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MESIC MOLECULES OF LIGHT NUCLEI IN THE OSCILLATOR REPRESENTATION

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1. Introduction

We study the ground state energies of mesic molecules of light nuclei in the framework of the oscillator representation method ref.[1,2]. This is one of the fundamental problems (see, for example ref.[3]). Muon transfer from the mesic hydrogen to other nuclei is of considerable interest when studying mesic atom interaction processes ref.[3-10].

At the present time, the transition of muons from mesic atoms of the hydrogen isotopes H (p, d, t) to light nuclei (He, Li, Be) is investigated [6-10]. The ground state muonic hydrogen with light nuclei can lead to the formation of hydrogen-nuclei mesic molecules [4]. A system like that is accompanied by the decay (dissociation) of the molecule into the hydrogen nuclei (isotopes) and mesic molecules of the light nuclei, because there are no bound states for the lower term.

The molecular mechanism of the muon transfer from the ground state of the mesic hydrogen to helium is confirmed experimentally [5,6]. Gershtein et al. [7] and Kravtsov et al.[8] have theoretically investigated the bound-state energies of the systems $(H\mu\,^3He)$ and $(H\mu\,^4He)$.

Muon transfer from the mesic hydrogen to lithium is of special interest (see, for example ref.[9]). The bound-state energies of mesic molecules of light nuclei (lithium) have first been calculated by using the Born-Oppenheimer approximation in ref.[10] and in ref.[11] these calculation were made on the basis of surface function expansions. In ref.[11] bound-state energies of the $(H\mu\,^7Be)$ system were caculated as well.

The main purpose of these investigations is mainly the construction of highly accurate numerical solutions of the Schrödinger equation for the three–body Coulomb system.

Research of the dependence of the binding energy of the mesic molecules $(H\mu N_Z)$ on the masses and charges of particles has both theoretical and experimental significance. It is one of the central points in understanding the molecular mechanism of the muon transfer from mesic hydrogen to other nuclei and the formation dynamics of the few-body Coulomb systems. It provides us, e.g., with the detailed information on the strong $H-N_Z$ interaction at very low energies (a few kev), information relevant to astrophysical questions, e.g., the pp-cycle in the sun.

The present paper should be considered the direct sequal of [2] where the oscillator representation method is formulated. Here we have calculated the bound-state energies of mesic molecules of light nuclei (He, Li, Be) by using the method of oscillator representation (OR). The OR method in the zeroth approximation provides an analytical study of the formation dynamics of the three-body Coulomb systems with the accuracy less then 1 per cent. As a consequence, the universal research of the dependence of bound state energy of the three-body Coulomb system on the masses and charges of particles becomes possible. In the papers[1,2], the OR method was used to calculate the energy spectrum for a wide class of potentials and to determine the region of stability for the three-body Coulomb system with unit-charges and arbitrary masses.

The paper is organized as follows. In section 2 we calculate the bound-state energies of mesic molecules of light nuclei (He, Li, Be). In section 3, we study the dependence of the binding energy of mesic molecules of light nuclei on the mass and charge and determine the stability boundary for mesic molecules of light nuclei with masses $m_N = 2Zm_p$ where m_p is the mass proton and Z is the charge of nuclei.

2. The binding energy of the mesic molecules of light nuclei

In this section, we shall study the ground state energies of mesic molecules of light nuclei in the framework of the oscillator representation method [1,2]. The binding energies [7-12] of the mesic molecules $(H\mu N_Z)$ are determined with respect to the ground state of the $(H\mu)$ atom

$$E_{bin} = E_{H\mu N_Z} - E_{H\mu} \,. \tag{1}$$

The details of calculation of the ground state energy of three-body Coulomb systems are given in ref.[2]. The ground state energy of the mesic molecules $(H\mu N_Z)$ can be represented in the form

$$E = -\frac{1}{2}\alpha^2 \frac{m_H m_N}{m_H + m_N} U \,,$$

where m_H and m_N are the masses of the hydrogen isotopes and nuclei, respectively. According to (1), the binding energies of the mesic molecules $(H\mu N_Z)$ are written in the form

$$E_{bin} = -\frac{\alpha^2}{2}\frac{m_H m_\mu}{m_H + m_\mu}\left(\frac{m_N}{m_\mu}\frac{m_H + m_\mu}{m_H + m_N}U - 1\right) \,, \tag{2}$$

where U is the energy parameter and m_μ is the muon mass. The energy parameter U is determined from the following equation as a function of masses and charges of particles

$$\varepsilon_0(U) = 0 \,, \tag{3}$$

where $\varepsilon_0(U)$ being the energy of oscillators in the zeroth approximation of the OR, can be written in the form (the details see ref.[2])

$$\varepsilon_0(U) = \min_{\Omega,\omega,\theta,d}\left\{\frac{2\theta}{2\theta+1}\left[\frac{d}{4}\left(\frac{\Omega}{\omega}+\frac{\omega}{\Omega}\right)+\frac{\theta}{2}\left(\frac{d}{d-2}\frac{\Omega}{\omega}+\frac{d\omega}{2\Omega}\right)+\frac{4Z}{\omega}\right.\right.$$
$$\left.\left.+\frac{Ud}{\omega\Omega}-\frac{d(d-4)}{4(d-2)}\frac{\Omega}{\omega}\right]-\frac{c}{2}\frac{d(d+2)(d+4)\Gamma(2\theta)}{4^\theta\Gamma^2(\theta)}f(\Omega,\omega,\theta,d)\right\} \tag{4}$$

where

$$f = \int_0^1 dy y^{d/2}(1-y)^2\int_{-1}^1 d\tau(1-\tau^2)^\theta\left[\frac{Z}{\sqrt{(1-y)^2\Omega^2 + c_N^2y^2\omega^2 - 2c_Ny(1-y)\tau\omega\Omega}}\right.$$
$$\left.+\frac{Z}{\sqrt{(1-y)^2\Omega^2 + c_H^2y^2\omega^2 + 2c_Hy(1-y)\tau\omega\Omega}}\right] \,,$$

and the notation is used

$$\lambda = \sqrt{\frac{m_H m_\mu m_N}{m_H + m_\mu + m_N}} \,, \qquad M = \frac{m_H m_N}{m_N + m_H}$$
$$c_j = \frac{\lambda}{m_j} \,, \qquad c = c_H + c_N \qquad j = H, N \,.$$

The charge of light nuclei is denoted by Z.

For the calculation, we have used the following values of masses:

$$m_\mu = 206.77m_e, \quad m_p = 1836.15m_e, \quad m_d = 3670.48m_e, \quad m_t = 5496.9m_e$$
$$m_{\,^3He} = 5495.92815m_e, \quad m_{\,^4He} = 7294.3561m_e, \quad m_{\,^6Le} = 10961.91216m_e,$$

$$m_{^7Li} = 12786.40853 m_e, \quad m_{^7Be} = 12787.17851 m_e,$$

where m_e is the electron mass. The results of calculations are given in Table 1.

Taking into account the values of the energy parameter U, which is represented in Table 1., we can determine the binding energy from (2) for each mesic molecule of light nuclei. The results our calculations are given in Tables 2 and 3.

Table 2 represents the absolute values of the binding energies (in eV) of mesic molecules of Helium isotopes, determined in papers [7,8], and our results. In Table 3 absolute values of the binding energies (in eV) of the $(H\mu\, ^6Li)$, $(H\mu\, ^7Li)$ and $(H\mu\, ^7Be)$ systems are given.

From Table 2, one can see that the results of our calculations of the binding energies for the mesic molecules of Helium isotopes are in agreement with other theoretical calculations. But in Table 3 our results are not in good agreement with the other theoretical investigations. We want to note that the accuracy of the zeroth approximation for the $(H\mu\, ^7Be)$ system is not sufficient.

2.The stability of mesic molecules of light nuclei

We consider the systems $(H\mu\, N_Z)$, where N_Z is the nuclei with charge Z and mass

$$m_N = 2Z m_p \;, \tag{5}$$

where m_p is the proton mass. We should calculate the energy parameter U as a function of Z for this system $(H\mu\, N_Z)$. The binding energy of the given system is determined also as a function of Z. The dependence of the binding energy of the system $(H\mu\, N_Z)$ as a function of the variable Z is represented on Fig 1.

The critical value of the variable Z_{cr} is determined from the condition

$$E_{bind}(Z_{cr}) = 0 \;. \tag{6}$$

From Fig.1 one can see that the critical charges Z_{cr} for the mesic molecules $(p\mu\, N_Z)$, $(d\mu\, N_Z)$ and $(t\mu\, N_Z)$ are different and bounded in the interval

$$5,5 < Z_{cr} < 7,3 \;.$$

Table 1. The values of the parameters of our model.

system	Ω	ω	d	θ	U
$(p\mu\, ^3He)$	.23719	.10957	4.1001	.10297	.13863
$(d\mu\, ^3He)$	.19336	.10621	4.1000	.10215	.09128
$(t\mu\, ^3He)$	.17423	.10195	4.1028	.10016	.07448
$(p\mu\, ^4He)$	.23029	.10771	4.1000	.10233	.13035
$(d\mu\, ^4He)$	.18513	.10531	4.1052	.10199	.08245
$(t\mu\, ^4He)$	.17210	.10789	4.1000	.10198	.06542
$(p\mu\, ^6Li)$	.21706	.10740	4.1018	.10236	.11929
$(d\mu\, ^6Li)$	.17335	.10730	4.1045	.10141	.07191
$(t\mu\, ^6Li)$	.16302	.11148	4.1138	.1001	.05509
$(p\mu\, ^7Li)$	.21394	.10686	4.1000	.10241	.11698
$(d\mu\, ^7Li)$	.17053	.11041	4.1000	.10377	.06944
$(t\mu\, ^7Li)$	.15821	.11075	4.1040	.10000	.05252
$(p\mu\, ^7Be)$	.21226	.10777	4.1007	.10226	.11616
$(d\mu\, ^7Be)$	.17113	.10950	4.1020	.10065	.06900
$(t\mu\, ^7Be)$	.15231	.11059	4.1033	.10001	.05215

Table 2. The absolute values of the binding energies of mesic molecules of Helium isotopes (in eV).

system	[7]	[8]	[12]	OR
$(p\mu\, ^3He)$	67.70	69.0		67.4
$(d\mu\, ^3He)$	69.96	70.6	70.74	70.0
$(t\mu\, ^3He)$	71.91	72.3		73.8
$(p\mu\, ^4He)$	74.36	75.4		73.0
$(d\mu\, ^4He)$	77.96	78.4		76.0
$(t\mu\, ^4He)$	80.76	81.3		79.0

Table 3. The absolute values of the binding energies of the $(H\mu Li)$ and $(H\mu Be)$ systems(in eV).

system	[11]	[10]	OR
$(p\mu\,{}^6Li)$	24.3	17.6	24.1
$(d\mu\,{}^6Li)$	23.8	18.5	27.0
$(t\mu\,{}^6Li)$	35.3	19.8	33.0
$(p\mu\,{}^7Li)$	20.8	21.0	20.0
$(d\mu\,{}^7Li)$	25.9	22.0	23.0
$(t\mu\,{}^7Li)$	37.5	23.3	29.0
$(p\mu\,{}^7Be)$	11.7		9.0
$(d\mu\,{}^7Be)$	29.3		14.0
$(t\mu\,{}^7Be)$			17.0

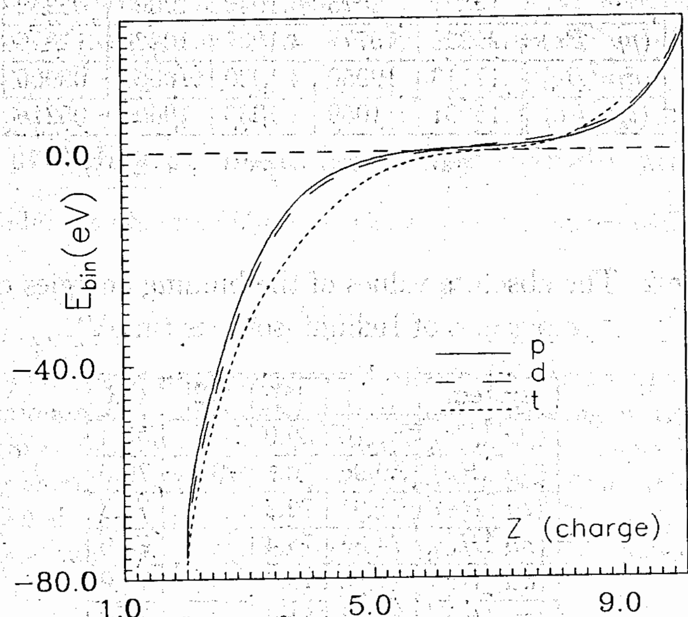

Fig.1 The dependence of the binding energies of the system $(H\mu\ N_Z)$ on the charges Z of nuclei.

This lowest and upper estimations for the critical values of Z are qualitative. In principle, the oscillator representation method can imrove the accuracy of calculation by taking into account higher perturbation corrections connected with the interaction Hamiltonian [2]. So, by taking into account higher perturbation orders one can be determine the exact values of the critical charges for any three-body Coulomb systems.

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Динейхан М., Ефимов Г.В. E4-93-465
Мезомолекулы легких ядер в осцилляторном представлении

Методом осцилляторного представления вычислены энергии основных состояний мезомолекул $(H\mu N_z)$, состоящих из изотопов водорода (p,d,t) и ядра N_z с зарядом $Z = 2, 3, 4, \ldots$ с массой $M_Z = 2Zm_p$. Получена зависимость энергии связи мезомолекул как функция заряда ядра. Определено критическое значение заряда ядра, связанное с существованием резонанса в процессе $(H\mu) + N_Z \to (H\mu N_Z) \to H + (\mu N_Z)$.

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Dineykhan M., Efimov G.V. E4-93-465
Mesic Molecules of Light Nuclei in the Oscillator Representation

The mesic molecules $(H\mu N_z)$, where H is the hydrogen isotopes (p,d,t) and N_z are nuclei with charges $Z = 2, 3, 4, \ldots$ and masses $M_Z = 2Zm_p$, are studied. The energy values of the ground state for the molecules of light nuclei have been calculated by using the oscillator representation method. The dependence of the binding energies of the muonic molecules on nuclear charges and the critical values of nuclear charges are obtained.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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