow \rho^+\pi^-)^{(10)}$. According to ref. [14], $BR(\bar{p}p \rightarrow \rho^+\pi^-)^{(10)} = (0.65 \pm 0.3) \cdot 10^{-2}$ and therefore the $\rho\pi$ contribution is also small.

9 Conclusion

Let us briefly summarize the results of the present paper. In Secs. 2 and 3 we have discussed two models - Model A and Model B - describing different on-shell contributions to the reaction $\bar{p}p \rightarrow \phi\pi^0$ (see Figs. 3 and 5). We argue that from the theoretical point of view Model B is more substantiated than Model A. Nevertheless, as shown in Secs. 4-6, the values of $BR(\bar{p}p \rightarrow \phi\pi^0)$ given by Model B are much less than experimental data, while Model A is in qualitative agreement with the data. If the main contribution to the reaction $\bar{p}p \rightarrow \phi\pi^0$ is given by the $(K^*\bar{K} + \bar{K}^*K)$ intermediate states, then Model A predicts that the ratio $\sigma_{\bar{p}p\rightarrow\phi\pi^0}/\sigma^{(11)}_{\bar{p}p\rightarrow K^{*+}K^-}$ will be practically constant if p_{lab} is rather small (see Fig. 6) and an analogous prediction takes place for the ratio $\sigma_{\bar{p}p\rightarrow\phi\pi^0}/\sigma^{(11)}_{\rho^+\rho^-}$ if the $\rho^+\rho^-$ contribution is dominant (see Fig. 7). In Sec. 7 it is shown that if in Model A the off shell form factor for the K meson is dropped , then Model A predicts that the ratio $BR(\bar{p}p \rightarrow \phi\pi^0)/Br(\bar{p}p \rightarrow K^{*+}K^-)^{(10)}$ for the annihilation from the P state is in the range $[0.02 \div 0.67]$. Finally, as shown in Sec. 8, the upper bound for the rescattering contribution to the reaction $\bar{p}p \rightarrow f_2'\pi^0$ from the S state is of order 10^{-6}.

By analogy with calculations in Sec. 7 we can expect that the upper bound for the rescattering contribution to the reaction $\bar{p}p \rightarrow f_2'\pi^0$ from the P states is also of order 10^{-6}. Therefore the role of rescattering in this reaction is negligible, and any violation of the OZI-rule in the reaction $\bar{p}p \rightarrow f_2'\pi^0$ will be an evidence of some unusual phenomena.

At the same time, at present we cannot exclude the possibility that the violation of the OZI-rule in the reaction $\bar{p}p \rightarrow \phi\pi^0$ observed by several experimental groups (see refs.[1-3]) can be explained as the effect of rescattering. However some assumptions lying in the basis of Model A seem questionable. First, it is necessary to check numerically that if the widths of the K^* and ρ mesons are neglected then the results will not essentially change (especially this concerns the question of neglecting Γ_ρ). Second, as argued in Sec. 2, Model A doesn't fully correspond to our assumption that the ϕ meson is created from the K and $\bar{K}$ mesons. Therefore, as pointed out in refs. [10, 7], we have to take into account the off shell form factor for the K meson, but the data agree with Model A if this form factor is not very important. The rescattering mechanism seems also questionable from the following simple estimate. Since the K^* meson lives approximately $1/\Gamma_*$ in the frame of reference where it is at rest, it is easy to see then when the K^* meson decays the distance between the K^* and K mesons in their c.m. frame is $2mk'/\Gamma_* m_* E_K(k') \approx 6Fm$. It seems doubtful that the K^* and K mesons can effectively interact being separated by such a distance. On the other hand the analogous distance between the ρ^+ and ρ^- mesons is of about $2Fm$, but the question arises whether it is possible to use the concept of ρ meson in such a process.

To shed light on the problem of the OZI-rule violation in the reaction $\bar{p}p \to \phi\pi^0$ it seems important to carry out calculations not only in the on-shell approximation, but taking also into account the off-shell contribution. It is also interesting to investigate why the OZI-rule violations in the reaction $\bar{p}p \to \phi\pi\pi$ isn't strong even in the channel with $I = S = 1$. We suppose to investigate these problems in subsequent publications.

Acknowledgments

The authors are grateful to M.G.Sapozhnikov for the statement of the problem and numerous discussions.

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Received by Publishing Department
on April 28, 1994.