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CONSERVED CURRENTS FOR $ed-e'NN$, $ed-e'd'$ AND $\gamma d-NN$ REACTIONS IN THE FRAMEWORK OF THREE-DIMENSIONAL FIELD-THEORETICAL FORMULATION

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1. Introduction

One of the aims of studying the scattering reactions with strong and electromagnetic interacting particles in the relativistic quantum field theory is such a type of construction of corresponding potentials and amplitudes, which satisfies the first principle of relativity, causality, unitary, gauge invariance etc. The general form of Bethe-Salpeter equations, where all these principles are observed, are presented in ref. [1,2,3]. However, as a rule, by solution of these or other field-theoretical equations two basic assumptions are used:

1) The number of particles from intermediate states of field-theoretical potentials by practical calculations is limited. This assumption is necessary for obtaining a closed system of equations and it brings an additional difficulties to make the sought amplitudes gauge invariant [1,2,3,4].

The employment of particle number restriction by constructing the microscopical potentials is usually verified only through the comparisons with experimental data and with results from other models. In this paper we will suggest an additional test for examining the accuracy of intermediate particle number restriction approach. This test is derived on the base of the same vertices which are used by construction of interaction potentials.

2) The input vertex functions for the building of Bethe-Salpeter potential or its quasipotential representations depends on two, three or more number of variables. In contrast to these the phenomenological vertices which are used as rule by calculation of microscopical potentials, are defined as one-variable functions. Therefore, in order to construct of the microscopical potentials in quasipotential or other three-dimensional approach on the base of one-variable phenomenological vertices, an additional assumptions about particles off mass shell behavior are used.

An other three-dimensional field-theoretical formulation of scattering reactions deriving from the covariant generalization of old perturbation theory [5], where all particle are on mass shell are presented in refs. [5,6,7,8]. However, in this formulation arise the nonphysical degrees of freedom which increase the number of independent variables in vertices or the total number of invariant form-factors and amplitudes [8].

In this paper we consider the alternative field-theoretical formulation of equations for amplitudes of coupled $ed - e'NN$, $ed - e'd'$ and $\gamma d - NN$ reactions. This relativistic approach is a three-dimensional one from the beginning and by the construction of potentials in resulting equations, we found one-variable covariant vertex functions. The base of suggesting equations is a field-theoretical spectral decomposition of $\gamma d - NN$ and $NN - NN$ transition amplitudes which after distinguish of $\gamma d - NN$ and $NN - NN$ amplitudes admit the form of so called Low-type equations [9-17]. The relativistic Low equations was often used by investigations of πN scattering problems in the framework of Chew-Low model [10,11] and in more general cases [12,13]. In refs. [14,15,16,17] we have suggested the general procedure of linearization of relativistic Low-type equations. These linearized equations have the form of Lippmann-Schwinger equations with linear energy dependent potential and we have used their solutions for description of low energy πN and NN scattering phase shifts. Besides this in refs. [16,17] we have demonstrated that the Low-type equations can be considered as a matrix representation of microscopic causality condition between two particle field operators which were given by Bogoliubov [18,19].

The present paper consists of six sections. The section 2 deals with the formal derivation of field-theoretical spectral decomposition of $\gamma d - NN$ and $ed - e'NN$ amplitudes, which afterwards are rewritten in the form of Low-type equations. The linearized representation of these Low-type equations is given in sect. 3. In sect. 4 we demonstrate the gauge invariance of suggesting equations. In sect. 5 we derive the condition for examining the accuracy of restriction of intermediate particle numbers by constructing the microscopic potential. Finally sect. 6 contains a brief summary.

2. FIELD-THEORETICAL SPECTRAL DECOMPOSITION OF DEUTERON ELECTROMAGNETIC DISINTEGRATION AMPLITUDE

We start from the general S-matrix element of the deuteron electro and photo disintegration reactions

$$< out; \mathbf{p}'_e\mathbf{p}'_{N1}\mathbf{p}'_{N2}|\mathbf{p}_e\mathbf{p}_d; in >=$$
$$= (2\pi)^4 i\delta^{(4)}(p'_e + p'_{N1} + p'_{N2} - p_e - p_d) < out; \mathbf{p}'_{N1}\mathbf{p}'_{N2}|\mathbf{J}_{\mathbf{p}'_e}(0)|\mathbf{p}_e\mathbf{p}_d; in > \tag{1}$$

$$< out; \mathbf{p}'_{N1}\mathbf{p}'_{N2}|\mathbf{k}\lambda\mathbf{p}_d; in >=$$
$$= (2\pi)^4 i\delta^{(4)}(p'_{N1} + p'_{N2} - k - p) < out; \mathbf{p}'_{N1}\mathbf{p}'_{N2}|j_\mu(0)|\mathbf{p}_e\mathbf{p}_d; in > \epsilon^{(\lambda)}_\mu(\mathbf{k}) \tag{2}$$

where we have used the conventions and normalization conditions from the book by Itzykson and Zuber [20]. So for electron and photon current operators we have $\mathbf{J}_{\mathbf{p}'_e}(x) = \overline{u}(\mathbf{p}_e)(i\gamma_\nu\nabla^\nu - m_e)\psi(x)$ and $j_\mu(x) = (\partial^2_x)A_\mu(x)$, $\epsilon^{(\lambda)}_\mu(\mathbf{k})$ is a photon polarization vector, $p_a = (\sqrt{\mathbf{p}_a^2 + m_a^2}, \mathbf{p}_a)$ denotes the on-mass-shell momentum of particle $a = N, d, e$ and for the sake of simplicity of notation following spin and isospin indices of nucleon, deuteron and electron will be omitted.

According to the reduction technique in quantum field theory [20], the deuteron electro-disintegration amplitude (1) can be transformed as

$$< out; \mathbf{p}'_{N1}\mathbf{p}'_{N2}|\mathbf{J}_{\mathbf{p}'_e}(0)|\mathbf{p}_e\mathbf{p}_d; in >=< out; \mathbf{p}'_{N1}\mathbf{p}'_{N2}|\left\{\mathbf{J}_{\mathbf{p}'_e}(0), b^\dagger{}_{\mathbf{p}_e}(0)\right\}|\mathbf{p}_d; in >$$
$$+i\int d^4x \exp(-ip_e x) < out; \mathbf{p}'_{N1}\mathbf{p}'_{N2}|T(\mathbf{J}_{\mathbf{p}'_e}(0)\mathbf{J}^\dagger_{\mathbf{p}_e}(x))|\mathbf{p}_d; in > \tag{3}$$

where $b^\dagger{}_{\mathbf{p}'_a}(x^0)$ denotes the Heisenberg field operator of the fermion $a = e, N$ with on-mass-shell momentum p_a

$$b^\dagger{}_{\mathbf{p}_a}(x^0) = \int d^3x exp(-ip_a x)\overline{u}(\mathbf{p}_a)\gamma_0\Psi(x) \tag{4}$$

The second part of the right side in eq. (3) consists of the second or more highly order contributions of electromagnetic interactions. Therefore, we neglect this term and after using the canonical commutation relation for electron fields in quantum electrodynamic, we obtain well known formula for the deuteron electro-disintegration amplitude

$$f_{ed\to e'N'N'} = -e^2\overline{u}(\mathbf{p}'_e)\gamma_\mu u(\mathbf{p}_e)\frac{1}{q^2} < out; \mathbf{p}'_{N1}\mathbf{p}'_{N2}|j_\mu(0)|_e\mathbf{p}_d; in > \tag{5a}$$

where $q^2 = (p'_{N1} + p'_{N2} - p_d)^2$ is the photon four-momentum.

The deuteron photo-disintegration amplitude, according the formula (2), can be written as

$$f_{\gamma d\to N'N'} =< out; \mathbf{p}'_{N1}\mathbf{p}'_{N2}|j_\mu(0)|\mathbf{p}_d; in > \epsilon_\mu^{(\lambda)}(\mathbf{k}) \tag{5b}$$

The deuteron electron and photon disintegration amplitude (5a) and (5b) are depicted in Fig. 1a and Fig. 1b. Note that these amplitudes depend nontriviall only on external nucleons and deuteron three-momenta. We suppose that the photon four-momentum in expressions (5a) and (5b) is defined through the momenta of external deuteron and nucleons $q = p'_{N1} + p'_{N2} - p_d$ i. e. photon is defined off-mass-shell so, that $\mathbf{q} = \mathbf{k}$ but $q^2 = k^2 = 0$ only on-energy-shell surface. After this definition we call the scattering amplitude (5a) and (5b) as half-off-mass-shell amplitude. For unified consideration of deuteron photo and electron scattering reactions we introduce following expression

$$f^{\mu}_{xd-N'N'} =< out; \mathbf{p}'_{N1}\mathbf{p}'_{N2}|j^\mu(0)|\mathbf{p}_d; in > \tag{6}$$

We continue the transformation of this expression on the base of the reduction technique of the quantum field theory [19,20]. Thus, after reduction one of nucleons from the "out" state we find

$$< out; \mathbf{p}'_{N1}\mathbf{p}'_{N2}|j_\mu(0)|\mathbf{p}_d; in >=< out; \mathbf{p}'_{N1}|\left[b_{\mathbf{p}'_{N2}}(0), j_\mu(0)\right]|\mathbf{p}_d; in > \tag{7}$$

$$+i\int d^4x \exp(-ipx) < out; \mathbf{p}'_{N1}\mathbf{p}'_{N2}|T\left(\eta_{\mathbf{p}'_{N2}}(0)j_\mu(x)\right)|\mathbf{p}_d; in >$$

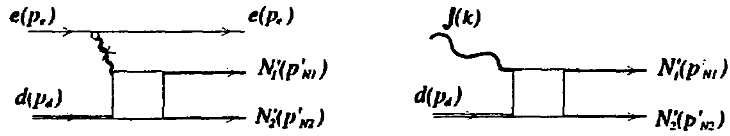
where $\eta_{\mathbf{p}'_N}(x) = \overline{u}(\mathbf{p}_N)(i\gamma_\nu\nabla^\nu - m_N)\Psi(x)$ denotes the nucleon current operator and for sake of the simplicity we omit the antisymmetrization procedure of identical particles as it is done for example in ref. [15,16,17].

We substitute the completeness condition of asymptotic "out" states into the second term of eq. (7) and after integration over the x variable we obtain

$$< out; \mathbf{p}'_{N1}\mathbf{p}'_{N2}|j_\mu(0)|\mathbf{p}_d; in >= Y_{N2,\mu}(\mathbf{p}_{N1}, \mathbf{p}_d)$$

$$- \sum_{n=d,NN,...} < \mathbf{p}'_{N1}|\eta_{\mathbf{p}'_{N2}}(0)|n; out > \frac{(2\pi)^3\delta^{(3)}(\mathbf{p}'_{N1} + \mathbf{p}'_{N2} - \mathbf{P}_n)}{p^{0'}_{N1} + p^{0'}_{N2} - E_n + i\epsilon} < out; n|j_\mu(0)|\mathbf{p}_d >$$

$$+ \sum_{m=N,\pi N,...} < \mathbf{p}'_{N1}|j_\mu(0)|m; out > \frac{(2\pi)^3\delta^{(3)}(\mathbf{p}'_{N2} - \mathbf{p}_d + \mathbf{P}_n)}{p^{0'}_{N2} - p^0_d + E_m} < out; m|\eta_{\mathbf{p}'_{N2}}(0)|\mathbf{p}_d > \tag{8}$$



(a) (b)

Fig.1. Diagrammatic representation of a) the deuteron electron-disintegration and b) photo-disintegration amplitude. Off-mass-shell particle here and everywhere below are marked with additional cross.

(a) (b)

Fig.2. Equal-time commutator (a) corresponding to the simplest one nucleon exchange term (11) and (b) corresponding to the four-point γd-N'N' vertex with off-mass-shell photon and one nucleon.

where $Y_{N2,\mu}$ denotes the equal time commutator which in current algebra is called as seagull term

$$Y_{N2,\mu}(\mathbf{p}_{N1},\mathbf{p}_d) =< \mathbf{p}'_{N1}|\left[b_{\mathbf{p}'_{N2}}(0), j_\mu(0)\right]|\mathbf{p}_d > \qquad (9)$$

This equal time commutator can be directly calculated on the base of simple phenomenological Lagrangian models as it has been done, for example, for πN and NN scattering cases in refs. [14,16,17]. So the current operator for simplest model of electromagnetic interaction of π-meson $\Phi(x)$ and nucleon $\Psi(x)$ field operators can be written as

$$j_\mu(x) = \overline{\Psi}(x)\frac{1}{2}(1+\tau_3)\gamma_\mu\Psi(x) + \left(\Phi(x)\times\frac{\partial\Phi(x)}{\partial x_\mu}\right)_3 \qquad (10)$$

and using the canonical commutation relations we obtain

$$Y^{(0)}_{N2,\mu}(\mathbf{p}_{N1},\mathbf{p}_d) = \overline{u}(\mathbf{p}'_{N2})\gamma_0\frac{1}{2}(1+\tau_3)\gamma_\mu < \mathbf{p}'_{N1}|\Psi(0)|\mathbf{p}_d > \qquad (11)$$

This expression of the simplest equal time commutator for the $\gamma d - N'N'$ system is depicted in Fig. 2a as a t-channel one off-mass-shell nucleon exchange diagram. Besides this, a photon-nucleon vertex in (11) is given in tree approximation. By calculation of equal time commutators on the base of more general models of meson-nucleon Lagrangians other terms arise which could not be represented as the off-mass-shell particle exchange expressions. These complicated terms represent four-point $\gamma d - NN$ vertices (Fig. 2b). According to the dispersion theory [18,19,20] the equal time commutator (9) represents the finite number polinom of p_{N2} four-momentum and consists of one-variable $(p_{N1}-p_d)^2$ vertex functions.

Relation (8) is the sought spectral decomposition of deuteron electromagnetic disintegration amplitude through the complete system of asymptotic "out" states. Hence, the intermediate states and propagators in this relation are defined on-mass-shell. This relation can be rewritten in a more compact form as Low-type equation. For this purpose we single out terms with NN and d intermediate states and define the sum of the rest of the terms as inhomogeneous or potential part of Low equation. Thus we get

$$< out;\mathbf{p}'_{N1}\mathbf{p}'_{N2}|j_\mu(0)|\mathbf{p}_d;in >= W_{N2,\mu}(\mathbf{p}'_{N1},\mathbf{p}'_{N2};\mathbf{p}_d)$$

$$-\sum_{n=d,NN} < \mathbf{p}'_{N1}|\eta_{\mathbf{p}'_{N2}}(0)|n;out > \frac{(2\pi)^3\delta^{(3)}(\mathbf{p}'_{N1}+\mathbf{p}'_{N2}-\mathbf{P}_n)}{p^{0\prime}_{N1}+p^{0\prime}_{N2}-E_n+i\epsilon} < out;n|j_\mu(0)|\mathbf{p}_d > \qquad (12)$$

where

$$W_{N2,\mu}(\mathbf{p}_{N1},\mathbf{p}'_{N2};\mathbf{p}_d) = Y_{N2,\mu}(\mathbf{p}'_{N1},\mathbf{p}_d) + (2\pi)^3 V_{N2,\mu}(\mathbf{p}'_{N1},\mathbf{p}'_{N2};\mathbf{p}_d) \qquad (13)$$

and in $V_{N2,\mu}$ we take into account all disconnected parts of transition amplitudes in eq. (8), or, in other words, we carry out the cluster decomposition [21] which enables us to obtain

$$V_{N2,\mu}(\mathbf{p}'_{N1},\mathbf{p}'_{N2};\mathbf{p}_d) =$$

$$-\sum_{n=\pi d,\ldots} < \mathbf{p}'_{N1}|\eta_{\mathbf{p}'_{N2}}(0)|n;out >_c \frac{\delta(\mathbf{p}'_{N1}+\mathbf{p}'_{N2}-\mathbf{P}_n)}{p^{0\prime}_{N1}+p^{0\prime}_{N2}-E_n+i\epsilon} < out;n|j_\mu(0)|\mathbf{p}_d >_c$$

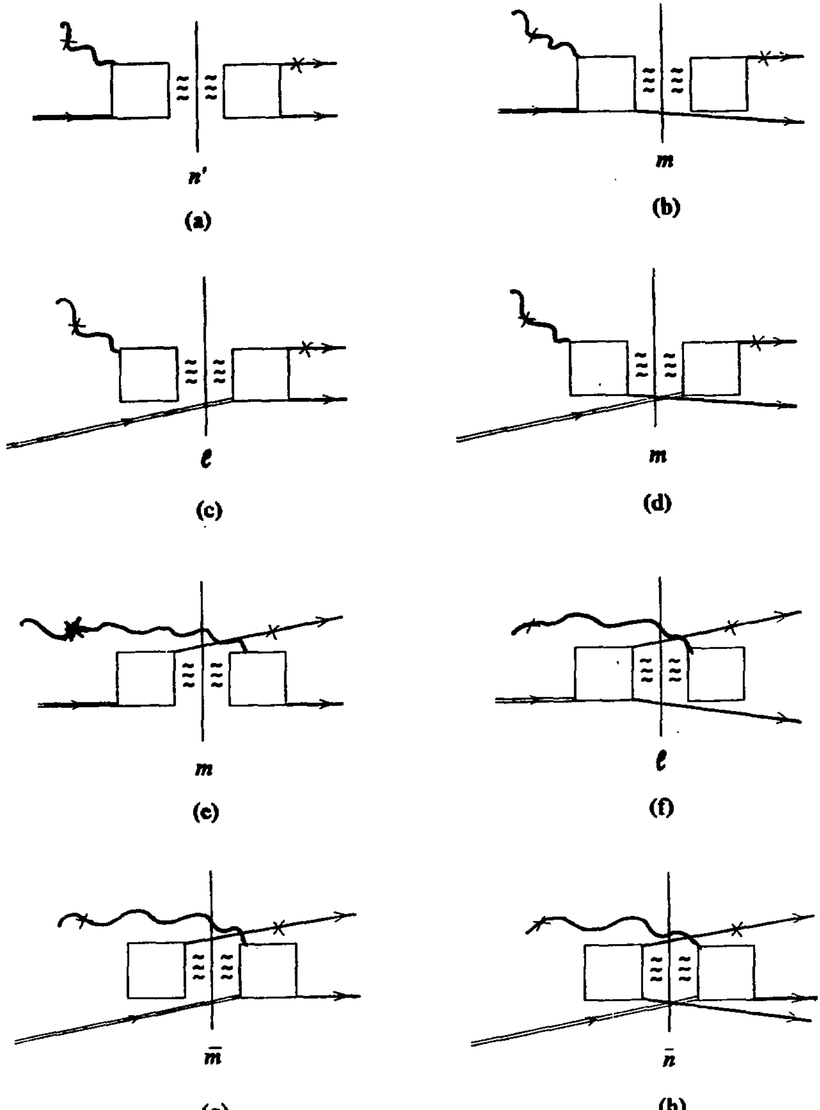


Fig.3. On-mass-shell particle exchange part V_η (14) of γd-NN transition potential with following intermediate states n' = NΔ,πd,πNN, ...; m = N,πN,...; ℓ =ρ,$\pi\pi$,...; and corresponding set of antiparticle intermediate states $\bar{n}$ = $\bar{N}\bar{N}$,$\bar{d}$,$\bar{N}\bar{\Delta}$,...; $\bar{m}$ = $\bar{N}$,$\pi\bar{N}$,...; $\bar{\ell}$=$\bar{\rho}$,$\bar{\pi}\bar{\pi}$.

$$+\sum_{m=\pi N,\rho N...} <0|\eta_{\mathbf{p}'_{N2}}(0)|m;out> \frac{\delta(\mathbf{p}'_{N2}-\mathbf{P}_m)}{p^{0\prime}_{N2}-E_m} <out;m,\mathbf{p}'_{N1}|j_\mu(0)|\mathbf{p}_d>_c$$

$$-\sum_{l=\rho,\pi\pi,...} <\mathbf{p}'_{N1}|\eta_{\mathbf{p}'_{N2}}(0)|\mathbf{p}_d,l;out>_c \frac{\delta(\mathbf{p}'_{N1}+\mathbf{p}'_{N2}-\mathbf{P}_l-\mathbf{p}_d)}{p^{0\prime}_{N1}+p^{0\prime}_{N2}-E_l-p^0_d} <out;l|j_\mu(0)|0>$$

$$+\sum_{\overline{m}=\overline{N},\pi\overline{N}...} <0|\eta_{\mathbf{p}'_{N2}}(0)|\mathbf{p}_d,\overline{m};out> \frac{\delta(\mathbf{p}'_{N2}-\mathbf{P}_{\overline{m}}-\mathbf{p}_d)}{p^{0\prime}_{N2}-E_{\overline{m}}-p^0_d} <out;\overline{m},\mathbf{p}'_{N1}|j_\mu(0)|0> \quad (14)$$

$$+\sum_{m=N,\pi N,...} <\mathbf{p}'_{N1}|j_\mu(0)|m;out>_c \frac{\delta(\mathbf{p}'_{N2}-\mathbf{p}_d+\mathbf{P}_m)}{p^{0\prime}_{N2}-p^0_d+E_m} <out;m|\eta_{\mathbf{p}'_{N2}}(0)|\mathbf{p}_d>_c$$

$$-\sum_{l=\rho,\pi\pi,...} <0|j_\mu(0)|l;out> \frac{\delta(\mathbf{p}'_{N1}+\mathbf{p}'_{N2}-\mathbf{p}_d+\mathbf{P}_l)}{p^{0\prime}_{N1}+p^{0\prime}_{N2}-p^0_d+E_l} <out;l,\mathbf{p}'_{N1}|\eta_{\mathbf{p}'_{N2}}(0)|\mathbf{p}_d>_c$$

$$+\sum_{\overline{m}=\overline{N},\pi\overline{N}...} <\mathbf{p}'_{N1}|j_\mu(0)|\mathbf{p}_d,\overline{m};out>_c \frac{\delta(\mathbf{p}'_{N2}+\mathbf{P}_{\overline{m}})}{p^{0\prime}_{N2}+E_{\overline{m}}} <out;\overline{m}|\eta_{\mathbf{p}'_{N2}}(0)|0>$$

$$-\sum_{\overline{n}=\overline{d},\overline{NN},...} <0|j_\mu(0)|\mathbf{p}_d,\overline{n};out> \frac{\delta(\mathbf{p}'_{N1}+\mathbf{p}'_{N2}+\mathbf{P}_{\overline{n}})}{p^{0\prime}_{N1}+p^{0}_{N2}+E_{\overline{n}}} <out;\overline{n},\mathbf{p}'_{N1}|\eta_{\mathbf{p}'_{N2}}(0)|0>$$

where the subscript "c" denotes the connected part of transition amplitude and the additional minus before some terms results from the permutation of nucleon fields.

We plot in Figs. 3a-3h the time-ordered diagrams which corresponds to the three-dimensional terms of $V_{N2,\mu}$ (14). Thus, the diagrams on Fig. 3a, Fig. 3d, Fig. 3e and Fig. 3h describe the $\gamma d - NN$ transition in s, $\overline{u}$,u and $\overline{s}$ channels. But the diagrams in Fig. 3b, Fig. 3g and Fig. 3c, Fig. 3f respect to the $\gamma d - NN$ transition through the transformation of one off-mass-shell nucleon and photon in $m = \pi N, \ldots$ and $l = \rho, \pi\pi, \ldots$ on-mass-shell states.

In the lowest order the inhomogeneous term $W_{N2,\mu}$ (13) contains one nucleon (Figs. 2a and 3e) and one anti-nucleon (Figs. 3d,g) exchange terms. To take into account one ρ,ω-meson exchange terms in the considered approach the connected amplitudes $< \mathbf{p}'_{N1}|\eta_{\mathbf{p}'_{N2}}(0)|\mathbf{p}_d,\rho(\omega);out>$ and $<out;\rho(\omega),\mathbf{p}'_{N1}|\eta_{\mathbf{p}'_{N2}}(0)|\mathbf{p}_d>$ should be calculated. The construction of these amplitudes is similar to the $\gamma d - NN$ and $d - \gamma NN$ transition amplitudes in which photon is replaced with $\rho(\omega)$-meson. So if in eq. (8) we replace on-mass-shell nucleon momentum with on-mass-shell $\rho(\omega)$-mesons four-momentum, then we obtain Low-type equation for $\rho(\omega)d - N'N'$ or $d - N'N'\rho(\omega)$ transition amplitudes.

In Fig. 4 the $\gamma d - NN$ transition diagrams with two-body meson exchange currents are plotted. These diagrams are often used for calculations of deuteron electromagnetic disintegration reactions [22,23,24,25,26]. The analogous meson-exchange currents can be easily extracted from the on-mass-shell particle exchange potential $V_{N2,\mu}$ (14). So the diagram from the Fig. 4a could be extracted from the second (Fig. 3b) terms of expression (14) with $m = \pi N$ intermediate state. The diagram on the Fig. 4b is included when constructing the deuteron wave function. The diagram on Fig. 4c contains the vertex functions with two or three off-mass-shell particles. Therefore, the contributions of this diagram together with the sum of other more complicated terms are included into equal time term (9) (Fig. 2b). And finally if we assume that in diagrams on Fig. 4d,e only one external nucleon and photon are off-mass-shell, then the diagrams on Fig. 4d,e are included into fourth or seventh (Fig. 3d or 3g) and into third or sixth (Fig. 3c or 3f) terms. But if we assume that in the diagrams 4d,e we have a more number of off-mass-shell particles, then the contributions of these diagrams are included also into the seagull term (9) (Fig. 2b) [16,19].

3. EQUATIONS FOR THE DEUTERON ELECTROMAGNETIC INTERACTIONS AMPLITUDES

The relativistic Low-type equation (8) or (12) can be transformed in a more convenient form of relativistic Lippmann-Schwinger type equation without one-deuteron intermediate state. For this purpose let us consider the coupled set of Low-type equations for processes of nucleon-nucleon scattering and $\gamma d - NN$ transition in which we keeping only a lowest order of e i. e. we neglect contributions from $\gamma d - \gamma d$ channel. We use also the field-theoretical spectral decomposition formula for the NN scattering amplitude [15,16,17]

$$f_{N'N'-NN} \equiv < out; \mathbf{p'}_{N1}\mathbf{p'}_{N2}|\overline{\eta}_{\mathbf{p}_{N1}}(0)|\mathbf{p}_{N2} > \tag{15}$$

This spectral decomposition formula has the analogy to (8) form [15,16,17] in which the photon current operator $J_\mu(0)$ is replaced by nucleon source operator $\overline{\eta}_{\mathbf{p}_{N1}}(0)$ and one-deuteron "in" state is replaced with one-nucleon state. It must be pointed out that the equal time anticommutator in NN scattering potential $< \mathbf{p'}_{N1}|\{b_{\mathbf{p'}_{N2}}(0), \overline{\eta}_{\mathbf{p}_{N1}}(0)\}|\mathbf{p}_{N2} >$ reproduce in exact form the one-boson-exchange model of NN potential [28] if it is calculated in the framework of the simplest meson-nucleon phenomenological Lagrangian models [16,17].

Thus, the field-theoretical spectral decomposition of NN (15) and $\gamma d - NN$ (8) amplitude can be rewritten as a set of coupled Low-type equations

$$f_{\alpha\beta} = W_{\alpha\beta} + f^{\dagger}_{\alpha d} G_d(E_\beta) f_{d\beta} + \sum_{\beta'=1,2} f^{\dagger}_{\alpha\beta'} G_{\beta'}(E_\beta) f_{\beta'\beta} \tag{16}$$

where α,β denotes the number of coupled channel $\alpha, \beta = 1,2 \equiv NN, \gamma d$ and E_β is total energy of asymptotic states β,

$$G_d(E_\beta) = \frac{1}{E_\beta - E_d}; \qquad G_{\beta'}(E_\beta) = \frac{1}{E_\beta - E_{\beta'} + i\epsilon} \tag{17a}$$

are Green functions of corresponding noninteracted systems and we have used the following definitions for multichannel amplitudes

$$f_{\alpha\beta} = \begin{pmatrix} < out; \mathbf{p'}_{N1}\mathbf{p'}_{N2}|\overline{\eta}_{\mathbf{p}_{N1}}(0)|\mathbf{p}_{N2} > & < out; \mathbf{p'}_{N1}\mathbf{p'}_{N2}|j_\mu(0)|\mathbf{p}_d; in > \\ < out; \mathbf{p'}_d|j_\mu(0)|\mathbf{p}_{N1}\mathbf{p}_{N2}; in > & 0 \end{pmatrix} \tag{17b}$$

$$f_{d\beta} = \begin{pmatrix} < \mathbf{p'}_d|\overline{\eta}_{\mathbf{p}_{N1}}(0)|\mathbf{p}_{N2} > \\ < \mathbf{p'}_d|j_\mu(0)|\mathbf{p}_d > \end{pmatrix} \tag{17c}$$

The inhomogeneous term $W_{1'1}$ of NN system is defined in our previous works [16,17]. By the construction of $W_{2'1}$ we use the complete system of "*out*" states and if for sake of simplicity we omit the s channel singularities in $W_{\alpha\beta}$, then we obtain

$$W_{1'2}(\mathbf{p}'_{N1},\mathbf{p}'_{N2};\mathbf{p}_d) = W^{\dagger}_{2'1}(\mathbf{p}'_d;\mathbf{p}_{N1},\mathbf{p}_{N2}) \tag{18}$$

Then following the refs. [15,16,17] one can show that the system of the relativistic three-dimensional equations (15) can be linearized. Thus, we can construct following set of Lippmann-Schwinger type equations equivalent to eqs. (15)

$$t_{\alpha\beta}(E) = U_{\alpha\beta}(E) + \sum_{\beta'=1,2} U_{\alpha\beta'}(E)G_{\beta'}(E)t_{\beta'\beta}(E) \tag{19}$$

where on-energy shell surface solution of eq. (18) coincides with solution of eq. (16) $t_{\alpha\beta}(E_\alpha)|_{on\ energy\ shell} = f_{\alpha\beta}$ and the linear energy depending potential is singlevalued determined with the inhomogeneous term of eqs. (16).

$$U_{\alpha\beta}(E = E_\beta) = W_{\alpha\beta} \tag{20a}$$

$$U_{\alpha\beta}(E) = A_{\alpha\beta} + EB_{\alpha\beta} \tag{20b}$$

where $A_{\alpha\beta}$ and $B_{\alpha\beta}$ are hermitian matrices which are constructed according to eqs. (9),(13),(14) for the $\gamma d - NN$ system and in [15,16,17] for the nucleon-nucleon case

$$A_{\alpha\beta} = \begin{pmatrix} A_{1'1} & W_{1'2} \\ W_{2'1} & 0 \end{pmatrix}; \qquad B_{\alpha\beta} = \begin{pmatrix} B_{1'1} & 0 \\ 0 & 0 \end{pmatrix} \tag{20c}$$

Now it is easy to derive the explicit expression for the sought deuteron electromagnetic disintegrations amplitude $t_{1'2}$ and t_{d2} through the nucleon-nucleon wave functions and $\gamma d - NN$ transitions potential (20a,b,c) and (13)

$$< \mathbf{p}'_{N1},\mathbf{p}'_{N2}|t_{1'2}(E_{N'N'})|\mathbf{p}_d > = < \Psi_{\mathbf{p}'_{N1},\mathbf{p}'_{N2}}|U_{1'2}(E_{N'N'})|\mathbf{p}_d > \tag{21}$$

$$< \mathbf{p}'_d|t_{d'2}(E_{d'})|\mathbf{p}_d > = < \Psi_{\mathbf{p}'_d}|U_{1'2}(E_{d'})|\mathbf{p}_d > \tag{22}$$

where $< \Psi_{\mathbf{p}'_{N1},\mathbf{p}'_{N2}}|$ and $< \Psi_{\mathbf{p}'_d}|$ are continuous and discrete wave functions for pure NN interactions which are defined by

$$< \Psi_{\mathbf{p}'_{N1},\mathbf{p}'_{N2}}|\mathbf{p}_{N1},\mathbf{p}_{N2} > = < \mathbf{p}'_{N1},\mathbf{p}'_{N2}|\mathbf{p}_{N1},\mathbf{p}_{N2} > +$$

$$\frac{1}{E_{\mathbf{p}_{N1},\mathbf{p}_{N2}} - E_{\mathbf{p}'_{N1},\mathbf{p}'_{N2}} + i\epsilon} < \mathbf{p}'_{N1},\mathbf{p}'_{N2}|\mathcal{T}_{1'1}(E_{\mathbf{p}'_{N1},\mathbf{p}'_{N2}})|\mathbf{p}_{N1},\mathbf{p}_{N2} > \tag{23}$$

$$< \Psi_{\mathbf{p}_{d'}}|\mathbf{p}_{N1},\mathbf{p}_{N2} > = \frac{1}{E_d - E_{\mathbf{p}'_{N1},\mathbf{p}'_{N2}}} < \mathbf{p}_{d'}|\mathcal{T}_{1'1}(E_d)|\mathbf{p}_{N1},\mathbf{p}_{N2} > \tag{24}$$

where $\mathcal{T}_{1,1}$ is the solution of relativistic Lippmann-Schwinger type equation for NN scattering problem [16,17] with $U_{1',1}$ potential

$$\mathcal{T}_{1'1}(E) = U_{1'1}(E) + U_{1'1}(E)G_{NN}(E)\mathcal{T}_{1'1}(E) \tag{25}$$

The nucleon-nucleon wave functions (23) and (24) satisfy completeness conditions

$$\int d^3\mathbf{p}_{N1} d^3\mathbf{p}_{N2} |\Psi_{\mathbf{p}_{N1},\mathbf{p}_{N2}}><\Psi_{\mathbf{p}_{N1},\mathbf{p}_{N2}}| + |\Psi_{\mathbf{p}_d}><\Psi_{\mathbf{p}_d}| = (1 - B_{1'1})^{-1} \qquad (26)$$

Besides this, the hermitian potential (20a,b) provides more general completeness conditions for the solution of multichannel equations (19)

$$<\beta'|\left\{\sum_{\alpha=NN,\pi d} |\Phi_\alpha><\Phi_\alpha| + |\Phi_d><\Phi_d|\right\}|\beta>= (1-B)^{-1}_{\beta'\beta} \qquad (27)$$

where

$$<\Phi_\alpha|\beta>=<\beta|\alpha> + G_\beta(E_\alpha)t_{\beta\alpha}(E_\alpha) \qquad (28a)$$

$$<\Phi_d|\beta>= G_\beta(E_d)t_{\beta d}(E_d) \qquad (28b)$$

Equation (23) and (24) represent our final result for the relativistic and three-dimensional amplitude for deuteron electron (Fig. 5a) and photo (Fig.5b) disintegration reactions. So if we multiply the equation (23) on the $-e^2\overline{u}(\mathbf{p}'_e)\gamma_\mu u(\mathbf{p}_e)q^{-2}$ or on the photon polarization vector, then according to eq. (5a) or (2) we obtain the amplitudes of $ed-NN$ or $\gamma d-NN$ reactions. If instead of $N'N'$ final states we consider one-deuteron d' state, then eq. (24) enables us to calculate the deuteron form-factors for the transitions $ed - e'd'$ and $\gamma d - d'$. In contrast to the other relativistic approaches the suggested formulation of coupled $ed - e'd'$,$\gamma d - d'$,$\gamma d - N'N'$ and $ed - e'N'N'$ reactions possesses the following attractive features:

1. The sought relativistic invariant amplitude (6) or (23) and (24) depend only on the three-momenta of on-mass-shell final two nucleon and initial deuteron. All transition amplitudes in basis formula of field-theoretical spectral decomposition (8) as well as other relations for sought amplitude are defined through the renormalized, physical matrix elements. In particular, the on-mass-shell particle exchange term V_μ (14) consists of transition matrix elements between "*out*" and "*in*" states and Green functions of noninteracting real particle. The equal time commutator (9) contains the corrections coming from the renormalization procedure [16,19] in every order of perturbation theory. But in the final form we can assume that this commutator is expressed through the finite physical form-factors which are independent of the renormalization procedure.

2. In the effective potential of Low-type equations and their linearized representation the on-mass-shell and off-mass-shell degrees of freedom are separated from each other. Thus all on-mass-shell particle exchange terms are included into V_μ (14) and nontrivial off-mass-shell contributions are contained in the equal-time commutator (9). Besides this, in suggested the formulation, as well as in other three-dimensional approaches [5,6,7,16], the anti-nucleon degrees of freedom are separated.

3. By calculation of Low-type equations (12) as input vertex function one can take the seagull terms (9) and vertex functions $<\mathbf{p}'_{N1}|\overline{\eta}_{\mathbf{p}_{N1}}(0)|\mathbf{p}_d>$, $<\mathbf{p}'_N|j_\mu(0)|\mathbf{p}_N>$, $<0|\overline{\eta}_{\mathbf{p}_{N1}}(0)|\mathbf{p}'_{\overline{N}}\mathbf{p}_d; in>$, $<out;\mathbf{p}'_N\mathbf{p}_{\overline{N}}|j_\mu(0)|0>$ etc. It should be pointed out that all these matrix elements are expressed through the one-variable vertices.

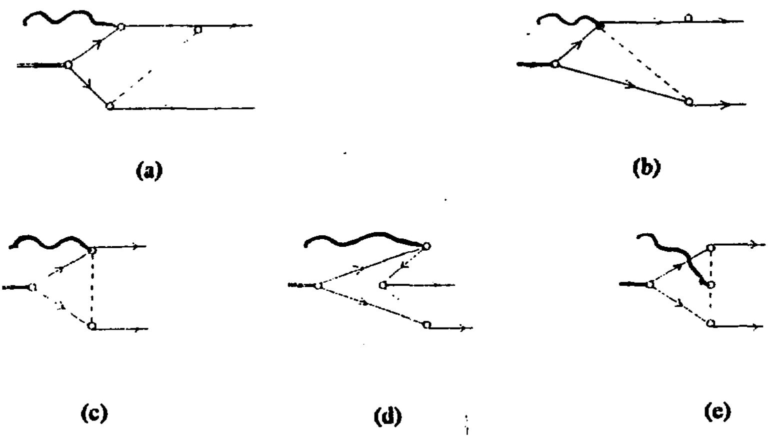


Fig.4. The dominant meson-exchange current for two nucleon system a,b) nonstatic corrections to the one-body current c, d, c) the leading order two-nucleons current in static limit.

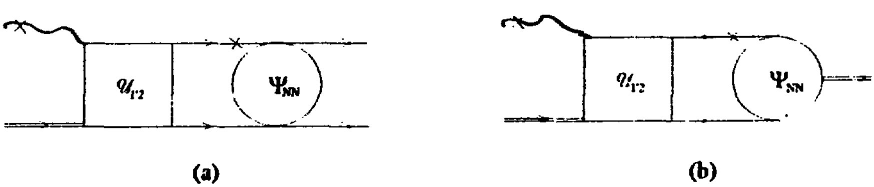


Fig.5. Diagrammatic representation of a) γd-NN transition amplitude given by eq (23) and b) γd-d vertex function, as it is defined by eq (24).

All relations for the coupled $\gamma d - NN$ or $ed - e'N'N'$ channels amplitudes which are derived in this paper, imply an obvious generalization for the more complete case of coupled $\gamma d - NN - \pi d - \Delta N - \Delta\Delta$ channels. The base of this generalization is the three-body relativistic theory of coupled $NN - \pi NN$ systems [29]. In ref. [30] the recipe of building relativistic equations is suggested for coupled $NN - \pi d - \Delta N$ channels as well as the ordinary set of coupled two-body equations.

4. GAUGE INVARIANCE

Let us consider the gauge conservation problem for the final form of γd scattering matrices (5a,b) and (6) with off-mass-shell photon. It is clear that for this it is enough to check the gauge invariance of initial expression (8) of spectral decomposition of half off-mass-shell $\gamma d - NN$ amplitude (6). Thus, if we take into account the photon currents conservation condition $\partial^{\mu} j_{\mu}(x) = 0$, then we obtain

$$\partial_{\mu} f^{\mu}_{1'2} \equiv \Delta w_{1'2} = < \mathbf{p'}_{N1} | [\dot{b}_{\mathbf{p'}_{N2}}(0), j_0(0)] | \mathbf{p}_d >$$

$$-i \sum_{n=d,NN,...} < \mathbf{p'}_{N1} | \eta_{\mathbf{p'}_{N2}}(0) | n; out > (2\pi)^3 \delta^{(3)}(\mathbf{p'}_{N1} + \mathbf{p'}_{N2} - \mathbf{P}_n) < out; n | j_0(0) | \mathbf{p}_d > \quad (29)$$

$$+i \sum_{m=N,\pi N,...} < \mathbf{p'}_{N1} | j_0(0) | m; out > (2\pi)^3 \delta^{(3)}(\mathbf{p'}_{N2} - \mathbf{p}_d + \mathbf{P}_m) < out; m | \eta_{\mathbf{p'}_{N2}}(0) | \mathbf{p}_d >$$

where $\dot{b}_{\mathbf{p}}(x^0) \equiv d/dx^0 b_{\mathbf{p}}(x^0) = -i \int d^3\mathbf{x} \exp(ipx) \overline{u}(\mathbf{p}) \eta(x)$

Hence, from the expression (8) we see that for the amplitude of deuteron photo disintegration reactions, which is independent of the zero components of photon field operators, the invariance under the gauge transformation for every choice of intermediate states in relations (8) and (29) is valid. Therefore, the photon-deuteron scattering amplitudes (23) and (24) with real photon field in asymptotic state is gauge invariant for arbitrary set of intermediate states. But for the electron-deuteron reactions the gauge invariance is fulfilled only by an infinite set of intermediate particle in eqs. (8) and (29).

As the simplest way to restore the gauge invariance for the ed scattering amplitude (6) with a finite number of particle in intermediate states, we suggest the following redefinition of $f_{1'2}$ (6) amplitude

$$\tilde{f}_{1'2} = f_{1'2} - \delta_{\eta 0} \frac{i}{q^0} \Delta w_{1'2}. \quad (30)$$

The additional term in (30) vanishes if we take into account in (8) and (29) the infinite set of intermediate states. But for the finite number of intermediate states in eq. (8) and (29) the redefined amplitude (30) insure the gauge invariance. Unlike to the gauge restoring recipes of refs. [1,2,3], in the formula (9) we have modified only zero component of current matrix elements. This modification corresponds to the redefinition of $U_{1'2}(E)$ potentials in resulting expressions (23) and (24) of electron-deuteron scattering amplitudes

$$\tilde{U}_{1'2}(E_{NN}) = U_{1'2}(E_{NN}) - \delta_{\eta 0} \frac{i}{q^0} \Delta w_{1'2} \quad (31)$$

After these redefinition the expressions of amplitudes for the reactions $ed - e'd'$ and $ed - e'NN$ (23) and (24) would be gauge invariant by every choice of intermediate states.

5. TEST FOR THE INTERMEDIATE PARTICLE NUMBER RESTRICTION IN MICROSCOPICAL POTENTIALS

Relation (29) presents an identity for equal time commutator of current operators and its matrix representation between complete set of "*out*" and "*in*" states. But for a finite set of intermediate states which will be, as a rule assumed by practical calculation, such a type of identities one could consider as an additional requirement for input vertex functions by construction of potential in the equations suggested. If we single out one-particle exchange terms in expression (29), then we get

$$< \mathbf{p}'_{N1}|\left[\dot{b}_{\mathbf{p}'_{N2}}(0), j_\mu(0)\right]|\mathbf{p}_d >= \mathcal{R}_\mu(\mathbf{p}'_{N1}\mathbf{p}'_{N2};\mathbf{p}_d) + i\sum_{n=d,NN}\left\{\ldots\right\} \quad (32)$$

$$-i\sum_{l=\rho}\left\{\ldots\right\} + i\sum_{\overline{m}=\overline{N}}\left\{\ldots\right\} + i\sum_{m=N}\left\{\ldots\right\} - i\sum_{l=\rho}\left\{\ldots\right\} + i\sum_{\overline{m}=\overline{N}}\left\{\ldots\right\} - i\sum_{\overline{n}=\overline{d}}\left\{\ldots\right\}$$

where the exact expressions of one intermediate particle terms inside of the curly brackets as well as the representation of sum of all other multi-particle intermediate terms which we have denoted as rest part $\mathcal{R}_\mu$ are derived from (29) after using cluster decomposition [21]. In fact, these expressions differ from the corresponding terms in V_μ (14) with only free Green functions in (14).

The second term of the right side of identity (32) contains the sought amplitudes of $\gamma d - d'$ and $\gamma d - NN$ transitions with off-shell photon field. But using the completeness condition (27), these s-channel terms can be rewritten as

$$\sum_{n=d,NN}\left\{f^{\dagger}_{1'n}f_{n2}\right\} = \sum_{\gamma,\gamma'=1,2} W^{\dagger}_{1'\gamma}(1-B)^{-1}_{\gamma\gamma'}W_{\gamma'2} \quad (33)$$

where we have used the following conditions

$$f_{\alpha\beta} = \sum_{\gamma} < \Phi_\alpha|\gamma > W_{\gamma\beta}; \quad f_{d\beta} = \sum_{\gamma} < \Phi_d|\gamma > W_{\gamma\beta} \quad (34)$$

Now substituting eq. (33) into identity (32) we obtain

$$\mathcal{R}_\mu(\mathbf{p}'_{N1}\mathbf{p}'_{N2};\mathbf{p}_d) =< \mathbf{p}'_{N1}|\left[\dot{b}_{\mathbf{p}'_{N2}}(0), j_\mu(0)\right]|\mathbf{p}_d > +i\sum_{\gamma,\gamma'=1,2} W^{\dagger}_{1'\gamma}(1-B)^{-1}_{\gamma\gamma'}W_{\gamma'2} \quad (35)$$

$$+i\sum_{l=\rho}\left\{\ldots\right\} - i\sum_{\overline{m}=\overline{N}}\left\{\ldots\right\}_i \sum_{m=N}\left\{\ldots\right\} + i\sum_{l=\rho}\left\{\ldots\right\} - i\sum_{\overline{m}=\overline{N}}\left\{\ldots\right\} + i\sum_{\overline{n}=\overline{d}}\left\{\ldots\right\}$$

The relation (35) one can consider as a test for the verification of limitation of intermediate particles number by construction of V_μ (14) in the framework of one-particle exchange model. So if in eq. (35) $\mathcal{R}_\mu \simeq 0$, then we can suppose that our choose of strong and electromagnetic vertices functions is successful. This requirement can be easily used if we allow for, that equal time commutator in (35) consists of one-variable $t = (p_d - p_{N1})^2$ vertices and other terms in (35) as well as V_μ (14) are nontrivial functions of more complicated variables.

6. CONCLUSION

The main achievement of the considered field-theoretical three-dimensional construction of gauge-invariant currents for scattering reactions is that here from the beginning as input functions one-variable relativistic invariant vertex functions are required. So the two-particle irreducible potentials $NN-NN$ and $\gamma d-NN$ reactions consist of equal-time commutators and on-mass-shell particle exchange terms which can be constructed directly from the phenomenoligical one-variable vertex functions. The analogical structure also has additional ΔW potential (29) which insures the gauge invariance for electron-deuteron scattering reactions. The equal time commutators in the considered potentials were the subject of investigation of current algebra [20,21]. Thus, these parts of potentials satisfy the corresponding sum rules and have the well known asymptotic behavior. Moreover, in the suggested formulation we can estimate the fitting vertices and effective potentials on the base of additional condition (35). This condition must be satisfied if the restriction of particle numbers in intermediate states in effective potential of solving equations is justifiable.

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