

СООБЩЕНИЯ  
ОБЪЕДИНЕННОГО  
ИНСТИТУТА  
ЯДЕРНЫХ  
ИССЛЕДОВАНИЙ

ДУБНА



C34/a

J-23

630/2-76

23/11-76

E4 - 9358

D.Janssen, R.V.Jolos

ON THE ANOMALOUS  $0_2^+$  EXCITED STATES  
IN SPHERICAL EVEN-EVEN NUCLEI

1975

**E4 - 9358**

**D.Janssen, R.V.Jolos**

**ON THE ANOMALOUS  $0_2^+$  EXCITED STATES  
IN SPHERICAL EVEN-EVEN NUCLEI**

Recent accumulation of experimental data /1/ shows that the properties of the  $0_2^+$  excited states in some Ge, Se and Mo isotopes differ very much from those of most of the two-phonon  $0^+$  states investigated up to now. These unusual  $0_2^+$  states lie just above or even below the first  $2_1^+$  states (Table 1). The ratios of the E2-transition probabilities are slightly larger than the vibrational limit (2) (Table 2). While in the usual spherical nuclei this ratio is approximately equal to unity. It is interesting that these unusual  $0_2^+$  states appear in a small number of isotopes:  $^{72,74}\text{Se}$  and  $^{96,98,100}\text{Mo}$ . The other germanium and selenium isotopes are usual vibrational nuclei. The heavy molybdenum isotopes are the deformed ones.

To understand these facts it is necessary to answer the following questions:

1. How to explain, in principle, in the framework of the collective model the appearance of the  $0_2^+$  states with such unusual properties?

2. Why these  $0_2^+$  states are found only in  $^{72,74}\text{Se}$ ,  $^{96,98,100}\text{Mo}$  and are not in other nuclei?

A small value of the static quadrupole moment of the first  $2^+$  state in  $^{70}\text{Ge}$  and the

Table 1  
The energies of the  $0_2^+$  states

	$^{70}\text{Ge}$	$^{72}\text{Ge}$	$^{72}\text{Se}$	$^{74}\text{Se}$	$^{96}\text{Mo}$	$^{98}\text{Mo}$	$^{100}\text{Mo}$
$\frac{E(0_2^+)}{E(2_1^+)}$	1.17	0.83	1.08	1.34	1.48	0.93	1.29

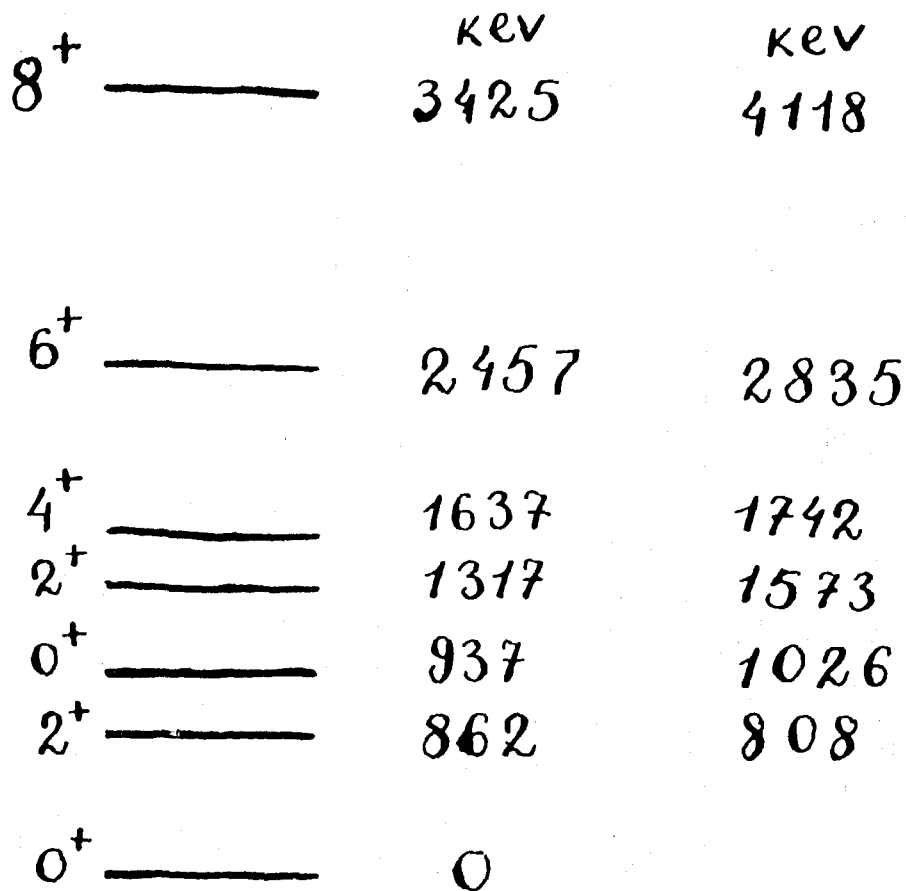


Fig. 1. Energy spectrum of  $^{72}\text{Se}$ .

fact that the ratio of the effective deformations of  $0_2^+$  and  $0_1^+$  states:

$$\frac{\langle 0_2^+ | \beta^2 | 0_2^+ \rangle}{\langle 0_1^+ | \beta^2 | 0_1^+ \rangle} = \frac{\sum_i B(E2; 0_2^+ \rightarrow 2_i^+)}{\sum_i B(E2; 0_1^+ \rightarrow 2_i^+)}$$

in these nuclei are approximately equal or smaller than unity (with the exception of  $^{72}\text{Se}$ ) both indicate that  $0_2^+$  states in  $^{70,72}\text{Ge}$ ,  $^{74}\text{Se}$ ,  $^{96,98,100}\text{Mo}$  are not deformed ones. It is possible that the  $0_2^+$  state in  $^{72}\text{Se}$  is deformed but in this nucleus the mixing of the spherical and deformed components is essential.

The explanation of unusual properties of the  $0_2^+$  states in molybdenum isotopes was suggested in /2/. In this work the consideration of the low-lying quadrupole excitations in Mo was based on the collective Hamiltonian which general structure was determined by the form of the microscopical Hamiltonian. The numerical values of the parameters were fixed so as to describe the energies of the lowest states in the best way. In this approach the ratios of the E2-transition probabilities depend on one parameter only.

It was shown that small energies of the  $0_2^+$  states can be explained by the large value of the coefficient  $W_{31}$  for the third order anharmonic terms in the collective Hamiltonian

$$W_{31} ([b_2^+ b_2^+ b_2 |_{00} \hat{F} + \text{h.c.}),$$

where  $\hat{F}$  is the factor which takes into account the Pauli principle.

It is important also that the role of the fourth order anharmonicities is relatively small.

Using this idea good results have been obtained not only for the energies but for E2-transition probabilities in  $^{96,100}\text{Mo}$  isotopes also.

In  $^{98}\text{Mo}$  the ratio  $E(0_2^+)/E(2_1^+)$  is minimal in comparison with  $^{96,100}\text{Mo}$ . As a result the largest value of  $W_{31}$  is necessary to describe this fact. Unfortunately it is impossible to explain the ratios of the E2-transition probabilities in  $^{98}\text{Mo}$  with the same value of  $W_{31}$ . We can explain the electromagnetic properties of  $^{98}\text{Mo}$  only, if we take a smaller value of  $W_{31}$  than it is required for the description of  $E(0_2^+)/E(2_1^+)$  ratio. It means, that there must exist an additional interaction which lowers the energies of the  $0_2^+$  states.

There is the other indication on an additional interaction: the appearance of the two  $2^+$  states in the two-phonon region instead of one as in the most of spherical nuclei. It was shown in <sup>/3/</sup> that the introduction of the phonon-phonon interaction of the type

$$\sum_{IM} g_I [b_2^+ b_2^+]_{IM} [b_2 b_2]_{IM}$$

explains the appearance of the two collective  $2^+$  states in the same energy region. The situation with  $0_2^+$  states in  $^{72,74}\text{Se}$  in many respects is the same as in molybdenum isotopes. Using the same collective Hamiltonian as in <sup>/2/</sup> we can describe energy spectrum of  $^{72}\text{Se}$  (fig. 1). It is interesting to note that the results obtained in <sup>/2/</sup> for E2-tran-

Table 2  
 The experimental values of the  $\frac{B(E2; 0_2^+ \rightarrow 2_1^+)}{B(E2; 2_1^+ \rightarrow 0_1^+)}$

	$^{70}\text{Ge}$	$^{72}\text{Se}$	$^{74}\text{Se}$	$^{96}\text{Mo}$	$^{100}\text{Mo}$
$\frac{B(E2; 0_2^+ \rightarrow 2_1^+)}{B(E2; 2_1^+ \rightarrow 0_1^+)}$	0.56	7.3	2.04	2.4	1.8

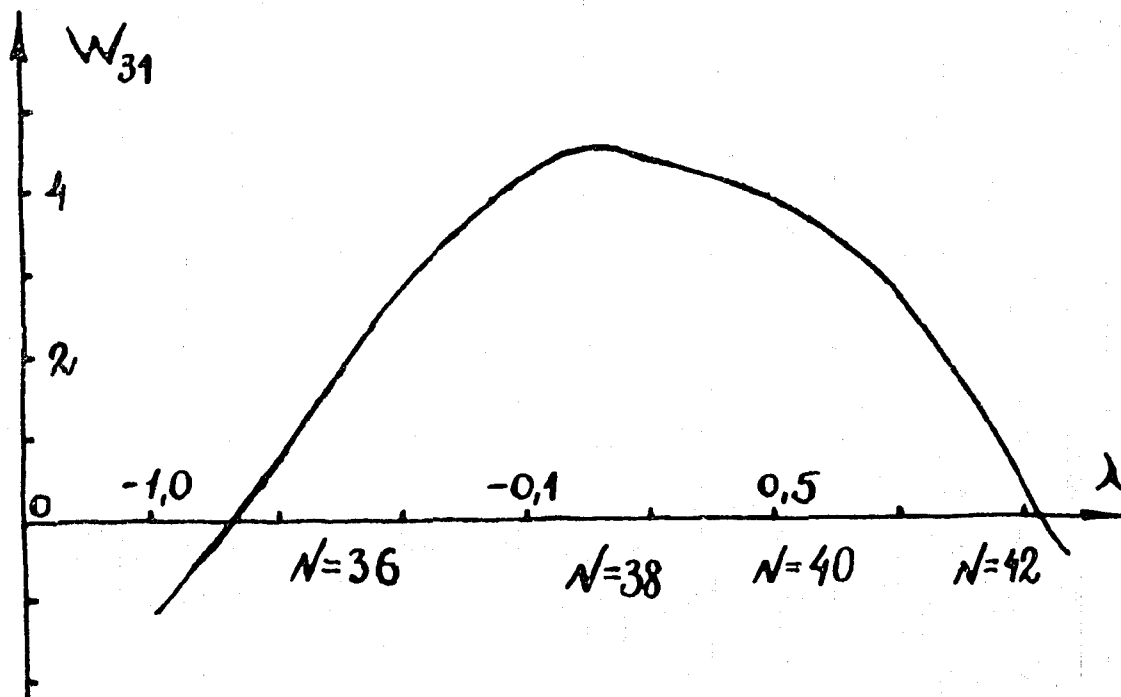


Fig. 2. Dependence of  $W_{31}$  coefficient in Ge and Se isotopes on neutron number  $N$  ( $W_{31}$  in arbitrary units).

sition probabilities when  $W_{31}$  has the largest value are in quantitative agreement with the experimental data for  $^{72}\text{Se}$  (Table 3).

Thus, the phenomenological approach shows that the reason for the unusual behaviour of  $0_2^+$  states in some spherical nuclei is the large value of the third order anharmonic terms in the collective Hamiltonian. Moreover the presence of the phonon-phonon interaction in the collective Hamiltonian which can be due to the coupling of quadrupole collective and single-particle or pairing-vibrational degrees of freedom is principally important. This interactions lowers the energies of the  $0_2^+$  excited states but practically does not influence the results for E2-transition probabilities.

Consider the situation from the microscopical point of view. The results of calculations of the neutron part of  $W_{31}$  for different values of the chemical potential  $\lambda$  (which corresponds to the neutron numbers  $N = 34-44$ , i.e., Ge and Se isotopes) are presented in fig. 2. Only the single-particle states of the unfilled shell have been taken into account in the calculations as we are interested in the variable part of  $W_{31}$ . For this reason we do not consider the proton part of  $W_{31}$ . It is seen that  $W_{31}$  has a maximum for neutron numbers  $N = 38, 40$ , i.e., for  $^{70,72}\text{Ge}$  and  $^{72,74}\text{Se}$ , where the low-lying  $0_2^+$  states have been found. The large contribution in  $W_{31}$  in the region of maximum gives the single particle state  $g_{9/2}$ . With increasing  $N$  from  $N = 40$ ,  $\lambda$  becomes closer and closer to  $g_{9/2}$ . As a result the coefficient  $u^2(g_{9/2}) - v^2(g_{9/2})$  decreases, and correspondingly the contribution of  $g_{9/2}$  to  $W_{31}$  becomes smaller and smaller.



Table 3

E2-transition probabilities in  $^{72}\text{Se}$ 

$\frac{B(E2; I_1 \rightarrow I_2)}{B(E2; J_1 \rightarrow J_2)}$	exp.	theor.	vibrational limit
$\frac{2_2^+ \rightarrow 2_1^+}{2_2^+ \rightarrow 0_1^+}$	$1.5 \pm 0.3$	0.4	
$\frac{2_2^+ \rightarrow 2_1^+}{2_2^+ \rightarrow 0_2^+}$	1	0.3	
$\frac{0_2^+ \rightarrow 2_1^+}{2_1^+ \rightarrow 0_1^+}$	7.3	4.2	2
$\frac{4_1^+ \rightarrow 2_1^+}{2_1^+ \rightarrow 0_1^+}$	1.7	2.3	2

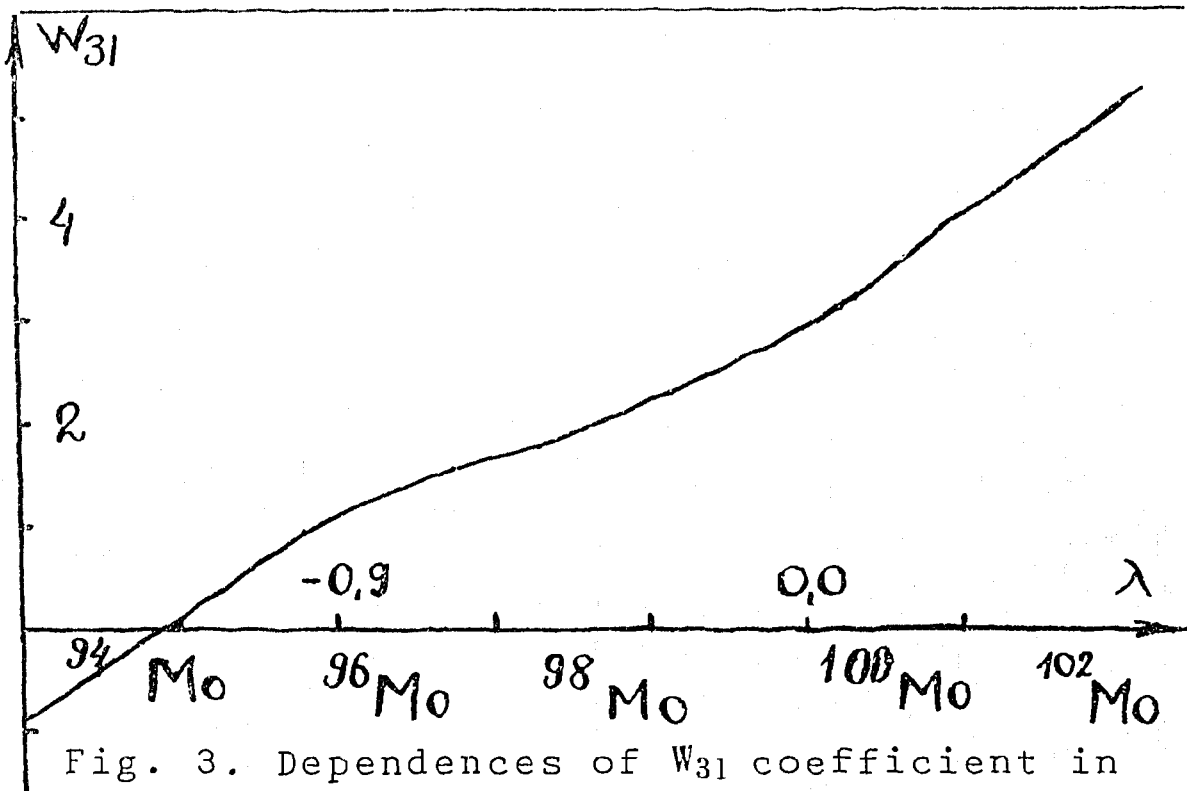


Fig. 3. Dependences of  $W_{31}$  coefficient in isotopes on neutron number  $N$ . ( $W_{31}$  in arbitrary units).

The analogous results for molybdenum isotopes are presented in fig. 3. The single-particle scheme has been taken from ref. /4/. It is seen that the coefficient  $W_{31}$  increases with increasing the number of neutrons in the unfilled shell and has no maxima for  $^{96,98,100}\text{Mo}$  in contrast to the situation in Ge-Se region. It corresponds to the fact that heavy molybdenum isotopes are deformed ones. The values of  $W_{31}$  in  $^{96,98,100}\text{Mo}$  are large enough to lower essentially the energies of the  $0_2^+$  states.

Thus, the microscopical calculations support the assumption on the increase of the role of the third order anharmonicity in  $^{70,72}\text{Ge}$  and  $^{72,74}\text{Se}$  in comparison with the neighbouring isotopes and about the important role of this effect in  $^{96,98,100}\text{Mo}$ .

In the recent work /5/, the coupling of the collective quadrupole and pairing-vibrational degrees of freedom have been considered. Let us consider the single-particle neutron scheme for Ge and Se isotopes (fig. 4). It was shown in /5/ that when the chemical potential lies near the level with  $j = 1/2$  (i.e., for  $N=38,40$ ) then the coupling of the collective quadrupole and pairing-vibrational degrees of freedom increases sharply.

The single-particle neutron scheme for molybdenum isotopes is shown in fig. 5. This scheme is similar to that in fig. 4: the state with  $j = 1/2$  lies higher than the single-particle state ( $g_{7/2}$ ) with large value of the angular momentum, and large value of diagonal matrix element of the operator  $r^2 Y_{2\mu}$ . Such state gives large contribution to the coefficient  $W_{31}$ . However

Fig. 4. Single-particle neutron scheme for Ge-Se region.

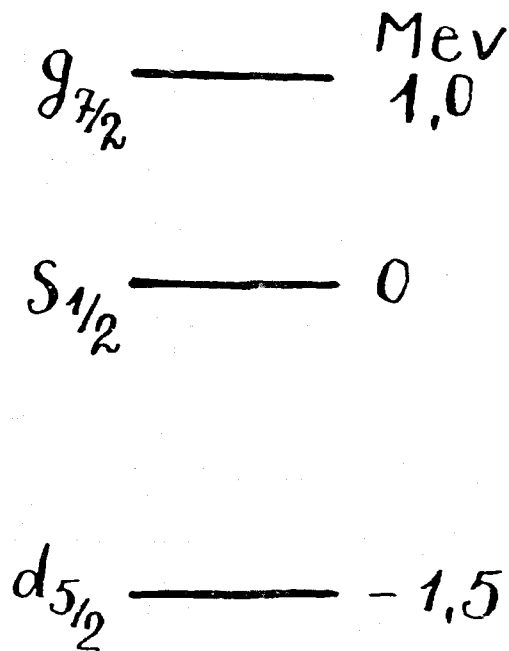
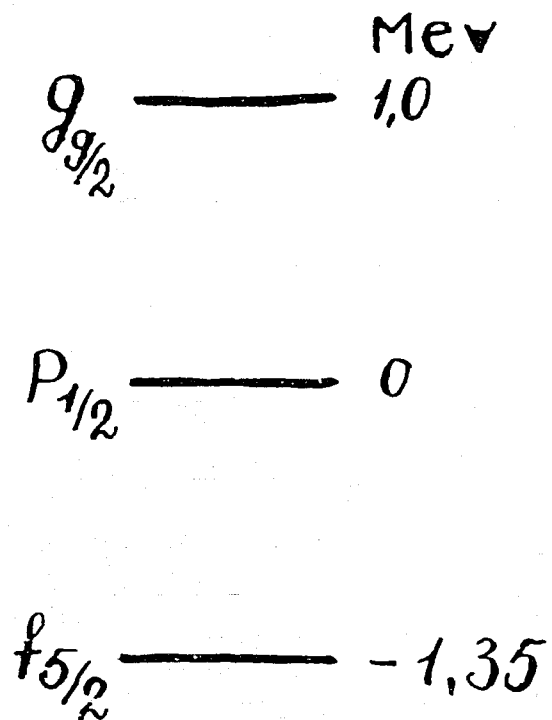


Fig. 5. Single-particle neutron scheme for Mo isotopes.

unlike the situation in fig. 4,  $g_{7/2}$  state has the same parity as the  $s_{1/2}$  and  $d_{5/2}$  single-particle states lying below. For this reason the two-quasiparticle component ( $g_{7/2} d_{5/2}$ ) gives the contribution to the phonon wave function and the coefficient  $W_{31}$ . But this contribution is relatively small in comparison with that of the component ( $g_{7/2} g_{7/2}$ ) due to the fact that

$$\frac{\langle d_{5/2} || r^2 Y_2 || g_{7/2} \rangle}{\langle g_{7/2} || r^2 Y_2 || g_{7/2} \rangle} \approx \frac{1}{6} .$$

The chemical potential lies near  $s_{1/2}$  in  $^{98,100}\text{Mo}$ . For this reason the coupling between the collective quadrupole and pairing-vibrational degrees of freedom is the largest in these isotopes.

Thus, the microscopical consideration shows that in  $^{70,72}\text{Ge}$ ,  $^{72,74}\text{Se}$  and  $^{98,100}\text{Mo}$  isotopes the third order anharmonicity in the collective quadrupole Hamiltonian and the coupling between the quadrupole and the pairing vibrational degrees of freedom are very important. In the framework of the phenomenological approach it was shown that these two facts can explain the energies and the electromagnetic properties of the unusual  $0\frac{1}{2}$  states.

## REFERENCES

1. J.H. Hamilton. Proc. Int. Conf. on Reactions Between Complex Nuclei, Nashville, USA (1974) vol. 2, p. 501; J. Barrette Nucl. Phys., A235, 154 (1974).

2. R.V.Jolos, F.Donau, V.G.Kartavenko, D.Janssen. *Jadernajy Fisica*, 20, 310 (1974).
3. R.V.Jolos, D.Janssen. Communication of JINR, P4-8743, Dubna, 1975.
4. S.Takeda, S.Yamaji, K.Matzuda et al. *Journ. Phys. Soc. Jap.*, 34, 1115 (1973).
5. S.Iwasaki, T.Marumori, F.Sakata and K.Takada. Preprint INS, 1975.

Received by Publishing Department  
on December 2, 1975.