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1. INTRODUCTION

The question of a microscopic derivation of the Coriolis interaction concept became of interest recently, especially in connection with such properties of deformed nuclei, as anomalous band structures, aligned bands and back-bending. A new result is the influence of the residual interaction in the p-p and p-h channels, which manifests itself in the so-called Coriolis attenuation. This connection was recently found out by I.Hamamoto[1] and by K.Hara[2].

The aim of this paper is to show how dynamical attenuation comes in if the microscopically consequent angular momentum projection technique is used as a starting point. In chapter 2 the concept of deformed states, depending on the spin, is used. This is a variant of the so-called projection after variation method. For odd-mass deformed nuclei a coupled system for the admixture of one quasi-particle states with $\Delta K = \pm 1$ and of three quasi-particle states is derived. These equations are the generalization of the well-known Thouless-equations. Formerly they have been found by Klein et al.[3] in their self-consistent method of collective motion, but the connection to attenuation has not been discussed.

In chapter 3 starting from our former investigations of band mixing the connection between Thouless-equations and attenuation is established. It is shown that for factorized interactions only the multipoles with odd time-reversal behaviour contribute. Further it is shown, that the decoupling parameter for $K=1/2$ bands is also influenced by the dynamical effect in the same way, as the non-diagonal matrix elements.

2. INFLUENCE OF ROTATION ON THE INTERNAL STRUCTURE IN ODD-MASS NUCLEI

In a rotating system the self-consistent field is distorted and the assumed axial symmetry is broken. In odd-mass nuclei this leads for low rotations to $\Delta K = \pm 1$ one quasi-particle admixtures, as well as to three quasi-particle admixtures caused by the variation of the self-consistent field. To see this explicitly, we start from the expression for the mixed state /4/

$$|IM\nu\rangle = |IMK\rangle + c^{I}_{K'K}\,|IMK'\rangle \qquad (2.1)$$

with $|IMK\rangle$, $|IMK'\rangle$ - angular momentum projected states

$$|IMK\rangle = P^{I}_{MK}\,\alpha^{+}_{K}|\Phi_0\rangle\,. \qquad (2.2)$$

The rotating core state $|\Phi_0\rangle$ includes, as is shown in /5/ , in the lowest order two-quasi-particle admixtures

$$|\Phi_0\rangle = |0\rangle + \sum_{\alpha} c^{I}_{\alpha} A^{+}_{\alpha}|0\rangle + \sum_{\alpha,\beta>0} c^{I}_{\alpha} c^{I}_{\tilde{\beta}} A^{+}_{\alpha} A^{+}_{\tilde{\beta}}|0\rangle\,, \qquad (2.3)$$

where

$|0\rangle$ - BCS vacuum, $A^{+}_{\alpha} = \alpha^{+}_{k_\alpha}\alpha^{+}_{k'_\alpha}$ *), $\alpha_{\tilde{k}} = s_k \alpha_{-k}$, $s_k = (-)^{1/2+K}$.

Hence from eq. (2.1)

$$|IM\nu\rangle = |IMK\rangle + \sum_{\alpha>0} c^{I}_{\alpha}\,|IMK+K_\alpha\rangle + c^{I}_{K'K}|IMK'\rangle +$$

$$+ c^{I}_{K'K}\sum_{\alpha>0} c^{I}_{\tilde{\alpha}}\,|IMK'-K_\alpha\rangle + \sum_{\alpha,\beta>0} c^{I}_{\alpha} c^{I}_{\tilde{\beta}}|IMK+K_\alpha-K_\beta\rangle\,, \qquad (2.4)$$

$$K_\alpha \equiv K_\alpha - K'_\alpha\,, \quad K_{\tilde{\alpha}} = -K_\alpha\,,$$

where now

$$|IMK\rangle = P^{I}_{MK}\,\alpha^{+}_{K}|0\rangle\,, \quad |IMK+K_\alpha\rangle = P^{I}_{MK+K_\alpha}\,\alpha^{+}_{K}A^{+}_{\alpha}|0\rangle\,.$$

The coefficients $c^{I}_{KK'}$ and c^{I}_{α} follow from the variational principle

$$\frac{\partial}{\partial c^{*I}_{K'K}} \langle IM\nu|H - E_{I\nu}|IM\nu\rangle = 0\,,$$

$$\frac{\partial}{\partial c^{*I}_{\alpha}} \langle IM\nu|H - E_{I\nu}|IM\nu\rangle = 0 \qquad (2.5)$$

with $E_{I\nu}$ - the eigenvalue for the state $|IM\nu\rangle$. From eqs. (2.4) and (2.5) we obtain

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*) $k \equiv \{K,\sigma\}$ -Nilsson quantum numbers.

$$c^{I}_{K'K}\langle IMK'|\bar H|IMK'\rangle + \sum_{\alpha>0} c^{I}_{\alpha}\langle IMK'|\bar H|IMK+K_{\alpha}\rangle +$$

$$+\sum_{\alpha>0}\overset{*}{c}{}^{I}_{\tilde\alpha}\langle IMK'-K_{\alpha}|\bar H|IMK\rangle = -\langle IMK'|\bar H|IMK\rangle \qquad (2.6)$$

and

$$\sum_{\beta>0} c^{I}_{\beta}\langle IMK+K_{\alpha}|\bar H|IMK+K_{\beta}\rangle + \sum_{\beta>0}\overset{*}{c}{}^{I}_{\tilde\beta}\langle IMK+K_{\alpha}-K_{\beta}|\bar H|IMK\rangle +$$

$$+\,c^{I}_{K'K}\langle IMK+K_{\alpha}|\bar H|IMK'\rangle = -\langle IMK+K_{\alpha}|\bar H|IMK\rangle, \qquad (2.7)$$

where

$$\bar H \equiv H - E_{I\nu} \;.$$

The matrix elements in this system are simply evaluated in zero order of nonadiabaticity [4,5]

$$\langle IMK'|\bar H|IMK'\rangle = E_{K'} - E_{K} \;,$$

$$\langle IMK'|\bar H|IMK+K_{\alpha}\rangle = \langle K'|H A^{+}_{\alpha}|K\rangle \,,$$

$$\langle IMK'-K_{\alpha}|\bar H|IMK\rangle = \langle K'|A_{\tilde\alpha}H|K\rangle \,, \qquad (2.8)$$

$$\langle IMK+K_{\alpha}|\bar H|IMK+K_{\beta}\rangle = \langle 0|A_{\alpha}(H-E_{0})A^{+}_{\beta}|0\rangle \equiv \langle\alpha|(H-E_{0})|\beta\rangle ,$$

$$\langle IMK+K_{\alpha}-K_{\beta}|\bar H|IMK\rangle = \langle 0|A_{\alpha}A_{\tilde\beta}H|0\rangle \equiv \langle\alpha\tilde\beta|H|0\rangle .$$

Further, the coupling terms equal in first order [4,5]

$$-\langle IMK'|\bar H|IMK\rangle = \frac{i}{\mathcal{J}}\langle K'|J_{y}|K\rangle\sqrt{(I-K)(I+K+1)} \qquad (2.9)$$

$$-\langle IMK+K_{\alpha}|\bar H|IMK\rangle = \frac{i}{\mathcal{J}}\langle\alpha|J_{y}|0\rangle\sqrt{(I-K)(I+K+1)} \qquad (2.10)$$

with $\mathcal{J}$ - the projection moment of inertia of the core. In both formulae additional contributions which compensate spurious terms in the rotational energy are omitted. In eq. (2.7) we shall neglect the one quasiparticle coupling term, being of the order 1/N , if N is the number of two quasiparticle states, included into the sums. Then eq. (2.7) obtains a form, similar to the case of an even-even nucleus [5]

$$\sum_{\beta>0} c^{I}_{\beta}\langle\alpha|H-E_{0}|\beta\rangle + \sum_{\beta>0}\overset{*}{c}{}^{I}_{\tilde\beta}\langle\alpha\tilde\beta|H|0\rangle = \frac{i\langle\alpha|J_{y}|\beta\rangle}{\mathcal{J}}\sqrt{(I-K)(I+K+1)}. \qquad (2.11)$$

From the time-reversal relations

$$\langle\tilde\alpha|H|\tilde\beta\rangle^{*} = \langle\alpha|H|\beta\rangle ,\quad \langle\alpha\tilde\beta|H|0\rangle^{*} = \langle\tilde\alpha\beta|H|0\rangle ,$$

$$\langle\tilde\alpha|J_{y}|0\rangle^{*} = \langle 0|J_{y}|\alpha\rangle \qquad (2.12)$$

follows

$$\overset{*}{c}{}^{I}_{\tilde{\alpha}} = - c^{I}_{\alpha} \tag{2.13}$$

and (2.11) gets the form

$$\sum_{\beta>0} c^{I}_{\beta} (\langle\alpha|H-E_0|\beta\rangle - \langle\alpha\tilde{\beta}\ |H|0\rangle) = \sum_{\beta>0} c^{I}_{\beta} \langle\alpha|[H, A^{+}_{\beta} + A_{\tilde{\beta}}]|0\rangle =$$

$$= \frac{i\langle\alpha|J_y|0\rangle}{\mathcal{J}} \sqrt{(I-K)(I+K+1)}. \tag{2.11'}$$

Similar we may write from eqs. (2.6) and (2.9)

$$c^{I}_{K'K}(E_{K'} - E_{K}) + \sum_{\beta>0} c^{I}_{\beta} \langle K'|[H, A^{+}_{\beta} + A_{\tilde{\beta}}]|K\rangle =$$

$$= \frac{i\langle K'|J_y|K\rangle}{\mathcal{J}} \sqrt{(I-K)(I+K+1)}. \tag{2.14}$$

Introducing in eqs. (2.11') and (2.14)

$$c^{I}_{\alpha} = \frac{i}{\mathcal{J}} \sqrt{(I-K)(I+K+1)}\ \tilde{c}_{\alpha}, \quad c^{I}_{K'K} = \frac{i}{\mathcal{J}} \sqrt{(I-K)(I+K+1)}\ \tilde{c}_{K'K} \tag{2.15}$$

we get

$$\sum_{\beta>0} \tilde{c}_{\beta} \langle\alpha|[H, A^{+}_{\beta} + A_{\tilde{\beta}}]|0\rangle = \langle\alpha|J_y|0\rangle \tag{2.16}$$

and

$$\tilde{c}_{K'K}(E_{k'} - E_{k}) + \sum_{\beta>0} \tilde{c}_{\beta} \langle k'|[H, A^{+}_{\beta} + A_{\tilde{\beta}}]|k\rangle = \langle k'|J_y|k\rangle. \tag{2.17}$$

The system of eqs. (2.16) and (2.17) is the generalization of Thouless-equations to the odd-mass nucleus. Formerly it has been obtained by Klein et al. /3/ in their equation of motion theory.

The structure of eqs. (2.11) and (2.14) is simple to understand. Introducing the operator

$$c^{I}_{y} = \sum_{k,k'} c^{I}_{K'K} \alpha^{+}_{k'} \alpha_{k} + \sum_{\alpha>0} c^{I}_{\alpha} (A^{+}_{\alpha} + A_{\tilde{\alpha}}) \tag{2.18}$$

eq. (2.11) gives

$$\langle\alpha|[H, c^{I}_{y}]|0\rangle = \frac{i}{\mathcal{J}} \langle\alpha|J_y|0\rangle \sqrt{(I-K)(I+K+1)},$$

e.g., the operator c^{I}_{y} is connected with the angle operator by the relation

$$c^{I}_{y} = -\phi_{y} \sqrt{(I-K)(I+K+1)} \tag{2.19}$$

while eq. (2.14) is equivalent to

$$\langle k'|[H, \phi_y]|k\rangle = -i \frac{\langle k'|J_y|k\rangle}{\mathcal{J}}.$$

3. CORIOLIS COUPLINC AND DYNAMICAL ATTENUATION

It was shown recently /4/ that the usual phenomenological Coriolis coupling concept is an approximation to the solution of the equation

$$H\,|IM\nu\rangle = E_{I\nu}\,|IM\nu\rangle\ .$$

In a perturbation calculation the admixture of a $K+1$-band to a K-band is determined by eq. (2.14)

$$c^{I}_{K'K} = \frac{i\langle k'|J_y|k\rangle}{\mathcal{J}(E_{k'}-E_k)}\sqrt{(I-K)(I+K+1)}\,\Big(1-\frac{\sum_{\alpha>0}\tilde{c}_\alpha\langle k'|[H,A^+_\alpha+A_{\tilde\alpha}]|k\rangle}{\langle k'|J_y|k\rangle}\Big). \qquad (3.1)$$

Using the Coriolis-interaction

$$H_C = -\frac{1}{2\mathcal{J}}(J_+ j_- + J_- j_+)$$

the expression (3.1) may be written as

$$c^{I}_{K'K} = \frac{\langle IMK+1|H_c|IMK\rangle}{E_{K'}-E_K}\,R_{K'K} \qquad (3.2)$$

with the attenuation factor

$$R_{K'K} = \Big(1-\frac{\sum_{\alpha>0}\tilde{c}_\alpha\langle k'|[H,A^+_\alpha+A_{\tilde\beta}]|k\rangle}{\langle k'|J_y|k\rangle}\Big). \qquad (3.3)$$

This relation for the dynamical attenuation factor was derived recently by K.Hara, using the RPA-method /2/ . The influence of three quasi-particle coupling is simple to see for factorized interactions.

For the $Q-Q$ force in the $p-h$ channel

$$H_{QQ} = -\frac{\chi}{2}\sum_{\mu=\pm1} Q^+_{2\mu}Q_{2\mu}$$

with

$$Q_{2\mu} \equiv Q_\mu = \sum_{\alpha>0} q_{\mu\alpha}\eta^{(+)}_\alpha A^+_\alpha + (-)^\mu q^*_{-\mu\alpha}\eta^{(+)}_\alpha A_\alpha \ +$$

$$+\sum_{k,k'>0} q_{\mu kk'}\xi^{(-)}_{kk'}\alpha^+_k\alpha_{k'} + (-)^\mu q^*_{-\mu kk'}\xi^{(-)}_{kk'}\alpha^+_{\tilde k}\alpha_{\tilde k'}\ , \qquad (3.4)$$

where

$$\eta^{(+)}_\alpha = u_{k_\alpha}v_{k'_\alpha} + u_{k'_\alpha}v_{k_\alpha}\ , \qquad \xi^{(-)}_{kk'} = u_k u_{k'} - v_k v_{k'}\ ,$$

we get

$$[Q_\mu, A^+_\alpha] = (-)^\mu q^*_{-\mu\alpha}\eta^{(+)}_\alpha\ , \quad [Q_\mu, A_{\tilde\alpha}] = -(-)^\mu q^*_{-\mu\alpha}\eta^{(+)}_\alpha\ ,$$

e.g.,

$$[H_{QQ}, A^+_\alpha + A_{\tilde\alpha}] = 0\ , \qquad (3.5)$$

and thus no dynamical attenuation appears for the Q-Q-interaction in the p-h -channel. The reason for this result is the behaviour of Q-matrix elements under time-reversal

$$< \tilde{k}'_\alpha | q_\mu | \tilde{k}_\alpha > = < k'_\alpha | q^+_\mu | k_\alpha >^* = < k_\alpha | q_\mu | k'_\alpha >$$

leading to the $\eta^{(+)}_\alpha$ -factors in eq. (3.4). Therefore, for instance a spin-spin force

$$H_\sigma = \frac{\chi_\sigma}{2} \vec{\sigma}_1 \vec{\sigma}_2 \qquad (3.6)$$

because of

$$\sigma_\mu = \sum_{\alpha>0} \sigma_{\mu\alpha} \eta^{(-)}_\alpha A^+_\alpha + (-)^\mu \sigma^*_{-\mu\alpha} \eta^{(-)}_\alpha A_\alpha + \sum_{k,k'} \sigma_{\mu kk'} \xi^{(+)}_{kk'} a^+_k a_{k'} + (-)^\mu \sigma^*_{-\mu kk'} \xi^{(+)}_{kk'} a^+_{\tilde{k}} a_{\tilde{k}'}$$

contributes to dynamical attenuation, as was already shown by K.Hara /2/.

Interesting is the appearance of the factor $\xi^{(+)}_{kk'}$, which shows the possibility of dynamical effects to the decoupling parameter a in K=1/2 -bands. To derive the expression for the diagonal contribution of the Coriolis interaction to the K=1/2 -band energies, we start from the expression

$$E_{IK} = \frac{\int_0^\pi d\beta \sin\beta \, d^I_{KK}(\beta) < k | HR_y(\beta) | k >}{\int_0^\pi d\beta \sin\beta \, d^I_{KK}(\beta) < k | R_y(\beta) | k >} \, . \qquad (3.7)$$

For K=1/2 one has to take into account more careful the different behaviour of overlap integrals at $\beta \geq 0$ and $\beta \leq \pi$ /6/. Using the Gaussian overlap approximation, we obtain

$$\int_0^\pi d\beta \sin\beta \, d^I_{KK}(\beta) < k | HR_y(\beta) | k > = \int_0^\infty d\beta \, \beta \, d^I_{KK}(\beta) < k | HR_y(\beta) | k > + \int_0^\infty d\beta \, \beta \, d^I_{K'K''}(\pi - \beta) < k | HR_y(\pi - \beta) | k > \, . \qquad (3.8)$$

The second integral with a maximal contribution from $\beta \leq \pi$ just gives the decoupling effect, but an additional dynamical contribution appears.

For the evaluation of overlap integrals in eq. (3.8) we use

$$| k > = a^+_k | \Phi_0 > \, , \qquad (3.9)$$

where the rotating core-state vector $|\Phi_0>$ is introduced through eq. (2.3). After this the second integral in eq. (3.8) gives in the lowest order the contributions

$$\int_0^\infty d\beta \, \beta \, d^I_{KK}(\pi-\beta) < 0 | a_k HR_y(\pi-\beta) a^+_k | 0 > +$$

$$+ \int_0^\infty d\beta \, \beta \, d^I_{K-1K}(\pi-\beta) < 0 | a_k \sum_{\alpha>0} c^{*I}_\alpha A_{\tilde{\alpha}} HR_y(\pi) a^+_k | 0 > < 0 | R_y(\beta) | 0 > + \qquad (3.10)$$

$$+ \int_0^\infty d\beta \, \beta \, d^I_{KK-1}(\pi-\beta) < 0 | a_k HR_y(\pi) \sum_{\alpha>0} c^I_{\tilde{\alpha}} A^+_{\tilde{\alpha}} a^+_k | 0 > < 0 | R_y(\beta) | 0 > .$$

The action of the rotation operator $R_y(\pi)$ on an K-eigenstate is [6]

$$R_y(\pi)a_k^+|0> = (-)^{1/2-K} \alpha_{-k}^+|0>. \qquad (3.11)$$

Then in the first integral we obtain

$$<0|\alpha_k H R_y(\pi-\beta)\alpha_k^+|0> = (-)^{1/2-K}<k|e^{i\beta j_y}|-k> \times \qquad (3.12)$$

$$\times (E_k <0|R_y(\beta)|0>+<0|HR_y(\beta)|0>) .$$

Together with [6]

$$<0|HR_y(\beta)|0> = E_0 + h_1\beta^2$$

and

$$<0|R_y(\beta)|0> = \exp(-j\beta^2)$$

$$d^I_{KK}(\pi-\beta) = (-)^{I-K} \frac{(I+1/2)\beta}{2}$$

the first integral for $K=1/2$ equals

$$\frac{i}{2}(I+1/2)<1/2|j_y|-1/2>(-)^{I-1/2}\{\frac{E_k+E_0}{2j^2}+\frac{h_1}{j^3}\}. \qquad (3.13)$$

Together with the first integral in eq.(3.8) and the normalization integral in eq. (3.7) this leads to the expression

$$E_{I1/2} = E_0 + E_{1/2} + \frac{1}{2\mathcal{J}}a(I+1/2)(-)^{I+1/2} \qquad (3.14)$$

with $\mathcal{J}$ - the projection moment of inertia $\mathcal{J} = -\frac{2j^2}{h_1}$ and the usual decoupling parameter

$$a = 2i<1/2|j_y|-1/2>.$$

Additionally, from eq. (3.10) there are dynamical contributions. Using the relation

$$d^I_{KK'}(\pi-\beta) = (-)^{I+K} d^I_{K'-K}(\beta) \qquad (3.15)$$

the second integral in eq. (3.10) gives

$$-\sum_{\alpha>0} c^I_\alpha <0|\alpha_k[H,A_{\tilde{\alpha}}]\alpha^+_{-k}|0>(-)^{1/2-K}(-)^{I+K}\frac{1}{2j}. \qquad (3.16)$$

Similarly, applying relation (3.11), the third integral in eq. (3.10) gives

$$-\sum_{\alpha>0} c^I_\alpha <0|\alpha_k[H,A^+_\alpha]\alpha^+_{-k}|0>(-)^{1/2-K}(-)^{I+K}\frac{1}{2j}. \qquad (3.17)$$

From eq. (2.15) with $K=-1/2$ we obtain

$$c^I_\alpha = \frac{i}{\mathcal{J}}(I+1/2)\tilde{c}_\alpha \qquad (3.18)$$

and finally, taking account of the normalization integral in (3.7), which is just equal to $1/2j$, the correction to the decoupling energy in eq. (3.14) is

$$-\frac{i}{\mathcal{J}}\sum_{\alpha>0}\tilde{c}_\alpha <0|\alpha_{1/2}[H,A^+_\alpha + A_{\tilde{\alpha}}]\alpha^+_{-1/2}|0>(I+1/2)(-)^{I+K}$$

leading to an effective decoupling coefficient

$$a_{eff} = a\left(1 - \frac{\sum_{\alpha>0} \tilde{c}_\alpha \langle 0 | \alpha_{1/2} [H, A^+_\alpha + A_{\tilde{\alpha}}] \alpha^+_{-1/2} | 0 \rangle}{\langle 1/2 | j_y | -1/2 \rangle}\right). \quad (3.19)$$

Comparing this with the relation (3.1) we see, that in the case of the p-h -channel the same type of attenuation appears for the decoupling coefficient like for the Coriolis matrix element.

In this connection we note, that Piepenbring [7] found attenuation factors 0.47 - - 0.59 and 0.70 - 0.87 for the decoupling parameter a $(1/2^-[530])$ by a phenomenological analysis of rotational bands in ^{173}Lu and ^{175}Lu.

4. CONCLUSION

It was shown in this paper that in a consequent treatment of rotating odd-mass nuclei three quasi-particle states should be included in a band-mixing calculation. For the interaction in the p-h -channel an attenuation appears also for the decoupling coefficient. This may help to understand the special role of interactions in the p-h -channel, because, as I. Hamamoto [1] has shown, in the p-p channel there is no attenuation for the decoupling coefficient. Besides the discussed dynamical reasons for three quasi-particle admixture, one should take into account also the effect of blocking which also leads in the framework of a consequent HFB-calculation, as is shown by Ring et al. [8], to three quasi-particle components in the wave function of the odd-mass nucleus.

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