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OPTIMAL SYSTEMATICS OF SINGLE-HUMPED FISSION BARRIERS FOR STATISTICAL CALCULATIONS

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1. Introduction

The statics of fission is governed by the variation of a potential energy of the fissioning system as a function of deformation in the transition from the initial state to scission (see the last reviews [1]-[5]). Therefore, the most important characteristics of fissioning nuclei are fission barriers determined as differences between the saddle-point and ground state masses

$$B_f = M_{sp}(A,Z) - M_{gs}(A,Z). \quad (1)$$

Various models used to calculate B_f can be classified in three categories: microscopic, semiclassical or hybrid, and macroscopic (see [5] for a brief review). In the *microscopic models* the nucleus is studied as a many-body problem of an ensemble of nucleons moving in a self-consistent Hartree-Fock field with possible extensions (see [6] for a review). This method should provide the most accurate knowledge of the fissioning system. However the complexity of the effective nucleon-nucleon interaction and a great number of nucleons in a heavy nucleus make the calculations very difficult and too lengthy to be used in Monte Carlo calculations of competing fission and evaporation processes. Therefore, for statistical applications, the "regular" part of fission barriers is usually either calculated in the framework of such macroscopic approaches as different versions of the liquid-drop model (**LDM**) [7]-[9], droplet model [11, 12], single-Yukawa modified LDM [13], Yukawa-plus-exponential modified LDM [10, 14, 15], or phenomenologically approximated [16] in accordance with their experimental values.

To estimate the "irregular" microscopic part of B_f, either different hybrid approaches are used taking into account quantal corrections for shell and pairing effects, the finite range of the nuclear force, the effects of the diffuseness of the nuclear surface and other physics effects [7]-[14], or certain different phenomenological approximations are used [16, 17]. Apparently, to date the most adequate description of macroscopic fission barriers for hot and usually rotating nuclei has been done by Sierk [15]. Sierk also performed a global fit for macroscopic B_f for nuclei with an atomic number from 20 to 100 for the entire range of the angular momentum L for which a fission barrier exists and obtained a global representation of results which depends only on three variables: A, Z and L. This global approximation is provided be the Fortran-77 subroutine BARFIT that is available for users (see [15]) and may be easily incorporated in the statistical evaporation model calculations. But in certain cases this code may require too much computing time to obtain a satisfactorily statistics in the Monte Carlo simulation of reactions.

In the present work we compare different easy-computing approaches and models for fission barriers in order to find out their applicability for statistical calculations of nuclear reactions involving fission processes.

2. Macroscopic and microscopic approaches for fission-barrier heights

In the hybrid macroscopic-microscopic approach a fission barrier is given by a sum of a macroscopic smooth term and a microscopic term, each being in the general case a function of atomic Z and mass A numbers, excitation energy of the fissioning nucleus E^*, its angular momentum L and deformation (denoted by α)

$$B_f(A,Z,E^*,L,\alpha) = B_f^{macro}(A,Z,E^*,L,\alpha) + B_f^{micro}(A,Z,E^*,L,\alpha). \qquad (2)$$

Let us at the beginning do not take into account the excitation energy and angular momentum dependences of B_f.

BITG73 approximation [16]. For fast statistical calculations Barashenkov *et al.* [16] has proposed to use a simple phenomenological approximation for fission barriers. The authors of [16] suggested not to calculate fission barriers during Monte Carlo simulations of nuclear reactions but to use the known experimental values by singling out of them the phenomenological "irregular" part which depends on shell corrections, residual interactions and other nuclear structure effects, and by approximating the remaining "regular" part by a simple analytical expression

$$B_f^{BITG73}(A,Z) = B_f^0(A,Z) - \delta W_{gs}^{BITG73}(A,Z) + \delta W_{sp}^{BITG73}(A,Z). \qquad (3)$$

The "regular" part of the experimental fission barriers $B_f^0(A,Z)$ was well approximated by the function (in MeV)

$$B_f^0(A,Z) = 12.5 + \begin{cases} +4.7(33.5 - Z^2/A)^{3/4}, & \text{if } Z^2/A \leq 33.5 \;; \\ -2.7(33.5 - Z^2/A)^{2/3}, & \text{if } Z^2/A > 33.5 \;. \end{cases} \qquad (4)$$

The "irregular" part was divided into two terms: a correction to the nuclear ground state mass $\delta W_{gs}^{BITG73}(A,Z)$ and a correction to the nuclear saddle-point mass $\delta W_{sp}^{BITG73}(A,Z)$. For $\delta W_{gs}^{BITG73}(A,Z)$ the authors of [16] proposed to use the Cameron's shell and pairing corrections

$$\begin{aligned} \delta W_{gs}^{BITG73}(A,Z) &= \Delta(Z,N) = S(Z,N) + P(Z,N) \equiv \\ &\equiv [S(Z)+P(Z)] + [S(N)+P(N)]; (N = A - Z), \end{aligned} \qquad (5)$$

tabulated in [18], or the data

$$\Delta(Z,N) = S(Z) + P(Z) + S(N) + P(N), \qquad (6)$$

tabulated in a subsequent work by Cameron *et al.* [19] and, therefore, very convenient for numerical calculations of an evaporative cascade.

In fig. 1, these two sets of Cameron shell corrections are shown for a collection of odd-odd nuclei together with the Myers and Swiatecki's LDM shell corrections [7]. One can see that the discrepancy in the absolute values of two sets of Cameron's shell corrections is small and only for some nuclei amounts up to 2 MeV, while the discrepancy between Cameron's and Myers and Swiatecki's values is more significant. For $\delta W_{sp}^{BITG73}(A,Z)$ the following approximation (in MeV) was obtained:

$$\delta W_{sp}^{BITG73}(A,Z) = \left\{ \begin{array}{l} \text{-0.5 for even } Z \\ 0 \text{ for odd } Z \end{array} \right\} + \left\{ \begin{array}{l} 0 \text{ for even } N \\ 1 \text{ for odd } N \end{array} \right\}. \quad (7)$$

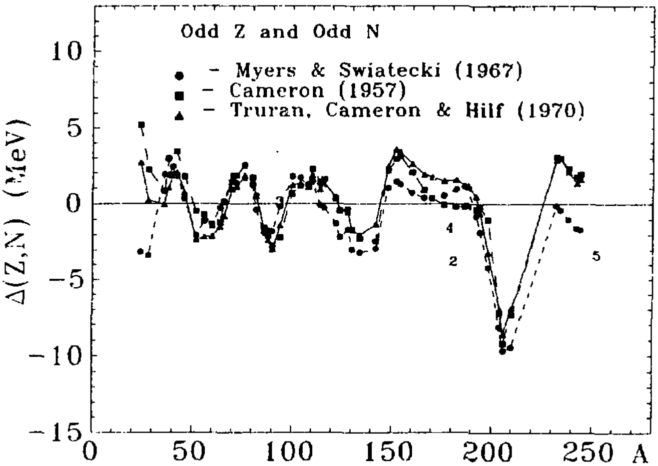


Fig. 1. Comparison of Cameron's [18], Truran, Cameron and Hilf's [19] and Myers and Swiatecki's [7] shell corrections.

BG77 approximation [17]. Another simple semiphenomenological approximation for fission barriers has been proposed by Barashenkov and Gereghi [17]. The authors of [17] proposed to use a formula analogous to (3) to calculate fission barriers

$$B_f^{BG77}(A,Z) = B_f^0(Z^2/A) - \Delta(A,Z) + \delta^{BG77}(A,Z). \quad (8)$$

For $\Delta(A,Z)$ it was proposed to use the same Cameron's corrections (5) or (6), while for $\delta^{BG77}(A,Z)$ it was suggested to use the following approximation:

$$\delta^{BG77}(A,Z) = \begin{cases} 0, & \text{for even } Z \text{ and even } N\,; \\ \delta_f, & \text{for odd } A\,; \\ 2\delta_f, & \text{for odd } Z \text{ and odd } N\,; \end{cases} \quad (9)$$

$$\delta_f = 1.248 \text{ MeV [20]}.$$

For the macroscopic part of the fission barriers $B_f^0(Z^2/A)$ it was proposed to use the LDM results in the parametrization of Cohen and Swiatecki [21]

$$\begin{aligned} B_f^0(Z^2/A) &= B_f^{LDM(CS63)} = \\ &= a_S A^{2/3} \begin{cases} 0.83(1-x)^3, & \text{for } 2/3 < x < 1 \ ; \\ 0.38(3/4-x), & \text{for } 1/3 < x < 2/3 \ , \end{cases} \end{aligned} \tag{10}$$

where the fissility parameter x is given by

$$x = \frac{E_C^0}{2E_S^0} = \frac{Z^2/A}{(a_C/2a_S)\{1-k[(N-Z)/A]^2\}}, \tag{11}$$

$$\begin{cases} a_S = 17.9439 \text{ MeV} \ , \\ a_C = 0.7053 \text{ MeV} \ , \\ k = 1.7826 \ , \end{cases} \tag{12}$$

(LDM parameters from [7])

and the surface E_S^0 and Coulomb E_C^0 energies of a spherical nucleus are given by

$$E_S^0 = a_S\{1-k[(N-Z)/A]^2\}A^{2/3}, \tag{13}$$

$$E_C^0 = a_C Z^2/A^{1/3}. \tag{14}$$

The phenomenological representation (10) approximates very well the macroscopic LDM fission barriers (see fig.2) which provides satisfactory agreement of B_f^{BG77} with the experimental data and, as B_f^{BG77} are easily computed, it may be successfully used in Monte Carlo simulations of nuclear reactions involving fission.

LDM approaches. In the LDM the experimental ground state mass for a nuclear equilibrium deformation α^0 is given by

$$M_{exp} = M_{LDM}^0(\alpha^0) + S^0(N,Z) + P^0(N,Z), \tag{15}$$

where M_{LDM}^0 is the LDM macroscopic mass, $S^0(N,Z)$ and $P^0(N,Z)$ are shell and pairing corrections, respectively. For a nucleus undergoing fission (superscript ^{f}) with a deformation α^f at the saddle-point we have

$$M^f = M_{LDM}^f(\alpha^f) + S^f(N,Z) + P^f(N,Z). \tag{16}$$

Substituting (15,16) into (1), we get

$$\begin{aligned} B_f &= [M_{LDM}^f(\alpha^f) - M_{LDM}^0(\alpha^0)] + \\ &+ [S^f(N,Z) - S^0(N,Z)] + [P^f(N,Z) - P^0(N,Z)]. \end{aligned} \tag{17}$$

Commonly, in the literature the last equation is written in the following form [1]:

$$B_f = B_f^0 - \delta W_{gs} + \delta W_{sp}, \tag{18}$$

where B_f^0 is the macroscopic LDM fission barrier, $\delta W_{gs} = S^0(N,Z)$ is the ground state shell correction and $\delta W_{sp} = S^f(N,Z) + P^f(N,Z) - P^0(N,Z)$ is shell and pairing (or more exactly, the increase in the pairing energy between the transition state and ground state) corrections at the saddle-point. Usually (see, e.g., [1]), one makes the assumption that for a nucleus undergoing fission the major shell structure effects are destroyed as the nucleus deforms from the equilibrium ground state shape to the saddle-point one, i.e., $S^f(N,Z) = 0$. In general, up to now it is not a common point of view in the literature what is to be used for δW_{sp} in (18). So, certain authors (see, e.g., [1, 22] neglect this term; others (see, e.g., [16, 17, 20]) use different phenomenological approximations for this term and, at last, the third group of authors (see, e.g., [23]) fits this term from the best description of experimental data.

In the notation of Nix [27], the potential energy of a deformed charged drop relative to the spherical drop, i.e., the macroscopic LDM fission barrier B_f^0 is given by

$$\begin{aligned} B_f^0 &= E_S - E_S^0 + E_C - E_C^0 = \\ &= [(B_S - 1) + 2x(B_C - 1)]E_S^0 \equiv b(x)E_S^0. \end{aligned} \tag{19}$$

Here E_C^0 and E_C are the Coulomb energies of a spherical and a deformed drop, respectively; E_S^0 and E_S are the total surface energy of a spherical and a deformed drop, respectively; x is the fissility parameter defined by (11); B_S and B_C are the relative surface and Coulomb energies depending on the deformation of the drop and are tabulated (together with $b(x)$) in [27] as functions of the fissility parameter x; E_S^0 and E_C^0 are defined by (13) and (14). The values for the constants a_S, a_C and k obtained from the best existing LDM fit to nuclear masses and fission barriers [7] are given by (12). In other models, the values of these constants differ from (12), which also results in changing $b(x)$ and B_f^0. Fig. 2 shows the function $b(x)$ calculated with Myers and Swiatecki's parameters (12), in the framework of the single-Yukawa modified LDM of Krappe and Nix [13] with the parameters

$$\begin{cases} a_S = 24.7 \text{ MeV}, \\ a_C = 0.7448 \text{ MeV}, \\ k = 4.0, \end{cases} \tag{20}$$

(for the nuclear radius parameter $r_0 = 1.16$ MeV and the range of the Yukawa function $a = 1.4$ fm), and in the framework of the Yukawa-plus-exponential modified LDM of

[1] In the present work, we confine ourselves to the analysis of only the one-humped fission barriers. In the case of transuranium nuclides the heights of double-humped fission barriers B_f^A and B_f^B are expressed by $B_f^i = \tilde{V}(\alpha_i) - \delta W_{gs} + \delta W_{sp}^i$, where $\tilde{V}(\alpha_i)$ is the macroscopic fission barrier and δW_{sp}^i is the shell correction for the i-th maximum of the potential energy which is counted off the liquid drop potential energy at the corresponding deformation, $\alpha_A \approx 0.3$ and $\alpha_B \approx 0.6$, $\delta W_{sp}^A \sim 2,80$ MeV and $\delta W_{sp}^B \sim 0.50$ MeV [24]; and δW_{gs} is calculated in the LDM [7]. (For a more detailed information about double-humped fission barriers see [23]-[26].)

Krappe, Nix and Sierk [14] with the parameters

$$\begin{cases} a_S = 21.7 \text{ MeV} , \\ a_C = 0.7322 \text{ MeV} , \\ a = 0.65 \text{ fm} , \\ r_0 = 1.18 \text{ fm} , \\ k = 2.04 ; \end{cases} \tag{21}$$

together with the approximation for $b(x)$ in the parametrization (10) of Cohen and Swiatecki [21]. One can see that for medium and heavy nuclei the old approximation of Cohen and Swiatecki (10) agrees very well with the LDM [7] prediction for $b(x)$, and being easily computed, may be successfully used in numerical calculations.

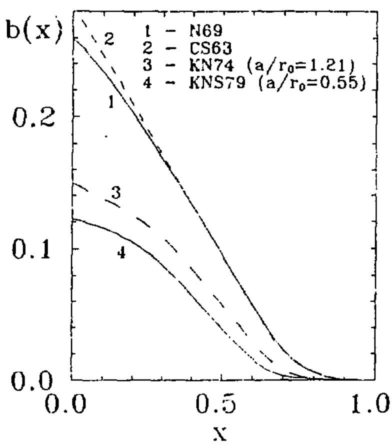


Fig. 2. Macroscopic fission barriers (units of E_S^0) as functions of the fissility parameter x for the LDM parameters [7] from [27] (marked as N69); in accordance with Cohen and Swiatecki's [21] parametrization (10) (marked as CS63); for the single-Yukawa modified LDM of Krappe and Nix [13] (marked as KN74); and for the Yukawa-plus-exponential modified LDM [14] (marked as KNS79).

For nuclei along Green's approximation to the line of β - stability [28]

$$N - Z = 0.4A^2/(A + 200). \tag{22}$$

in fig. 3 we compare macroscopic fission barriers calculated in the LDM with Myers and Swiatecki's parameters [7], in the single-Yukawa modified LDM [13], in the Yukawa-plus-exponential modified LDM [14], and in the LDM with parameters of Pauli and Ledergerber [8]

$$\begin{cases} a_S = 19.008 \text{ MeV} , \\ a_C = 0.720 \text{ MeV} , \\ k = 2.84 , \end{cases} \tag{23}$$

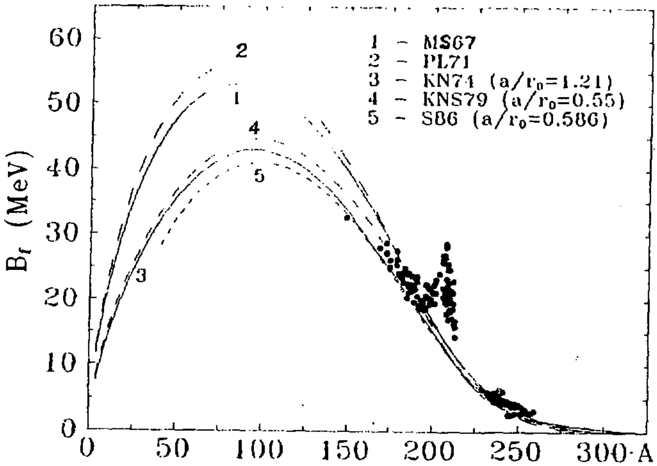


Fig. 3. Comparison of macroscopic fission barriers calculated for nuclei along Green's approximation to the line of β – stability in the LDM with Myers and Swiatecki's parameters [7] (marked as MS67); with Pauli and Ledergerber's LDM parameters [8] (marked as PL71); in the single-Yukawa modified LDM [13] (marked as KN74); in the Yukawa-plus-exponential modified LDM [14] (marked as KNS79), and obtained with Sierk's subroutine BARFIT [15] (marked as S86). The experimental points are from [11, 53].

For comparison, this figure also shows the experimental data from refs. [11, 53] and the results (marked by abbreviation S86) obtained (for $L = 0$) with the subroutine BARFIT of Sierk [15] which provides fission barrier heights as a function of Z, A, and angular momentum L by a multiparameter approximation of results obtained by the Yukawa-plus-exponential modified LDM with the following parameters

$$\begin{cases} a_S = 21.13 \text{ MeV} , \\ a_C = 0.7448 \text{ MeV} , \\ a = 0.68 \text{ fm} , \\ r_0 = 1.16 \text{ fm} , \\ k = 2.3 . \end{cases} \tag{24}$$

One can see that for medium nuclei fission barriers calculated with Pauli and Ledergerber's parameters (23) are the highest ($B_f^{0(max)}(PL71) = 55.13$ MeV) and those calculated in the LDM [7] are the second highest ($B_f^{0(max)}(MS67) = 52.99$ MeV, whereas those calculated in the single-Yukawa and Yukawa-plus-exponential models lying somewhat lower ($B_f^{0(max)}(KN74) = 43.03$ MeV, $B_f^{0(max)}(KNS79) = 52.99$ MeV, $B_f^{0(max)}(S86) = 41.03$ MeV). The three models KN74, KNS79, and S86 that include the finite range of nuclear force and the diffuse nuclear surface yield results that are very similar to each other, although for all nuclei the barriers calculated with the Yukawa-plus-exponential model [14] are slightly higher than those calculated with the single-Yukawa model [13], and the global approximation of Sierk [15] provides the lowest barrier heights.

In the present paper, we test seven models to calculate macroscopic fission barriers, namely, proposed in: [16] (marked as BITG73); [17] (marked as BG77); [7] (marked as MS67); [8] (marked as PL71); [14] (marked as KNS79); [13] (marked as KN74); and [15] (marked as S86). For δW_{gs} we test three sets of values, namely, the Cameron's corrections $\Delta(Z, N)$ defined by eqs. (5) and (6), and LDM shell corrections with Myers and Swiatecki's [7] parameters. For δW_{sp} we test two sets of values, namely, as defined by eqs. (7) and (9).

In table 1, fission barriers calculated by phenomenological [16] and semiphenomenological [17] methods with Cameron's [18] and Truran, Cameron and Hilf's [19] shell and pairing corrections are compared with the experimental values from the summary table IV of ref. [11].

One can see that both these methods give results quite consistent with experimental data, although for lighter nuclei the approach proposed in [16] predicts fission barriers slightly closer to the experimental data and vice versa for heavier nuclei, independently of what shell corrections from [18, 19] we use. The results obtained with different shell corrections from [18, 19] differ appreciably only for neutron-rich and neutron-deficient nuclei. In these cases the use of shell corrections from [19] seems to be more preferable.

The results of calculations of macroscopic fission barriers by models [7, 8, 13, 14, 15] with Myers and Swiatecki's [7] shell corrections for microscopic parts of barriers are compared in table 2 with the same experimental data. One can see that all these methods provide fission barriers quite close to the experimental data.

Figs. 4 and 5 show fission barriers calculated with methods proposed in [16, 17, 7, 13, 8, 14] for nuclei along Green's approximation to the line of β - stability together with the experimental data. As one can see, for heavy nuclei all methods provide fission barriers in good agreement with experimental data. For this option the semiphenomenological approach of Barashenkov and Gereghi [17] permanently overestimates the experimental data for nuclei lighter than Pb. Apparently, the Yukawa-plus-exponential modified LDM [14] provides the best agreement of calculated barriers with the experimental data for the nuclei along the line of β - stability.

Excitation energy dependence of fission barriers. The change of properties of atomic nuclei with increasing excitation energies influences strongly the nuclear fissility. The calculations by methods of Tomas-Fermi [31] and Hartree-Fock [30] predict that "thermal" effects must lead to the B_f decrease. The investigations [30]-[33] of the dependence of B_f on nuclear temperature T show that the dependences of Coulomb E_C^0 and surface E_S^0 energies on T are the following:

$$E_S(T) = E_S(0)[1 - \beta T^2]$$
$$E_C(T) = E_C(0)[1 - \alpha T^2], \qquad (25)$$

Table 1
Calculated in the framework of refs. [16, 17] (with shell and pairing corrections from [18] and [19]) and experimental (taken from the summary table IV of ref. [11]) values of fission barriers for isotopes of different nuclei

Isotope	Exp. [11]	Present calculations by different methods			
		BITG73[16] shell & pair. corr. from [18]	BITG73[16] shell & pair. corr. from [19]	BG77[17] shell & pair. corr. from [18]	BG77[17] shell & pair. corr. from [19]
$^{173}Lu_{71}$	27.3	27.65	25.62	32.40	30.36
$^{179}Ta_{73}$	26.2	24.04	29.93	27.72	25.49
$^{188}Os_{76}$	23.7	22.62	25.22	23.77	21.25
$^{187}Os_{76}$	22.5	22.36	25.24	23.82	20.86
$^{186}Os_{76}$	22.5	21.59	24.49	22.85	20.54
$^{191}Ir_{77}$	22.8	21.74	24.14	23.01	21.19
$^{189}Ir_{77}$	21.7	20.49	23.49	21.92	19.78
$^{198}Hg_{80}$	21.8	22.11	19.72	21.40	20.61
$^{201}Tl_{81}$	22.3	22.47	21.12	22.60	21.27
$^{209}Bi_{83}$	22.6	24.24	24.68	24.10	23.66
$^{207}Bi_{83}$	21.2	21.91	22.75	22.77	21.69
$^{212}Po_{84}$	18.6	19.62	19.44	19.24	20.05
$^{211}Po_{84}$	21.5	20.21	20.91	20.80	20.74
$^{210}Po_{84}$	20.4	19.93	21.47	21.21	21.48
$^{213}At_{85}$	16.8	15.32	17.45	17.39	18.00
$^{227}Ra_{88}$	8.30	8.22	8.91	8.53	8.22
$^{233}Th_{90}$	6.44	7.35	7.48	7.30	7.55
$^{232}Th_{90}$	5.95	6.65	6.71	6.46	7.33
$^{232}Pa_{91}$	6.18	6.09	6.77	6.95	6.45
$^{239}U_{92}$	6.29	6.71	6.32	6.24	7.01
$^{238}U_{92}$	5.60	6.13	5.63	5.51	6.86
$^{237}U_{92}$	6.40	6.56	6.32	6.28	6.69
$^{236}U_{92}$	5.44	6.00	5.54	5.57	6.50
$^{235}U_{92}$	5.75	6.32	6.05	6.23	6.31
$^{234}U_{92}$	5.30	5.66	5.28	5.40	6.09
$^{233}U_{92}$	5.49	6.05	5.72	6.11	5.87
$^{238}Np_{93}$	6.04	6.01	6.24	6.46	6.15
$^{237}Np_{93}$	5.49	5.46	5.47	5.75	5.97
$^{244}Pu_{94}$	4.60	5.76	4.73	4.75	6.32
$^{242}Pu_{94}$	4.70	5.48	4.57	4.67	6.10
$^{241}Pu_{94}$	6.20	5.93	5.26	5.45	5.97
$^{240}Pu_{94}$	4.85	5.38	4.58	4.73	5.83
$^{239}Pu_{94}$	5.48	5.83	5.28	5.52	5.67
$^{238}Pu_{94}$	4.70	5.29	4.51	4.82	5.49
$^{236}Pu_{94}$	4.55	5.00	4.28	4.68	5.10

Table 1 (*continued*)

Isotope	Exp. [11]	Present calculations by different methods			
		BITG73[16] shell & pair. corr. from [18]	BITG73[16] shell & pair. corr. from [19]	BG77[17] shell & pair. corr. from [18]	BG77[17] shell & pair. corr. from [19]
$^{244}Am_{95}$	6.[illegible]	5.70	5.48	5.81	5.77
$^{242}Am_{95}$	6.40	5.60	5.47	5.88	5.53
$^{241}Am_{95}$	6.00	5.05	4.80	5.17	5.38
$^{250}Cm_{96}$	4.10	5.35	3.66	4.04	5.69
$^{248}Cm_{96}$	4.25	5.56	4.22	4.44	5.60
$^{246}Cm_{96}$	4.35	5.37	4.08	4.44	5.45
$^{244}Cm_{96}$	4.25	5.13	3.94	4.38	5.25
$^{242}Cm_{96}$	4.25	5.06	3.97	4.47	4.99
$^{240}Cm_{96}$	4.15	5.00	3.92	4.57	4.68
$^{250}Bk_{97}$	5.80	5.51	4.90	5.37	5.27
$^{249}Bk_{97}$	4.35	5.15	4.52	4.85	5.22
$^{252}Cf_{98}$	3.65	4.94	3.26	3.81	4.95
$^{250}Cf_{98}$	3.95	5.18	3.84	4.23	4.88
$^{248}Cf_{98}$	3.85	5.01	3.72	4.25	4.75
$^{246}Cf_{98}$	3.85	4.80	3.60	4.21	4.57
$^{254}Fm_{100}$	3.35	4.50	3.00	3.66	4.36
$^{248}Fm_{100}$	2.75	4.40	3.41	4.13	4.04
$^{246}Fm_{100}$	2.55	4.36	3.50	4.28	3.83
$^{245}Fm_{100}$	2.62	4.84	4.23	5.10	3.71
$^{244}Fm_{100}$	2.62	4.33	3.50	4.44	3.57

where the nuclear temperature T is given by

$$T = \sqrt{E/a}\ ;\ E \equiv E^* - \Delta_f\ .$$

Here E^* and a are the excitation energy and level density parameter of a nucleus, respectively. $\Delta_f = \chi \cdot 14/\sqrt{A}$ [MeV] is the pairing energy of a fissioning nuclei; $\chi = 0$, 1, or 2, respectively, for odd-odd, odd-even, or even-even nuclei. For constants α and β in [30] it was found that

$$\begin{aligned} \alpha &= 1 \cdot 10^{-3}\mathrm{MeV}^{-2} \\ \beta &= 6.3157 \cdot 10^{-3}\mathrm{MeV}^{-2}. \end{aligned} \tag{26}$$

Barashenkov *et al.* [29] proposed to estimate the dependence of B_f on E^* by the following empirical relation

$$B_f(E) = B_f(0)/(1 + \sqrt{E/2A}) \tag{27}$$

Table 2

Calculated in the framework of different models (macroscopic B_f^0 – in accordance with [7, 8, 13, 14, 15] and shell corrections from [7]) and experimental (taken from the summary table IV of ref. [11]) values of fission barriers for isotopes of different nuclei

Isotope	Exp. [11]	Present calculations by different methods					
		MS67[7]	PL71[8]	KN74[13]	KNS79[14]	S86[15] without shell cor.	S86[15] with shell cor. from [7]
$^{173}Lu_{71}$	27.3	28.28	28.93	24.85	27.01	24.13	23.59
$^{179}Ta_{73}$	26.2	25.49	25.73	22.44	24.69	22.12	21.43
$^{188}Os_{76}$	23.7	21.25	20.89	18.77	20.31	19.03	18.06
$^{187}Os_{76}$	22.5	20.86	20.81	18.96	20.27	18.84	17.78
$^{186}Os_{76}$	22.5	20.54	20.79	19.21	20.30	18.65	17.57
$^{191}Ir_{77}$	22.8	21.19	20.65	18.93	20.45	17.98	18.31
$^{189}Ir_{77}$	21.7	19.78	19.84	18.65	19.72	17.63	17.11
$^{198}Hg_{80}$	21.8	20.61	20.17	19.99	20.85	14.60	18.81
$^{201}Tl_{81}$	22.3	21.27	20.71	20.74	21.55	13.61	19.71
$^{209}Bi_{83}$	22.6	23.66	22.44	22.04	23.37	11.94	22.30
$^{207}Bi_{83}$	21.2	21.69	20.94	21.03	21.94	11.70	20.49
$^{212}Po_{84}$	18.6	20.05	18.78	18.45	19.72	11.00	18.82
$^{211}Po_{84}$	21.5	20.74	19.70	19.60	20.68	10.90	19.60
$^{210}Po_{84}$	20.4	21.48	20.65	20.77	21.67	10.79	20.40
$^{213}At_{85}$	16.8	18.00	17.13	17.27	18.13	9.90	17.02
$^{227}Ra_{88}$	8.30	8.22	6.39	5.46	7.09	7.81	7.10
$^{233}Th_{90}$	6.44	7.55	5.82	5.00	6.39	6.34	6.51
$^{232}Th_{90}$	5.95	7.33	5.76	5.06	6.36	6.30	6.34
$^{232}Pa_{91}$	6.18	6.45	5.34	5.09	5.95	5.49	5.63
$^{239}U_{92}$	6.29	7.01	5.41	4.80	5.84	5.06	6.04
$^{238}U_{92}$	5.60	6.86	5.39	4.87	5.84	5.03	5.93
$^{237}U_{92}$	6.40	6.69	5.35	4.94	5.83	4.99	5.80
$^{236}U_{92}$	5.44	6.50	5.29	4.99	5.80	4.94	5.66
$^{235}U_{92}$	5.75	6.31	5.22	5.02	5.75	4.89	5.50
$^{234}U_{92}$	5.30	6.09	5.13	5.05	5.69	4.84	5.33
$^{233}U_{92}$	5.49	5.87	5.02	5.06	5.60	4.78	5.14
$^{238}Np_{93}$	6.04	6.15	5.09	4.95	5.56	4.34	5.37
$^{237}Np_{93}$	5.49	5.97	5.02	4.99	5.52	4.29	5.23
$^{244}Pu_{94}$	4.60	6.32	4.97	4.62	5.39	3.95	5.46
$^{242}Pu_{94}$	4.70	6.10	4.97	4.79	5.42	3.88	5.32
$^{241}Pu_{94}$	6.20	5.97	4.94	4.86	5.41	3.84	5.22
$^{240}Pu_{94}$	4.85	5.83	4.90	4.91	5.38	3.79	5.11
$^{239}Pu_{94}$	5.48	5.67	4.84	4.95	5.34	3.75	4.97
$^{238}Pu_{94}$	4.70	5.49	4.77	4.97	5.28	3.70	4.83
$^{236}Pu_{94}$	4.55	5.10	4.57	4.97	5.11	3.58	4.48

Table 2 (*continued*)

Isotope	Exp. [11]	Present calculations by different methods					
		MS67[7]	PL71[8]	KN74[13]	KNS79[14]	S86[15] without shell cor.	S86[15] with shell cor. from [7]
$^{244}Am_{95}$	6.21	5.77	4.77	4.74	5.25	3.38	5.05
$^{242}Am_{95}$	6.40	5.53	4.72	4.86	5.23	3.30	4.86
$^{241}Am_{95}$	6.00	5.38	4.68	4.90	5.20	3.25	4.75
$^{250}Cm_{96}$	4.10	5.69	4.46	4.36	4.87	3.05	4.91
$^{248}Cm_{96}$	4.25	5.60	4.55	4.56	5.00	2.99	4.89
$^{246}Cm_{96}$	4.35	5.45	4.58	4.70	5.07	2.93	4.80
$^{244}Cm_{96}$	4.25	5.25	4.56	4.82	5.07	2.85	4.65
$^{242}Cm_{96}$	4.25	4.99	4.47	4.88	5.01	2.75	4.43
$^{240}Cm_{96}$	4.15	4.68	4.31	4.89	4.88	2.65	4.15
$^{250}Bk_{97}$	5.80	5.27	4.36	4.53	4.76	2.58	4.63
$^{249}Bk_{97}$	4.35	5.22	4.39	4.62	4.81	2.55	4.61
$^{252}Cf_{98}$	3.65	4.95	4.16	4.49	4.54	2.22	4.38
$^{250}Cf_{98}$	3.95	4.88	4.23	4.66	4.64	2.16	4.35
$^{248}Cf_{98}$	3.85	4.75	4.24	4.78	4.68	2.08	4.26
$^{246}Cf_{98}$	3.85	4.57	4.20	4.84	4.66	2.00	4.11
$^{254}Fm_{100}$	3.35	4.36	3.89	4.56	4.26	1.55	3.96
$^{248}Fm_{100}$	2.75	4.04	3.89	4.84	4.31	1.33	3.71
$^{246}Fm_{100}$	2.55	3.83	3.77	4.82	4.21	1.25	3.51
$^{245}Fm_{100}$	2.62	3.71	3.69	4.7[illegible]	4.13	1.21	3.39
$^{244}Fm_{100}$	2.62	3.57	3.59	4.74	4.04	1.16	3.25

Earlier, by means of the classical thermodynamics Yamaguchi [34] has derived the following relation:

$$B_f(E) = B_f(0)(1 - E/E_0),$$

where $E_0 = aT_0^2$, $T_0 \simeq 9$ MeV.

At last, one has to mention the recent attempt of Newton, Popescu and Leigh [35] to interpolate the results of Garcias *et al.* [36] [2] on the evolution of the fission barriers as a function of temperature and angular momentum by a simple formula that can be incorporated in a statistical model code.

In the present paper we estimate the influence of "thermal" effects on B_f (see figs. 4 and 5) in two ways, namely, by means of relation (25) with α and β given by (26) for $a = A/10$ (the results are marked by abbreviation SCM76), and by phenomenological relation (27) (the results are marked by abbreviation BGIT74). One can see that the phenomenological approach (27) provides a stronger decrease in B_f with increasing

[2] Recently Garcias *et al.* [36] have made calculations of nuclear fission barriers by using a Thomas-Fermi model, which self-consistently incorporates the effects of rotations and temperature, with the Skyrme SkM* force. But this method is too difficult to be used in Monte Carlo calculations of fission processes.

excitation energy in comparison with the approach (25). This is most appreciable for medium weight and light nuclei with the energies of excitation above 50 MeV. "Thermal" effects may cause a tenfold increase of the nuclear fissility for $Z^2/A \leq 27$ (see, for example, [37]).

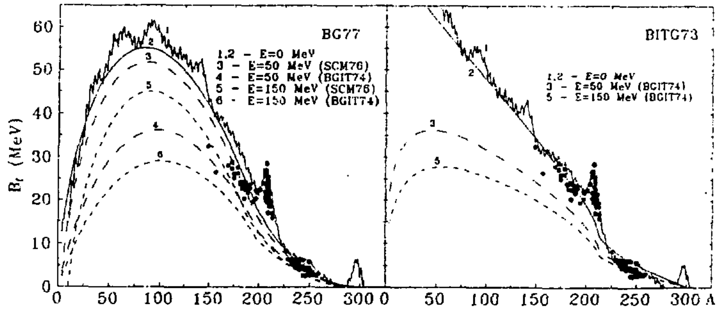


Fig. 4. Comparison of fission barriers predicted by phenomenological [16] (marked as BITG73) and semiphenomenological [17] (marked as BG77) approaches with the experimental data [11, 53] for nuclei along the line of β - stability. The solid lines are the results for zero excitation energy (smooth lines — only the macroscopic part of B_f; "irregular" lines — with Cameron's [18] shell and pairing corrections). The dashed lines show the results of calculations for excited nuclei for the values of excitation energies E indicated in the figure. The results of calculations for the dependence of fission barriers on the excitation energy as proposed in [29] are marked by the abbreviation BGIT74; and for the one proposed in [30], by the abbreviation SCM76.

The dependence of fission barrier heights on the rotation of nuclei . In ref. [38] from the measured angular correlations and distributions of fission fragments produced in the bombardment of ^{232}Th targets with different projectiles it has been found that in the case of proton-induced reactions the upper limits of the mean angular momenta transferred to the fissioning nuclei are small. As one can see from table 3, only a small fraction of the grazing angular momentum is left in the fissioning nuclei.

The high relative angular momentum in the entrance channels seems to be taken away by the cascade ejectiles or by fast pre-equilibrium particles [38]. Therefore, in calculations of such reactions in the first order it is possible to neglect the dependence of B_f on the angular momenta L of fissioning nuclei. On the contrary, in heavy-ion induced reactions the momenta of fissioning nuclei are high (see, e.g., [4]) and the dependence of B_f on L must be taken into account.

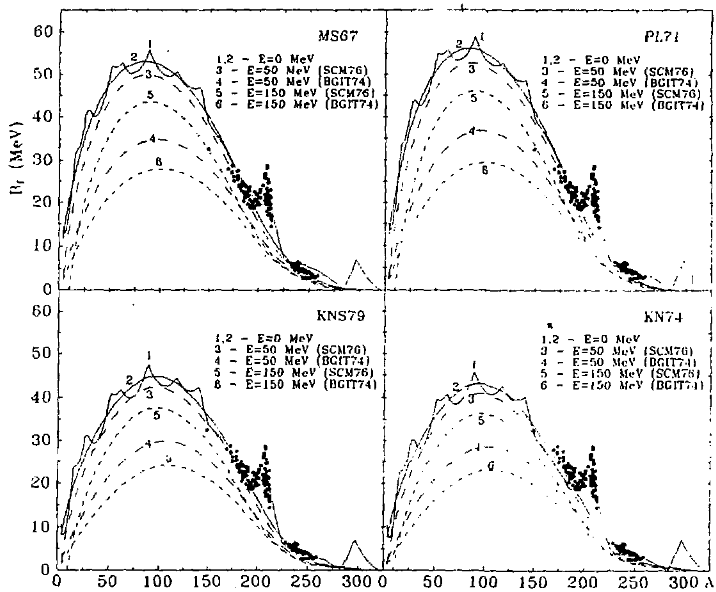


Fig. 5. Comparison of fission barriers calculated in the framework of the LDM [7] (abbreviation MS67), the LDM [8] (abbreviation PL71), the Yukawa-plus-exponential modified LDM [14] (abbreviation KNS79), and the single-Yukawa modified LDM [13] (abbreviation KN74) with the experimental data [11, 53] for nuclei along the line of β - stability. The remaining notation is the same as in fig. 4.

Several approaches for the description of fission barrier dependence on the angular momenta of rotating nuclei are used to date.

One of the most extensively used and perhaps the most successful theoretical model for this purpose is the Rotation-Liquid-Drop Model (**RLDM**) of Cohen, Plasil and Swiatecki [9]. However, questions have been raised about the general validity of the RLDM [39, 40].

Table 3
The mean angular momentum transfers $< L >$ of fissioning nuclei and the maximum possible angular momentum in the entrance channel, i.e., the grazing angular momentum l_{max} for different reactions [38]

System	E_i, MeV	l_{max}, $\hbar$	$< L >$, $\hbar$
$p + ^{232}Th$	140	25	4
	250	35	1
	500	49	1
	1000	70	1
$d + ^{232}Th$	70	25	13
	140	37	11
	500	72	5
	1000	102	5
$\alpha + ^{232}Th$	280	75	17
	1000	148	7
$\alpha + ^{197}Au$	280	72	28

Later on Mustafa *et al.* [10] have proposed a model which differs from the RLDM in the shape parametrization and in the calculation of the Coulomb, surface, and rotation energies. The authors of [10] used the two-center-model shape parametrization which allows for triaxial shape variations and a continuous transition from one-center to two-center shapes with a smooth neck. Mustafa *et al.* [10] calculated the surface energy with the Yukawa-plus-exponential folding function of Krappe, Nix and Sierk [14] which incorporates the effects of the finite range of nuclear force and the diffuse nuclear surface, and calculated both the Coulomb and rotation energies with surface diffuseness described by the Yukawa folding function.

A further development of this approach has been done by Sierk [15]. Sierk used a highly accurate numerical techniques, a flexible shape parametrization which allows accurate estimation of the convergence of results as a function of the number of degrees of freedom of the nuclear shapes considered, and a more perfect set of parameters for calculations in comparison with [10]. In addition, Sierk has approximated his results for many hundreds of nuclei in a useable form in two computer subroutines **BARFIT** and **MOMFIT** which provide accurate values for fission barrier heights and saddle-point moments of inertia as functions of Z, A and L, and can be easily incorporated in statistical evaporation models.

At last, one has to mention the following phenomenological approach frequently used in statistical calculations (see, e.g., [16, 41]) to estimate the dependence of B_f on L. In this approach one assumes that the nuclear rotation energy E_R is not available for excitation energy released in the fission and evaporation processes. This implies that the fission barrier $B_f(L)$ for a fissioning nucleus with the angular momentum L can be written as

$$B_f(L) = B_f(0) - (E_R^{gs} - E_R^{sp}). \quad (28)$$

Here E_R^{gs} and E_R^{sp} are nuclear rotational energies for the ground state and at the saddle-point, respectively

$$E_R^{gs} = \frac{L(L+1)\hbar^2}{2J_{rb}},$$

$$E_R^{sp} = \frac{L(L+1)\hbar^2}{2J_{sp}}, \tag{29}$$

$$J_{rb} = 0.4 M_n r_0^2 A^{5/3}. \tag{30}$$

L is the angular momentum of nucleus, M_n is the nucleon mass, and for the moment of inertia of a nucleus at the saddle-point J_{sp} the values calculated and plotted in [42] or tabulated in [21] are used.

As one can see from fig. 6, Strutinsky's [42] results for moments of inertia of nuclei at the saddle-point are very close to Cohen and Swiatecki's ones [21]; thus, concrete numerical calculations may be done with any of them.

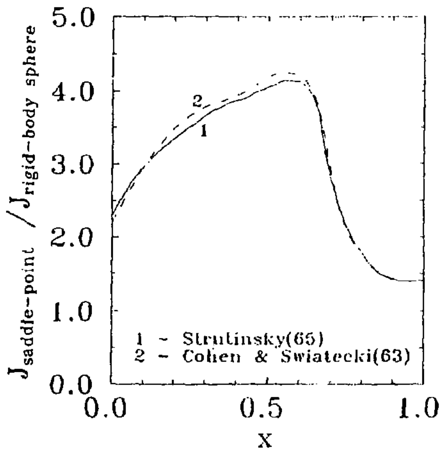
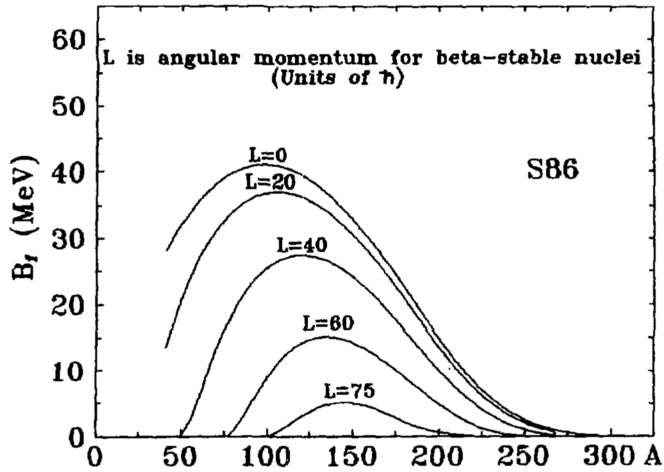


Fig. 6. Comparison of Strutinsky's [42] and Cohen and Swiatecki's [21] prediction for moments of inertia of nuclei at the saddle-point as functions of the fissility parameter x.

As an example, fig. 7 shows macroscopic fission barriers of rotating nuclei with different values of angular momentum L calculated in accordance with (28-30) with the LDM [7] parameters and Strutinsky's [42] values for J_{sp}, and, for comparison, fission barriers computed with Sierk's subroutine BARFIT as functions of the mass number for beta-stable nuclei.



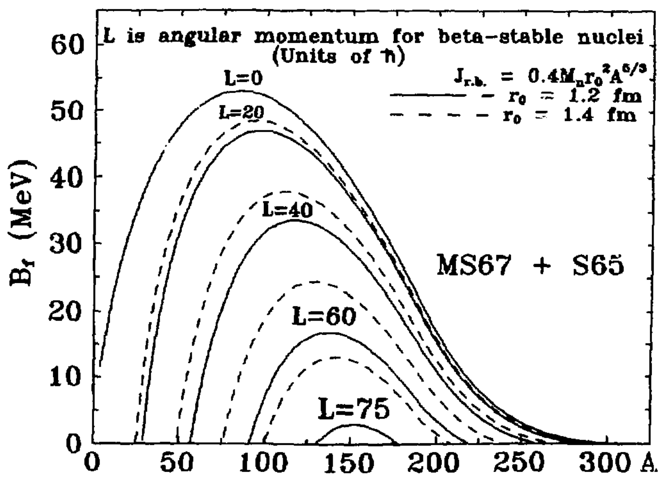


Fig. 7. Calculated fission barriers for rotating nuclei with different values of the nuclear angular momentum L as functions of the mass number for beta - stable nuclei. **Lower fig.**: the LDM [7] with Strutinsky's [42] values for moments of inertia of nuclei at the saddle-point, in accordance with(28-30); **upper fig.**: predictions of Sierk's [15] subroutine BARFIT.

One can see that for small values of the angular momentum L and $A \leq 200$ the phenomenological approach (28-30) with the LDM [7] predicts significantly higher values for fission barriers than the Yukawa-plus-exponential model [15]. But for $A > 200$ and/or high values of the nuclear angular momentum L the results obtained in both these approaches are similar.

Fig. 8 shows fission barrier heights calculated within different models for ^{153}Tb, ^{176}Os, and ^{229}Np nuclei as functions of the angular momentum L. One can see that fission barriers calculated phenomenologically by (28-30) in the LDM [7] with Strutinsky's values for moments of inertia of nuclei at the saddle-point are similar to those calculated in the Yukawa-plus-exponential model [5] and to Mustafa's *et al.* [10] predictions. Our concrete calculations show that Sierk's subroutine BARFIT [15] needs about a tenfold increase of computing time in comparison with calculations according to (28-30). Thus, in concrete Monte Carlo calculations which need much computing time to obtain a good statistics we can successfully use the phenomenological approach (28-30) to estimate the dependence of fission barriers on the angular momentum of nucleus.

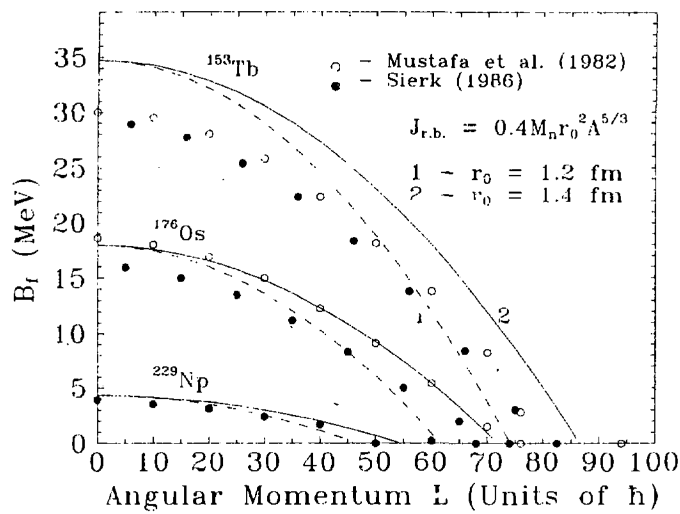


Fig. 8. Calculated fission barriers as functions of the angular momentum for ^{153}Tb, ^{176}Os and ^{229}Np. Solid and dashed lines are our calculations in accordance with (28-30) for the LDM [7] and Strutinsky's [42] values for moments of inertia of nuclei at the saddle-point for $r_0 = 1{,}4$ fm and $r_0 = 1.2$ fm, respectively. The open circles show the tabulated results of Mustafa *et al.* [10]. The solid circles are the results obtained with the subroutine BARFIT of Sierk [15].

3. Analysis of fissility of excited compound nuclei

The main relationship for particle emission and fission widths. In this section, we will use the fission barriers considered above to analyze the energy dependence of the fissility of different excited compound nuclei. In the Weisskopf statistical theory of particle emission [43] and the Bohr and Wheeler [44] theory of fission the partial widths Γ_j for the emission of a particle j ($j \equiv n, p, d, t, {}^3He, \alpha$) and Γ_f for fission are expressed by the following approximate formulae (units: $\hbar = c = 1$; see, e.g., [23, 41]):

$$\Gamma_j = \frac{(2s_j+1)m_j}{\pi^2 \rho_c(U_c)} \int_{V_j}^{U_j - B_j} \sigma^j_{inv}(E)\rho_j(U_j - B_j - E)EdE \ , \tag{31}$$

$$\Gamma_f = \frac{1}{2\pi\rho_c(U_c)} \int_0^{U_f - B_f} \rho_f(U_f - B_f - E)dE \ . \tag{32}$$

Here ρ_c, ρ_j, and ρ_f are the level densities of a compound nucleus, residual nucleus produced after the emission of the j-th particle, and of fissioning nucleus at the fission saddle point, respectively; m_j, s_j and B_j are the mass, spin and the binding energy of the j-th particle, respectively; B_f is the fission barrier height. In the present work, we calculate the binding energies of particles through the use of Cameron's [18] formulae; $\sigma^j_{inv}(E)$ is the inverse cross-section for absorption of the j-th particle with kinetic energy E by the residual nucleus. We use here for $\sigma^j_{inv}(E)$ the approximation proposed by Dostrovsky [45]:

$$\sigma^j_{inv}(E) = \sigma^j_{geom}\alpha_j \left(1 + \frac{\beta_j}{E}\right) \ , \tag{33}$$

where

$$\sigma^j_{geom} = \pi R_j^2 \ ; \ R_j = \tilde{r}_0 A_{fj}^{1/3} \ ; \ \tilde{r}_0 = 1.5 \text{ fm} \ ;$$

$$\alpha_n = 0.76 + 2.2A_{fj}^{-1/3} \ ;$$

$$\beta_n = (2.12A_{fj}^{-2/3} - 0.05)/\alpha_n \ .$$

For charged particles $\beta_j = V_j$, where V_j is the effective Coulomb barrier and the constants α_j are calculated for every concrete nucleus by interpolation under the values given in ref. [45]. The angular momentum L dependence of the level density is taken into account by the relation $\rho(E^*, L) = \rho(U, 0)$ where $U = E^* - E_R$ and E_R are respectively, the "thermal" and rotational energies of the nucleus;

$$U_c = E^* - E_R^c - \Delta_c \ ; \ U_j = E^* - E_R^j - \Delta_j \ ; \ U_f = E^* - E_R^f - \Delta_f \ .$$

Here E^* is the total excitation energy of the compound nucleus; E_R^c, E_R^j, and E_R^f are the rotational energies for the compound, residual, and fissioning nucleus at the saddle point, respectively, and are determined by the formulae (29,30);

$$\Delta_c = \chi \cdot 12/\sqrt{A_c} \; ; \; \Delta_j = \chi \cdot 12/\sqrt{A_{fj}} \; ; \text{ and } \Delta_f = \chi \cdot 14/\sqrt{A_c} \text{ (in MeV)}$$

are the pairing energies for the compound and residual nuclei, and for the fission saddle point, respectively, $A_{fj} = A_c - A_j$ where A_c and A_j are the mass numbers of the compound nucleus and of the j-th particle, respectively.

In the Fermi-gas approach for the nuclear level density

$$\rho(E^*) = Const \cdot exp\{2\sqrt{aE^*}\} \; ,$$

for particle emission Γ_j and fission Γ_f widths (31-32), one obtains (see, e.g., [46]):

$$\begin{aligned} \Gamma_j &= \frac{(2s_j+1)m_j\alpha_j\tilde{r}_0^2A_{fj}^{2/3}}{\pi a_j exp(2\sqrt{a_cU_c})}\{(U_j - B_j')[1 + (k_j - 1)exp(k_j)] - \\ &- [6 + (k_j^3 - 3k_j^2 + 6k_j - 6)exp(k_j)]/(4a_j)\} \; , \end{aligned} \tag{34}$$

$$\Gamma_f = \frac{1 + (k_f - 1)exp(k_f)}{4\pi a_f exp(2\sqrt{a_cU_c})} \; , \tag{35}$$

where $B_n' = B_n - \beta_n$; $B_{j\neq n}' = B_j + V_j$; $k_j = 2\sqrt{a_j(U_j - B_j')}$; $k_f = 2\sqrt{a_f(U_f - B_f)}$; a_c, a_j, and a_f are the level density parameter for the compound and residual nuclei, and for the fission saddle point, respectively.

In the case of transuranium nuclei, when double-humped fission barriers are used, we define the fission width by the expression (see, e.g., [23]):

$$\Gamma_f = \frac{\Gamma_A\Gamma_B}{\Gamma_A + \Gamma_B} \; , \tag{36}$$

where Γ_A and Γ_B are the partial widths for the corresponding saddle points. We calculate each of these widths by formula (35) with the own shell correction.

Comparison with Experiment. By now a lot of experimental data are available on the nuclear fissility and fission cross-section of heavy nuclei induced by different probes (see the reviews [1]-[4]). The fissility is the ratio of the fission cross-section to the inelastic interaction cross-section $P_f = \sigma_f/\sigma_{in}$. For a given excited compound nucleus the fissility may be estimated as the ratio of partial widths Γ_f/Γ_{tot}, where $\Gamma_{tot} = \Gamma_f + \sum_j \Gamma_j$.

We have analyzed, by using formulae (34-35) and fission barriers regarded above, practically all the data on nuclear fissility published in the review [1]. Let us show here only some exemplary results. As an example, measured [1] and calculated fissilities for ^{189}Ir, ^{188}Os, ^{169}Ta, and ^{173}Lu nuclides are shown in fig. 9. The calculations were performed with fission barriers from ref. [14] without taking into account the dependence of B_f on the excitation energy E^*, with Cameron's [19] shell and pairing corrections, the third Iljinov, Mebel's *et al.* systematics for the level density parameter without an explicit taking into account of collective effects, for the values of the ratio a_f/a_n indicated in the figure.

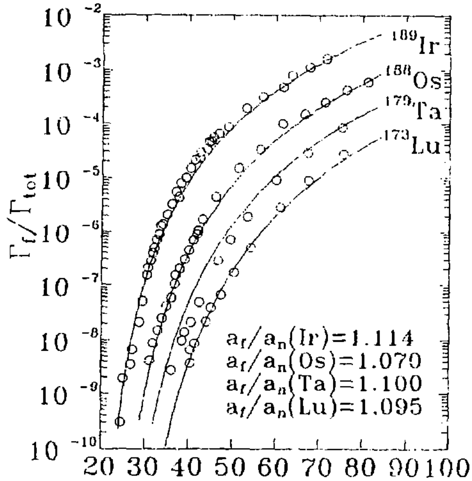


Fig. 9. Excitation energy dependence of the fissility Γ_f/Γ_{tot} of different nuclei. Curves are the results of our calculation with KNS79 [14] fission barriers, C70 [19] shell corrections, the third Iljinov, Mebel *et al.* [23] systematics for the level density parameter without the explicit taking into account of collective effects. Experimental points were taken from the review [1]. The used values for the ratio a_f/a_n are shown in figure.

One can see that in this approach one obtains a good description of experimental data. Our analysis shown that for every nuclide it is possible to select a concrete model for the fission barrier, shell and pairing corrections, a systematics for the level density parameter, and to fit the value of the ratio a_f/a_n for obtaining a very good description of the experimental data. But it is not possible to describe well the experimental fissilities for all nuclides with a fixed set of these options.

The calculated fissilities are the most sensitive to the used values of the ratio a_f/a_n. As an example, fig. 10 shows how the calculated fissility of the excited ^{189}Ir nuclide depends on the ratio a_f/a_n. One can see that for high excitation energies $E^* > 50$ MeV a small increase of the ratio a_f/a_n from 1.04 to 1.13 results in an increase of the calculated fissility more than one order of magnitude.

Fig. 11 shows the fissilities of the ^{189}Ir nuclide for the ratio $a_f/a_n = 1.114$, the third Iljinov, Mebel's *et al.* [23] systematics for the level density parameters, Cameron's [19]

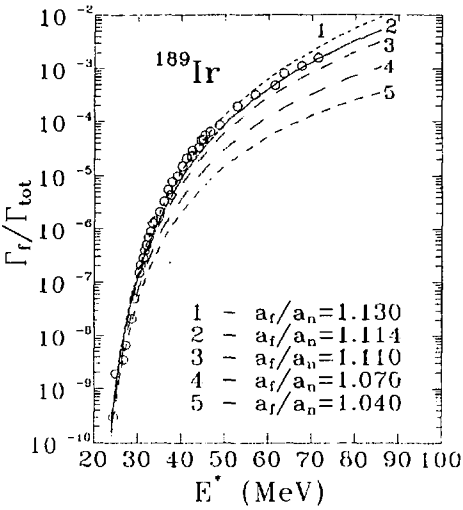


Fig. 10. Dependence of the calculated fissility of the excited ^{189}Ir compound nucleus on the used values for the ratio a_f/a_n. The remaining notation is the same as in fig. 9.

shell and pairing corrections calculated with different fission barriers, namely, BITG73 [16]; BG77 [17]; MS67 [7]; PL71 [8]; KN74 [13]; KNS79 [14]; and S86 [15] without taking into account the dependence of B_f on E^*. One can see that for this option all the used fission barriers provide a correct description of the shape of the calculated curves, and by fitting the value of the ratio a_f/a_n it is possible to obtain a good description also for the absolute value of the fissility for each regarded model for B_f.

An example of the dependence of the calculated fissilities on the form of the energy dependence of the fission barriers $B_f(E^*)$ is shown in fig. 12. Our analysis shows that for the interval of excitation energies regarded here it is possible to fit the value of the ratio a_f/a_n to describe the data with the dependences $B_f(E^*)$ proposed both by Barashenkov *et al.* [29] and by Sauer *et al.* [30], as well as without an explicit dependence of B_f on E^*. To elucidate better this question, it is necessary to analyze the fissilities and fission cross-sections in a larger range of incident/excitation energies.

An example of influence of the angular momentum on the fissility of an excited fissioning nucleus is shown in fig. 13. One can see that for small values of the angular momentum $L < 20$ (that is realized, e.g., in the case of nucleon-nucleus interactions at intermediate energies) we can neglect the dependence of the fission barriers on the angular momentum in calculations of the nuclear fissilities. On the contrary, for high values of L (that is realized, e.g., in heavy ion-induced reactions) taking into account

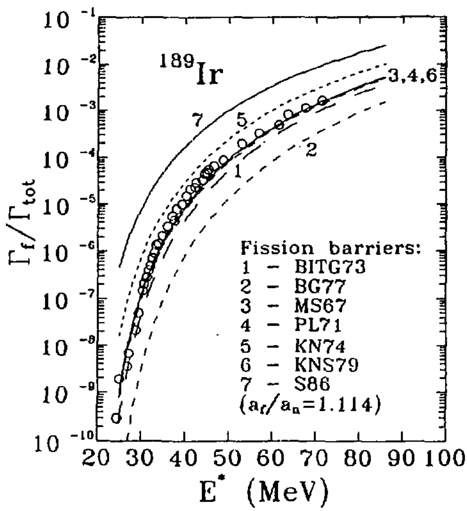


Fig. 11. Dependence of the fissility of the exited ^{189}Ir compound nucleus on the fission barriers used in the calculations. The remaining notation is the same as in fig. 9.

the dependence $B_f(L)$ not only increases the absolute values of the fissilities several orders of magnitude but also significantly changes the shape of the dependence of nuclear fissility on the excitation energy of a rotating fissioning nucleus.

Fig. 14 shows how the theoretical fissility depends on the systematics for the level density parameter used in the calculations. One can see that Malyshev's systematics for $a(Z,N)$ provides a good description of the shape (and by fitting the ratio a_f/a_n, of the absolute value) of the nuclear fissility as a function of E^* only for low values of E^*. Cherepanov and Iljinov's [47] and Iljinov, Mebel's *et al.* [28] systematics for $a(Z,N,E^*)$ allow one to obtain a good description of the data in a larger interval of E^*, reproduce very close results and seem to describe the data better than the popular systematics of Ignatyuk *et al.* [48].

4. The fission cross-section

In this section, we incorporate all the above-considered systematics for fission barriers, shell and pairing corrections, level density parameters, and formulae for the calculation of the fission width in the Cascade-Exciton Model (CEM) of nuclear reactions [50] and calculate the fission cross-section for intermediate-energy proton-induced reactions. A detailed description of the CEM may be found in [50]. Therefore, we state here only that the CEM assumes that the reactions occur in three stages.

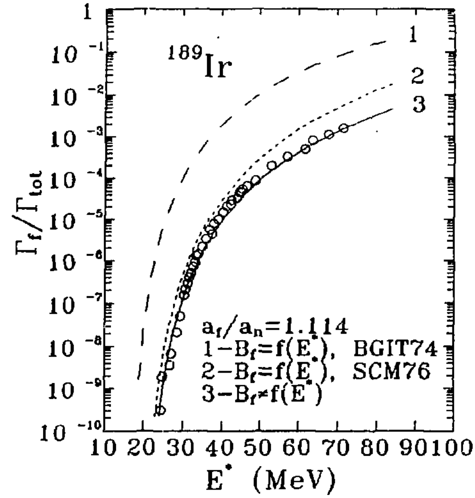


Fig. 12. Dependence of the fissility of the exited ^{189}Ir compound nucleus on the form of the excitation energy dependence of fission barriers $B_f(E^*)$. The remaining notation is the same as in fig. 9.

The first stage is the intranuclear cascade in which primary particles can be rescattered several times prior to absorption by, or escape from the nucleus. The excited residual nucleus formed after the emission of the cascade particles determines the particle-hole configuration that is the starting point for the second, pre-equilibrium stage of the reaction. The subsequent relaxation of the nuclear excitation is treated in terms of the exciton model of pre-equilibrium decay which includes the description of the equilibrium evaporative stage of the reaction. The CEM uses the Monte Carlo method to simulate all three stages of the reactions.

The fission cross-section σ_f is determined by the ratio of the number N_f of fission events to the total number N_t of Monte Carlo simulations

$$\sigma_f = \sigma_{in}P_f = \sigma_{in}\frac{N_f}{N_{in}} = \sigma_{geom}\frac{N_f}{N_t}\,, \qquad (37)$$

where $\sigma_{in} = \sigma_{geom}N_{in}/N_t$ is the total reaction cross-section; N_{in} is the total number of simulated inelastic interactions; σ_{geom} is the geometrical cross-section for the projectile-target interaction. In the case of low-fissioning nuclei (e.g., gold) $N_f << N_t$, and as a consequence, a large number of cascades should be calculated to obtain the value σ_f with a sufficient statistical accuracy, and the calculation of σ_f becomes extremely

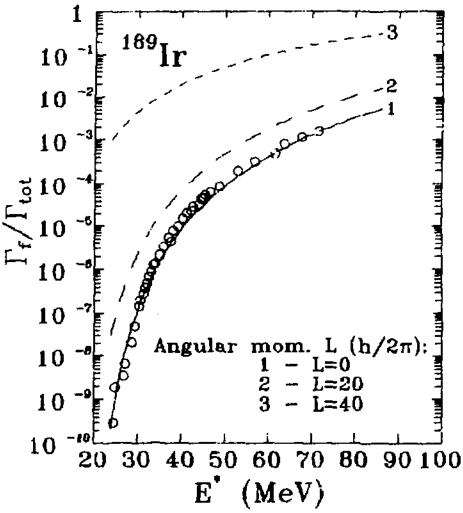


Fig. 13. Dependence of the fissility of the exited ^{189}Ir compound nucleus on the value of its angular momentum L. The dependence of fission barriers on the angular momentum $B_f(L)$ were calculated under the formulae (28-30) with $r_0 = 1.2$ fm. The remaining notation is the same as in fig. 9.

time-consuming. Therefore, here, besides the direct calculation of the fission cross-section through expression (37), following Barashenkov *et al.* [29], we have carried out Monte Carlo sampling by means of the statistical functions $W_n = \prod_{i=1}^{N} w_{ni}$ and $W_f = 1 - W_n$, where W_n is the Monte Carlo calculated probability for the nucleus to "drop" the excitation energy E^* by the chain (cascade) of N successive evaporations of particles; W_f is the probability for the nucleus to fission at one of the chain stages; $w_{ni} = 1 - w_{fi}$ is the probability of particle emission at the i-th stage of the evaporative cascade; w_{fi} is the corresponding fission probability which is easy to determine using the formulae (34-35) for the widths Γ_j and Γ_f. After the subsequent averaging of the W_f value over the total number N_{in} of the cascades followed, and the multiplication of the result by the corresponding total cross-section σ_{in} for inelastic interactions, we obtain the following expression for the fission cross-section:

$$\sigma_f = \frac{\sigma_{in}}{N_{in}} \sum_{i=1}^{N_{in}} (W_f)_i \, . \qquad (38)$$

As an example, the incident energy dependences of experimental and calculated within this formula fission cross-sections for proton-gold and -uranium interactions are

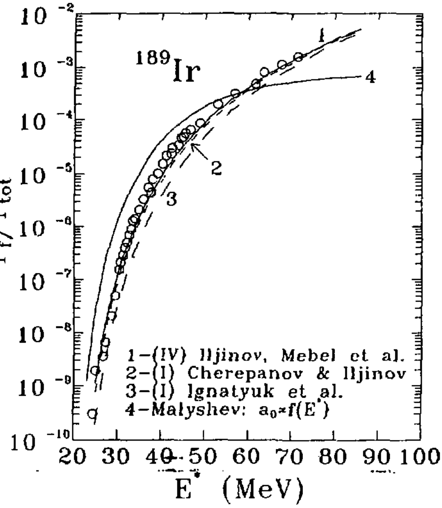


Fig. 14. Dependence of the fissility of the exited ^{189}Ir compound nucleus on the systematic. foı the level density parameter used in calculations; $a_f/a_n = 1.114$. The remaining notatic.ı is the same as in fig. 9.

shown in fig.15. We performed these calculations with Cameron's [18] shell and pairing corrections, third Iljinov, Mebel's *et al.* [23] systematics for the level density parameter, Krappe, Nix and Sierk's [14] fission barriers with the dependences $B_f(E^*)$ proposed by Barashenkov *et al.* [29], by Sauer *et al.* [33], as well as without a dependence of B_f on E^*. The values used for the ratio a_f/a_n are shown in the figure. One can see that by choosing the corresponding values for the ratio a_f/a_n the CEM reproduces correctly the shape and the absolute value of the fission cross-sections in the interval of bombarding energies regarded here, independently of the form of the dependence $B_f(E^*)$ used in the calculations. Analogous results have been obtained also for other targets. A more detailed analysis of fission processes in the framework of the CEM will be done in a separate paper.

4. Summary and conclusion

Thus, the review and comparative analysis of the models for description of fast-computing single-humped fission barriers for statistical calculations are given. Our analysis shows that the simple and not time-consuming phenomenological approaches of Barashenkov *et al.* [16, 17] provide equally good descriptions of the experimental fission barriers with Cameron's [18] and Truran, Cameron and Hilf's [19] shell and pairing corrections both very convenient for Monte Carlo calculations. Nevertheless, for neutron-rich and neutron-deficient nuclei the use of the shell corrections from ref. [19] seems to be more preferable. When one uses the Myers and Swiatecki's [7] shell corrections popular in the description of nuclear fission, the Yukawa-plus-exponential modified LDM [14] provides the best agreement of calculated B_f with the experimental data for the nuclei along the line of β-stability.

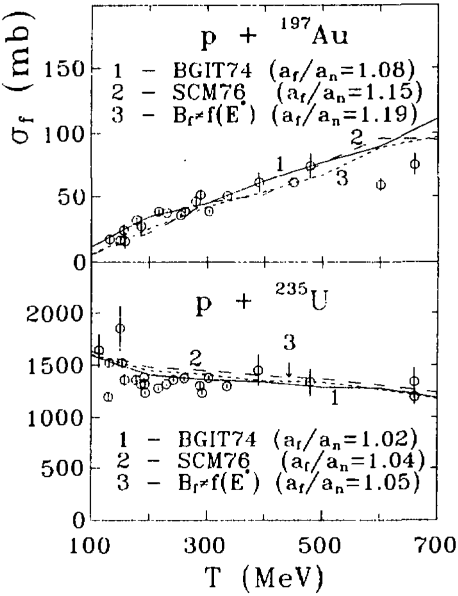


Fig. 15. The energy dependence of the fission cross section for nuclei of gold and uranium. Calculations were performed with KNS79 [14] fission barriers, Cameron's [18] shell and pairing corrections, third Iljinov, Mebel's *et al.* [23] systematics for the level density parameter, for the dependences of B_f on E^* proposed by Barashenkov *et al.* (BGIT74) [29], by Sauer *et al.* (SCM76) [30], as well as without the dependence of B_f on E^*. The values used for the ratio a_f/a_n are shown in the figure. The experimental points are taken from the summary table 159 of the monograph [51].

Our estimation of the reduction of the fission barrier heights with increasing excitation energy E^* has shown that the phenomenological approach (27) proposed by Barashenkov *et al.* [29] provides a significantly stronger decrease of B_f with increasing E^* in comparison with the approach (25) of Sauer *et al.* [30]. "Thermal" effects may cause about a tenfold increase of the nuclear fissility for medium weight and light nuclei with the excitation energies above 50 MeV.

It has been shown that if one takes into account the dependence of fission barriers on the angular momentum of a *fissioning nucleus*, the phenomenological approach with formulae (28-30) provides results similar to those obtained by Mustafa's *et al.* [10] and Sierk's [15] models, but needs about ten times shorter computing time in comparison with the subroutine BARFIT of Sierk and, therefore, is more convenient for Monte Carlo calculations. For all this, let us note that Strutinsky's [42] results for the moments of inertia of nuclei at the saddle-point are very close to the Cohen and Swiatecki's [21] ones, so the concrete numerical calculations may be done with any of them.

Nuclear fissility P_f for different excited compound nuclei as functions of the excitation energies E^* have been studied. We performed a detailed analysis of the dependence of theoretical fissilities on the models for B_f and functional forms $B_f = B_f(E^*)$ and $B_f = B_f(L)$, on the systematics for the level density parameter, and on the values of the ratio a_f/a_n used in calculations. It has been found out that for every nuclide it is possible to select a concrete model for B_f, $a(Z, N, E^*)$, shell and pairing corrections, and to fit the ratio a_f/a_n for obtaining an excellent description of experimental data. But it is impossible to describe well the experimental P_f simultaneously for all the nuclides with a fixed set of these options. The theoretical P_f are the most sensitive to the values of the ratio a_f/a_n used in calculations. We nave found out that Cherepanov and Iljinov's [47] and Iljinov, Mebel's *et al.* [23] systematics for $a(Z, N, E^*)$ allow one to obtain a good description of nuclear fissilities in the hole interval of excitation energies regarded here, reproduce very close results and seem to describe the data better than the popular systematics of Ignatyuk *et al.* [48].

It has been shown that the Cascade-Exciton Model of nuclear reactions is able to reproduce correctly the shape and the absolute value (let us recall that the CEM predicts the absolute values for all calculated characteristics and does not require any normalization to adjust the results) of the fission cross-sections for proton-nucleus interactions at intermediate energies. This fact, together with a good description of proton- and neutron-induced particle production published in [50, 52], indicate the predicative power of the CEM and the possibility of using the CEM to provide nuclear data at intermediate energies needed for different important applications, e.g., for the transmutation of long-lived radionuclides produced in reactors with a spallation source.

From our point of view, very voluminous but unco-ordinated experimental data on fission processes obtained by now in separate measurements do not permit one to discriminate various models for fission barriers and to determine simultaneously the value of the ratio a_f/a_n. New complex data on fission processes, measured simultaneously with the characteristics of all emitted particles and fragments for such reactions where the fission cross-section is of the same order of magnitude with the particle- and fragment-production cross-sections, and the analysis of all these data in a unique approach may clear up these questions. Such "complete" measurements are possible and desirable in the near future at the FOBOS setup in the JINR FLNR.

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