объединенный
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исследований
дубна

E4-94-106

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THE TFD TREATMENT OF THE QUASIPARTICLE-PHONON INTERACTION AT FINITE TEMPERATURE

Submitted to «Modern Physics Letters A»

1994

The study of phenomena occurring in hot nuclear systems in the framework provided by the nuclear many body theory has attracted attention of many authors ref.[1-7]. The problem of particular interest is collective excitations in these systems and their dependence on the temperature T. It is well proved for cold heavy nuclei that the main part of the width of collective giant resonance states is due to coupling of the RPA - states with more complex ones like 2p-2h or four quasiparticle or two phonon states ref.[8-10]. The same idea has been explored for giant resonances in hot nuclei as well [2]. But comparing the theoretical results with experimental data shows the discrepancy between both (see, e.g. the discussion of the problem in [6]). So, new attempts to resolve this problem seem to be desirable.

We formulate here an approach based on extension of the quasiparticle - phonon nuclear model (the QPM) [8, 9] to $T \neq 0$. The QPM gives reasonable description of spreading widths of giant resonances and other resonance - like structures in spectra of heavy nuclei at the excitation energies $E_x \leq 20 - 25$ MeV. Distinctive features of the QPM are the schematic (namely, separable) residual forces and the use of the RPA phonons as elementary blocks to describe excitations in even - even nuclei. Both ingredients simplify drastically formulas of the model as well as numerical calculations within it.

To extend the QPM to $T \neq 0$, we explore the formalism of the thermo field dynamics (the TFD) [10]. The main idea behind the TFD is to define a thermal vacuum $|0(\beta)\rangle$ such that the thermal expectation value of any operator

$$\ll A \gg = \frac{1}{Tr(exp(-\beta H))} Tr[A\ exp(-\beta H)]$$

($\beta = T^{-1}$) equals the expectation value with respect to the thermal state

$$\ll A \gg = \langle 0(\beta)|A|0(\beta)\rangle\ .$$

The extension of quantum field theory at $T = 0$ to finite temperature requires a doubling of the field degrees of freedom. In the TFD, a tilde conjugate operator $\tilde{A}$ is associated to any operator A acting in ordinary space through the tilde conjugation rules

$$(\tilde{AB}) = \tilde{A}\tilde{B}\ ; (c_1 A + c_2 B)^{\sim} = c_1^* \tilde{A} + c_2^* \tilde{B}\ ,$$

where A and B stand for any operators and c_1 , c_2 are c-numbers. The asterisk denotes the complex conjugate. The tilde operation commutes with the hermitian conjugation operation and any tilde and non-tilde operators are assumed to commute or anticommute with each other. For any system governed by the Hamiltonian H the whole Hilbert space now is spanned by the direct product of the eigen states of H and those of the tilde Hamiltonian $\tilde{H}$ having the same eigenvalues. The time - translation operator is not the energy operator H but the thermal Hamiltonian $\mathcal{H} = H - \tilde{H}$. This means that the properties of the system excitations are obtained by the diagonalization of $\mathcal{H}$. Since the TFD includes temperature as well as the space and time coordinates, there appears, in addition to these operator relations, a relation which determines the temperature. This relation is called the thermal state condition and it determines the thermal vacuum (or temperature dependent vacuum) $|0(\beta)\rangle$ which contains the thermally excited particles. This condition has the form

$$\alpha(x,t)|0(\beta)\rangle = \sigma_\alpha \tilde{\alpha}^+(x, t - i\beta/2)|0(\beta)\rangle \ ,$$

where σ_α is a certain phase factor. The Heisenberg equation, equal - time commutation relations, the tilde conjugation rules and the thermal state condition form the basic relations in the TFD.

Now we apply the outlined formalism to hot nuclear system governed by the Hamiltonian of the QPM. This Hamiltonian consists of the average fields for protons and neutrons, the monopole pp- and nn- pairing interactions and the separable multipole particle - hole interactions consisting of the isoscalar and isovector parts

$$H = H_{sp} + H_{pair} + H_{ph}$$

Within the QPM, one follows the standard way of transformation of the Hamiltonian of a system of interacting nucleons to that of interacting elementary excitation modes. The first step along this way is the Bogoliubov transformation from the creation and annihilation operators of particles and holes to the creation and annihilation operators of quasiparticles. To take into account the influence of temperature, we have to make additional canonical transformation to the operators of thermal quasiparticles that will annihilate the thermal vacuum (such operators have to exist due to the thermal state condition). So the final form of the transformation from the nucleon creation and annihilation operators a , a^+ , $\tilde{a}$, $\tilde{a}^+$ to thermal quasiparticles β , β^+ , $\tilde{\beta}$, $\tilde{\beta}^+$, which have $|0(\beta)\rangle$ as a vacuum, is the following [5, 11, 12] :

$$\begin{pmatrix} a_{jm} \\ a^+_{\overline{jm}} \\ \tilde{a}^+_{jm} \\ \tilde{a}_{\overline{jm}} \end{pmatrix} = \begin{pmatrix} A & B \\ -B & A \end{pmatrix} \begin{pmatrix} \beta_{jm} \\ \beta^+_{\overline{jm}} \\ \tilde{\beta}^+_{jm} \\ \tilde{\beta}_{\overline{jm}} \end{pmatrix} \quad (1)$$

$$A = x_j \begin{pmatrix} u_j & v_j \\ -v_j & u_j \end{pmatrix} \quad B = y_j \begin{pmatrix} u_j & v_j \\ -v_j & u_j \end{pmatrix}$$

To find the coefficients u_j, v_j, x_j, y_j, we minimize the grand thermodynamic potential $\Omega = \langle 0(\beta)|\mathcal{H}'|0(\beta)\rangle - TS$ (S is entropy of the system) for a system of nucleons governed by Hamiltonian $H' = H_{sp} + H_{pair}$ at $T = const.$ In other words, we find the condition of thermal equlibrium for a system.

After variation of Ω over u_j, v_j, x_j, y_j, we obtain for the coefficients u_j, v_j the relations that are well known in the theory of nuclear superfluidity at $T \neq 0$ with the Hamiltonian of Bardeen - Cooper - Shrieffer (see e.g. ref.[13]) and for the coefficients x_j, y_j the following expression :

$$x_j = \sqrt{1 - n_j}\ , y_j = \sqrt{n_j}$$

where n_j is the thermal Fermi occupation number

$$n_j = \frac{1}{1 + exp(\beta\varepsilon_j)}$$

(ε_j is the quasiparticle energy).

After the transformation (1) to the thermal quasiparticles the thermal Hamiltonian of the QPM takes the form

$$\mathcal{H} = \sum_{jm} \varepsilon_{jm}(\beta^+_{jm}\beta_{jm} - \tilde{\beta}^+_{jm}\tilde{\beta}_{jm}) -$$

$$-\frac{1}{2}\sum_{\lambda\mu}\sum_{\tau,\rho=\pm 1}(k_0^{(\lambda)} + \rho k_1^{(\lambda)})\{M^+_{\lambda\mu}(\tau)M_{\lambda\mu}(\rho\tau) - \tilde{M}^+_{\lambda\mu}(\tau)\tilde{M}_{\lambda\mu}(\rho\tau)\}\ ,$$

where

$$M^+_{\lambda\mu}(\tau) = \frac{(-)^{\lambda-\mu}}{\sqrt{2\lambda - 1}}\sum\nolimits^{\tau}_{jj'} f^{(\lambda)}_{jj'}\left\{[A^+_\beta(jj'; \lambda\mu) + \right.$$

$$\left. +(-)^{\lambda-\mu}A_\beta(jj'; \lambda - \mu)] + B_\beta(jj'; \lambda\mu)\right\} \quad (2)$$

$$A^+_\beta(jj';\lambda\mu) =$$

$$= \tfrac{1}{2}u^{(+)}_{jj'}(\sqrt{1-n_j}\sqrt{1-n_{j'}}[\beta^+_{jm}\beta^+_{j'm'}]_{\lambda\mu} - \sqrt{n_j}\sqrt{n_{j'}}[\tilde{\beta}^+_{\overline{jm}}\tilde{\beta}^+_{\overline{j'm'}}]_{\lambda\mu})-$$

$$-v^{(-)}_{jj'}\sqrt{1-n_j}\sqrt{n_{j'}}[\beta^+_{jm}\tilde{\beta}^+_{\overline{j'm'}}]_{\lambda\mu}$$

$$B_\beta(jj';\lambda\mu) =$$

$$= -v^{(-)}_{jj'}(\sqrt{1-n_j}\sqrt{1-n_{j'}}[\beta^+_{jm}\beta_{\overline{j'm'}}]_{\lambda\mu} + \sqrt{n_j}\sqrt{n_{j'}}[\tilde{\beta}_{jm}\tilde{\beta}^+_{\overline{j'm'}}]_{\lambda\mu})+$$

$$+u^{(+)}_{jj'}\sqrt{1-n_j}\sqrt{n_{j'}}([\beta^+_{jm}\tilde{\beta}_{j'm'}]_{\lambda\mu} + (-)^{\lambda-\mu}[\beta_{jm}\tilde{\beta}^+_{j'm'}]_{\lambda-\mu})$$

In these formulas we use the following notation: $f^{(\lambda)}_{jj'}$ is the reduced single - particle matrix element of the multipole operator; $\kappa^{(\lambda)}_0, \kappa^{(\lambda)}_1$ are the coupling constants of the isoscalar and isovector multipole - multipole interactions, respectively; $u^{(+)}_{jj'} = u_jv_{j'} + u_{j'}v_j$, $v^{(-)}_{jj'} = u_ju_{j'} - v_{j'}v_j$. The index τ is an isotopic index and takes two values, $\tau = n,p$. So the symbol $\sum^\tau$ means that the summation is taken only over neutron or proton single - particle states and changing the sign of τ means changing $n \leftrightarrow p$. The square brackets $[\;]_{\lambda\mu}$ stand for the coupling of single - particle angular momenta j,j' to the sum angular momentum λ. The bar over lower indices $\overline{jm}$ denotes the time reversal state.

One can easily see from (2) that the structure of the thermal Hamiltonian $\mathcal{H}$ in terms of the operators $\beta^+,\beta,\tilde{\beta}^+,\tilde{\beta}$ is the same as of the Hamiltonian H in terms of the Bogoliubov quasiparticles α^+,α (cf. [9]). The main difference is redefinition of vertices corresponding to terms of the same operator structure. For example, the coefficient at the term $A^+_\beta(j_1j_2;\lambda\mu)A^+_\beta(j_3j_4;\lambda\mu)$ depends now not only on the superfluid particle-hole factor $u^{(+)}_{j_1j_2}$ as at $T=0$ but on the particle - particle (or hole - hole) factor $v^{(-)}_{j_1j_2}$ as well. The thermal vacuum $|0(\beta)\rangle$ now formally plays the role which is similar to the role of the quasiparticle vacuum at $T=0$. So further derivation can be done in a parallel way with the $T=0$ case (see also [4, 5, 12]).

Firstly, we introduce a thermal phonon operator and redefine the thermal ground state as a phonon vacuum $|\Psi_0(\beta)\rangle$:

$$Q^+_{\lambda\mu i} = \frac{1}{2}\sum_{jj'}\Big(\psi^{\lambda i}_{jj'}[\beta^+_{jm}\beta^+_{j'm'}]_{\lambda\mu} + \tilde{\psi}^{\lambda i}_{jj'}[\tilde{\beta}^+_{\overline{jm}}\tilde{\beta}^+_{\overline{j'm'}}]_{\lambda\mu}+$$

$$+2\eta_{jj'}^{\lambda i}[\beta_{jm}^{+}\tilde{\beta}_{\overline{j'm'}}^{+}]_{\lambda\mu}+(-)^{\lambda-\mu}\phi_{jj'}^{\lambda i}[\beta_{jm}\beta_{j'm'}]_{\lambda-\mu}+$$

$$+(-)^{\lambda-\mu}\tilde{\phi}_{jj'}^{\lambda i}[\tilde{\beta}_{\overline{jm}}\tilde{\beta}_{\overline{j'm'}}]_{\lambda-\mu}+2(-)^{\lambda-\mu}\zeta_{jj'}^{\lambda i}[\beta_{jm}\tilde{\beta}_{\overline{j'm'}}]_{\lambda-\mu}\Big)$$

$$Q_i|\Psi_0(\beta)\rangle = 0$$

Then we suppose that the standard assumptions of the RPA are valid, namely, the number of thermal quasiparticles in the new vacuum state $|\Psi_0(\beta)\rangle$ is negligible and thermal phonon operators commute :

$$\langle\Psi_0(\beta)|\beta_{jm}^{+}\beta_{jm}|\Psi_0(\beta)\rangle \approx 0$$

$$\langle\Psi_0(\beta)|[Q_{\lambda\mu i}\ , Q_{\lambda'\mu'i'}^{+}]|\Psi_0(\beta)\rangle = \delta_{\lambda\lambda'}\delta_{\mu\mu'}\delta_{ii'}$$

With these assumptions it is easy to find the following constraint on the bifermionic amplitudes of the thermal phonon:

$$\frac{1}{2}\sum_{jj'}(\psi_{jj'}^{\lambda i})^2-(\phi_{jj'}^{\lambda i})^2+(\tilde{\psi}_{jj'}^{\lambda i})^2-(\tilde{\phi}_{jj'}^{\lambda i})^2+2(\eta_{jj'}^{\lambda i})^2-2(\zeta_{jj'}^{\lambda i})^2=\delta_{\lambda\lambda'}\delta_{ii'} \quad (3)$$

The energy $\omega_{\lambda i}$ of the one-phonon excitation $Q_{\lambda\mu i}^{+}|\Psi_0(\beta)\rangle$ and the amplitudes $\psi,\phi,\eta,\ \tilde{\psi},\tilde{\phi},\zeta$ can be found by minimizing the expectation value of $\mathcal{H}$ with respect to the one-phonon state at the constraint (3). The corresponding formulae can be found in [12] (they have been derived by other methods in [13, 15] as well).

Making the inverse transformation from the operators $Q_{\lambda\mu i}^{+}, Q_{\lambda\mu i}$ to bifermionic operators $[\beta_1^{+}\beta_2^{+}]_{\lambda\mu}$ etc. with the RPA values of the amplitudes ψ,ϕ,η, $\tilde{\psi},\tilde{\phi},\zeta$, we get the following expression for $\mathcal{H}$ in terms of thermal quasiparticles and phonons:

$$\mathcal{H}=\sum_{\lambda\mu i}\omega_{\lambda i}(Q_{\lambda\mu i}^{+}Q_{\lambda\mu i}-\tilde{Q}_{\lambda\mu i}^{+}\tilde{Q}_{\lambda\mu i})-$$

$$-\frac{1}{2\sqrt{2}}\sum_{\lambda\mu i}\sum_{\tau}\sum_{jj'}^{\tau}\frac{f_{jj'}^{(\lambda)}}{\sqrt{\mathcal{N}_{\tau}^{\lambda i}}}\Big\{((-)^{\lambda-\mu}Q_{\lambda\mu i}^{+}+Q_{\lambda-\mu i})B_{\beta}(jj';\lambda-\mu)- \quad (4)$$

$$-((-)^{\lambda-\mu}\tilde{Q}_{\lambda\mu i}^{+}+\tilde{Q}_{\lambda-\mu i})\tilde{B}_{\beta}(jj';\lambda-\mu)+h.c.\Big\}$$

In (4) we introduce the tilde operators $\tilde{Q}_{\lambda\mu i}$ and $\tilde{B}_{\beta}(jj';\lambda-\mu)$, for convenience. $\mathcal{N}_{\tau}^{\lambda i}(\tau=n,p)$ are something like normalization factors of the neutron and proton parts of the one-phonon wave function. The smaller is the value of $\mathcal{N}_\tau^{\lambda i}$ the higher is the collectivity of the phonon.

The second term in (4) (we denote it by $\mathcal{H}_{qph}$) is the interaction term of thermal quasiparticles and thermal phonons (i.e. phonons built of pairs of thermal quasiparticles). This term mixes the states with a different phonon number, and due to this mixing the strength of the RPA-state is fragmented over some excitation energy interval. In other words, the term $\mathcal{H}_{qph}$ produces a spreading width of a thermal one-phonon state.

To describe the fragmentation of thermal phonons, we use the variational method with a trial wave function of the form:

$$|\Psi_\beta(JM\nu)\rangle = \Big\{\sum_i R_i(J\nu)Q^+_{JMi}+$$

$$+\sum_{\lambda_1 i_1 \lambda_2 i_2} P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)\left[Q^+_{\lambda_1\mu_1 i_1}Q^+_{\lambda_2\mu_2 i_2}\right]_{JM}\Big\}|\Psi_0(\beta)\rangle \tag{5}$$

So we take into account the effect of the interaction between phonons on excited states but not on the vacuum state which is supposed to be the thermal phonon vacuum $|\Psi_0(\beta)\rangle$ as before. The wave function (5) has to be normalized

$$\sum_i (R_i(J\nu))^2 + 2\sum_{\lambda_1 i_1 \lambda_2 i_2}(P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu))^2 = 1 \tag{6}$$

To find the energy of the state (5) $\eta_{J\nu}$ and the coefficients R, P, we again minimize the expectation value of $\mathcal{H}$ over $|\Psi_\beta(JM\nu)\rangle$ at the constraint (6). Note that the term $\mathcal{H}_{qph}$ didn't contribute to the expectation value of $\mathcal{H}$ over a one phonon state (i.e. in the random phase approximation). We have

$$\langle\Psi_\nu(JM)|\mathcal{H}|\Psi_\nu(JM)\rangle = \sum_i \omega_{Ji}[R_i(J\nu)]^2+$$

$$+2\sum_{\lambda_1 i_1 \lambda_2 i_2}(\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2})[P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)]^2 + 2\sum_i \sum_{\lambda_1 i_1 \lambda_2 i_2} R_i(J\nu)P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji) \tag{7}$$

where

$$U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji) = \langle\Psi_0(\beta)|Q_{JMi}\mathcal{H}_{qph}[Q^+_{\lambda_1\mu_1 i_1}Q^+_{\lambda_2\mu_2 i_2}]_{JM}|\Psi_0(\beta)\rangle =$$

$$= U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji,n) + U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji,p)$$

is the coupling matrix element of one- and two-phonon configurations. The function $U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)$ is a quite complicated bilinear form of the phonon amplitudes $\psi, \phi, \eta, \tilde{\psi}, \tilde{\phi}, \zeta$, namely:

$$U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji,\tau) = -\tfrac{1}{\sqrt{2}}\sqrt{2\lambda_1+1}\sqrt{2\lambda_2+1}\times$$

$$\times\sum_{j_1 j_2 j_3}{}^{\tau}\Bigg[(-)^J\Gamma^{\lambda_2 i_2}_{j_1 j_2}\left\{\begin{array}{ccc}\lambda_2 & \lambda_1 & J\\ j_3 & j_2 & j_1\end{array}\right\}\mathcal{K}^{\lambda_1 i_1 J i}_{j_3 j_2 j_1}+$$

$$+(-)^{\lambda_1-\lambda_2}\Gamma^{\lambda_1 i_1}_{j_1 j_2}\left\{\begin{array}{ccc}\lambda_1 & \lambda_2 & J\\ j_3 & j_2 & j_1\end{array}\right\}\mathcal{K}^{\lambda_2 i_2 J i}_{j_3 j_2 j_1}+$$

$$+(-)^{J-\lambda_1}\Gamma^{Ji}_{j_1 j_2}\left\{\begin{array}{ccc}J & \lambda_1 & \lambda_2\\ j_3 & j_2 & j_1\end{array}\right\}\mathcal{L}^{\lambda_1 i_1 \lambda_2 i_2}_{j_3 j_2 j_1}\Bigg]$$

Where

$$\mathcal{K}^{\lambda_1 i_1 J i}_{j_3 j_2 j_1} = v^{(-)}_{j_1 j_2}\sqrt{1-n_{j_1}}\sqrt{1-n_{j_2}}(-)^{j_1+j_3+\lambda_1+J}\times$$

$$\times(\psi^{\lambda_1 i_1}_{j_1 j_3}\psi^{Ji}_{j_2 j_3} + +\phi^{\lambda_1 i_1}_{j_1 j_3}\phi^{Ji}_{j_2 j_3} + \eta^{\lambda_1 i_1}_{j_1 j_3}\eta^{Ji}_{j_2 j_3} + \zeta^{\lambda_1 i_1}_{j_1 j_3}\zeta^{Ji}_{j_2 j_3})+$$

$$+u^{(+)}_{j_1 j_2}\sqrt{1-n_{j_1}}\sqrt{n_{j_2}}(-)^{\lambda_1+j_1+j_2}\times$$

$$\times(\psi^{\lambda_1 i_1}_{j_1 j_3}\eta^{Ji}_{j_3 j_2} + \phi^{\lambda_1 i_1}_{j_1 j_3}\zeta^{Ji}_{j_3 j_2} + \eta^{\lambda_1 i_1}_{j_1 j_3}\tilde{\psi}^{Ji}_{j_3 j_2} + \zeta^{\lambda_1 i_1}_{j_1 j_3}\tilde{\phi}^{Ji}_{j_3 j_2})-$$

$$-u^{(+)}_{j_1 j_2}\sqrt{n_{j_1}}\sqrt{1-n_{j_2}}(-)^J(\eta^{\lambda_1 i_1}_{j_3 j_1}\psi^{Ji}_{j_2 j_3} + \zeta^{\lambda_1 i_1}_{j_3 j_1}\phi^{Ji}_{j_2 j_3} + \tilde{\psi}^{\lambda_1 i_1}_{j_3 j_1}\eta^{Ji}_{j_2 j_3} + \tilde{\phi}^{\lambda_1 i_1}_{j_3 j}\zeta^{Ji}_{j_2 j_3})-$$

$$-v^{(-)}_{j_1 j_2}\sqrt{n_{j_1}}\sqrt{n_{j_2}}(-)^{j_2+j_3}(\eta^{\lambda_1 i_1}_{j_3 j_1}\eta^{Ji}_{j_3 j_2} + \zeta^{\lambda_1 i_1}_{j_3 j_1}\zeta^{Ji}_{j_3 j_2} + \tilde{\psi}^{\lambda_1 i_1}_{j_3 j_1}\tilde{\psi}^{Ji}_{j_3 j_2} + \tilde{\phi}^{\lambda_1 i_1}_{j_3 j_1}\tilde{\phi}^{Ji}_{j_3 j_2})$$

$$\mathcal{L}^{\lambda_1 i_1 \lambda_2 i_2}_{j_3 j_2 j_1} = v^{(-)}_{j_1 j_2}\sqrt{1-n_{j_1}}\sqrt{1-n_{j_2}}(-)^{j_1+j_3+\lambda_1+\lambda_2}\times$$

$$\times(\psi^{\lambda_1 i_1}_{j_1 j_3}\phi^{\lambda_2 i_2}_{j_2 j_3} + \phi^{\lambda_1 i_1}_{j_1 j_3}\psi^{\lambda_2 i_2}_{j_2 j_3} + \eta^{\lambda_1 i_1}_{j_1 j_3}\zeta^{\lambda_2 i_2}_{j_2 j_3} + \zeta^{\lambda_1 i_1}_{j_1 j_3}\eta^{\lambda_2 i_2}_{j_2 j_3})+$$

$$+u^{(+)}_{j_1 j_2}\sqrt{1-n_{j_1}}\sqrt{n_{j_2}}(-)^{\lambda_1+j_1+j_2}\times$$

$$\times(\psi^{\lambda_1 i_1}_{j_1 j_3}\zeta^{\lambda_2 i_2}_{j_3 j_2} + \phi^{\lambda_1 i_1}_{j_1 j_3}\eta^{\lambda_2 i_2}_{j_3 j_2} + \eta^{\lambda_1 i_1}_{j_1 j_3}\tilde{\phi}^{\lambda_2 i_2}_{j_3 j_2} + \zeta^{\lambda_1 i_1}_{j_1 j_3}\tilde{\psi}^{\lambda_2 i_2}_{j_3 j_2})-$$

$$-u^{(+)}_{j_1 j_2}\sqrt{n_{j_1}}\sqrt{1-n_{j_2}}(-)^{\lambda_2}(\zeta^{\lambda_1 i_1}_{j_3 j_1}\psi^{\lambda_2 i_2}_{j_2 j_1} + \eta^{\lambda_1 i_1}_{j_3 j_1}\phi^{\lambda_2 i_2}_{j_2 j_3} + \tilde{\phi}^{\lambda_1 i_1}_{j_3 j_1}\eta^{\lambda_2 i_2}_{j_2 j_3} + \tilde{\psi}^{\lambda_1 i_1}_{j_3 j_1}\zeta^{\lambda_2 i_2}_{j_2 j_3})-$$

$$-v^{(-)}_{j_1 j_2}\sqrt{n_{j_1}}\sqrt{n_{j_2}}(-)^{j_2+j_3}(\eta^{\lambda_1 i_1}_{j_3 j_1}\zeta^{\lambda_2 i_2}_{j_3 j_2}+\zeta^{\lambda_1 i_1}_{j_3 j_1}\eta^{\lambda_2 i_2}_{j_3 j_2}+\tilde{\psi}^{\lambda_1 i_1}_{j_3 j_1}\tilde{\phi}^{\lambda_2 i_2}_{j_3 j_2}+\tilde{\phi}^{\lambda_1 i_1}_{j_3 j_1}\tilde{\psi}^{\lambda_2 i_2}_{j_3 j_2})$$

After variation of (7) at the constraint (6) one gets the homogeneous system of linear equations with the coefficients depending on energy $\eta_{J\nu}$. The system can be resolved if $\eta_{J\nu}$ is the root of the following secular equation:

$$det|(\omega_{Ji}-\eta_{J\nu})\delta_{ii'}-\frac{1}{2}\sum_{\lambda_1 i_1 \lambda_2 i_2}\frac{U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji')}{\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}-\eta_{J\nu}}|=0 \qquad (8)$$

Equation (8) for the energies of excited states built on the thermal ground (compound) state of a hot nucleus together with the expression for $U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji')$ is the main result of the present paper. Formally, eq.(8) has the same form as at $T=0$ (see, e.g. [9]). The difference is that now $U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)$ depends on the temperature through the thermal occupation numbers (not only directly but through the amplitudes ψ, ϕ , η etc. as well). Moreover, the energies of one-phonon states $\omega_{\lambda i}$ are calculated in the thermal RPA and the number of phonons of given multipolarity at finite temperature is twice as large as at $T=0$ because of new poles $\varepsilon_j - \varepsilon_{j'}$ which appear in the thermal RPA equation.

In [12], we have derived the expressions for the thermal RPA amplitudes $\psi, \phi, \eta, \tilde{\psi}, \tilde{\phi}$, ζ. In the leading order the non-tilde amplitudes ψ, ϕ are proportional to $(1-n_j)$, the mixed amplitudes η, ζ are proportional to $n_j^{1/2}$ and the tilde amplitudes $\tilde{\psi}, \tilde{\phi}$ are proportional to n_j. So, at $T \to 0$ only the terms containing ψ, ϕ will survive, and, as a result, one gets for $U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)$ the same expression as in refs.[8, 9]. It means that there is a natural correspondence between the results of our approach at $T \neq 0$ and at $T=0$.

In the above-stated consideration, we followed quite closely the way outlined in [4, 5]. The difference from, e.g. [4], is due to a specific form of a residual interaction (which is now taken to be separable for simplicity) and the phonon language when we go beyond the thermal RPA. It seems interesting to clarify in detail what is the difference between the second thermal RPA approach of [4] and our way to take into account the coupling with comlex configurations by mixing one- and two- thermal phonon states.

But it seems that the present TFD approach to the damping of nuclear excitations at finite temperature differs quite noticeably from that of papers [2, 3]. In those papers, the Matsubara formalism has been used but the main point was that the thermal particle - hole and the thermal phonon excitations have been considered on equal footing, i.e., both systems were heated and both Fermi and Bose thermal occupation numbers came into play, respectively. In the TFD approach, the thermal phonons are formed of the thermal quasiparticles but the phonon system itself is not heated and, therefore, Bose thermal occupation numbers didn't appear in our consideration. Certainly, one can project the initial Hamiltonian from bifermion to boson space using some kind of boson expansion and only after that make the TFD transformation. This way has been discussed by Hatsuda [5], for example. Then, Bose thermal occupation numbers appear naturally but the structure of bosons has to be calculated at $T = 0$. This is not the case e.g. in [2] where the structure of phonons have been calculated in *the thermal RPA*. It seems to us that this is a kind of double counting.

In conclusion, we have considered the TFD extension of the quasiparticle - phonon nuclear model to a finite temperature. Using the thermal Bogoliubov transformation we have derived the thermal QPM Hamiltonian $\mathcal{H}_{QPM}$ and then expressed it in terms of thermal quasiparticles and thermal RPA - phonons. At the last stage, we have diagonalized it approximately within the space of one- and two- thermal phonon states. We have derived an expression for the coupling matrix element between thermal phonons and an equation for the energies of excited states taking into account the interaction between one- and two thermal phonon configurations.

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Received by Publishing Department
on March 29, 1994.