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THE THERMOFIELD TRANSFORMATION IN THE QUASIPARTICLE — PHONON NUCLEAR MODEL

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1 Introduction

Theoretical studies of the so-called "hot" nuclei, i.e., highly excited nuclei with the excitation energy distributed uniformly over many degrees of freedom have quite a long history [1]. Nevertheless this branch of the nuclear structure theory is still quite actual. One of the reasons is the increasing amount of experimental data about various hot nuclear systems we get from the deep inelastic nucleus - nucleus collisions. Moreover the description of hot nuclear systems is a hard problem and it seems natural to use in the investigation different theoretical approaches. In studying hot nuclear systems theorists often follow the way of extending the known models for cold nuclei to the case $T \neq 0$. It was done, e.g., in the nuclear field theory [2] and in the theory of finite Fermi systems [3]. The properties of nuclear excitations from the energy range 0 - 25 MeV are successfully described also by the quasiparticle - phonon nuclear model (the QPM) [4-6]. In the present paper, we extend the QPM to $T \neq 0$ using the methods of the thermo field dynamics (the TFD) [7].

2 The TFD formalism

Following the papers [8-10], we firstly sketch out the formalism of the thermo field dynamics. Let us consider a system of nucleons with the Hamiltonian H in the thermal equilibrium at $T \neq 0$. The system is described by the partition function of a grand canonical ensemble, and the thermal average of any operator A can be written as

$$\ll A \gg = \frac{1}{Tr(exp(-\beta H))} Tr[A\, exp(-\beta H)] ,$$

where $\beta = T^{-1}$. The Fock space of the states of the system $|n\rangle$ is produced by action of the nucleon creation operators a^+_μ on the corresponding vacuum state $|0\rangle$. The standard anticommutation relations are valid for the operators a^+_μ and the corresponding annihilation operators a_μ. Let us formally double the space of the states of the system by introducing the so-called "tilde" states $|\tilde{n}\rangle$ produced by the tilde creation operators $\tilde{a}^+_\mu$ acting on the tilde vacuum state $|\tilde{0}\rangle$. The initial and the tilde creation and annihilation operators anticommute with each other:

$$\{a^+_\mu, a_\nu\} = \delta_{\mu\nu} \ , \ \{a_\mu, a_\nu\} = \{a^+_\mu, a^+_\nu\} = 0$$

$$\{\tilde{a}^+_\mu, \tilde{a}_\nu\} = \delta_{\mu\nu} \ , \ \{\tilde{a}_\mu, \tilde{a}_\nu\} = \{\tilde{a}^+_\mu, \tilde{a}^+_\nu\} = 0$$

$$\{\tilde{a}_\mu, a_\nu\} = \{\tilde{a}^+_\mu, a^+_\nu\} = \{\tilde{a}_\mu, a^+_\nu\} = \{\tilde{a}^+_\mu, a_\nu\} = 0$$

$$a_\mu|0\rangle = 0 \,, \tilde{a}_\mu|\tilde{0}\rangle = 0$$

For any operator A acting in the initial Fock space there exists its tilde counterpart $\tilde{A}$ acting in the space of tilde states. The tilde operation is defined by the rules

$$(\tilde{A^+}) = (\tilde{A})^+ \,; (\tilde{AB}) = \tilde{A}\tilde{B} \,; (c_1 A + c_2 B)^\sim = c_1^* \tilde{A} + c_2^* \tilde{B} \,; (\tilde{\tilde{A}}) = \pm A \,,$$

where c_1, c_2 are c-numbers and the sign + (-) is taken for a bosonic (fermionic) operator A.

Now the time - translation operator is not the energy operator H, but the so-called thermal Hamiltonian $\mathcal{H} = H - \tilde{H}$. $\tilde{H}$ is the Hamiltonian acting in the tilde space and having the same eigenvalues as H: $H|n\rangle = E_n|n\rangle$, $\tilde{H}|\tilde{n}\rangle = E_n|\tilde{n}\rangle$.

The doubling of the Fock space gives us a possibility to express the statistical ensemble average of A as a vacuum expectation value of A $\ll A \gg = \langle 0(\beta)|A|0(\beta)\rangle$ if the thermal (or temperature dependent) vacuum $|0(\beta)\rangle$ is defined by

$$|0(\beta)\rangle = \frac{1}{\sqrt{Tr(exp(-\beta H))}} \sum_n exp(-\frac{\beta E_n}{2})|n\rangle \otimes |\tilde{n}\rangle$$

The state $|0(\beta)\rangle$ is the vacuum for the thermal quasiparticle annihilation operators

$$\beta_{jm} = x_j a_{jm} - y_j \tilde{a}^+_{jm}$$

$$\tilde{\beta}_{jm} = x_j \tilde{a}_{jm} + y_j a^+_{jm}$$

$$\beta_{jm}|0(\beta)\rangle = \tilde{\beta}_{jm}|0(\beta)\rangle = 0$$

The transformation $\{x, y\}$ is a unitary one and due to this the following anticommutation relations are valid for $\beta_{jm}, \beta^+_{jm}, \tilde{\beta}_{jm}, \tilde{\beta}^+_{jm}$:

$$\{\beta^+_\mu, \beta_\nu\} = \delta_{\mu\nu} \,, \{\beta_\mu, \beta_\nu\} = \{\beta^+_\mu, \beta^+_\nu\} = 0$$

$$\{\tilde{\beta}^+_\mu, \tilde{\beta}_\nu\} = \delta_{\mu\nu} \,, \{\tilde{\beta}_\mu, \tilde{\beta}_\nu\} = \{\tilde{\beta}^+_\mu, \tilde{\beta}^+_\nu\} = 0$$

$$\{\tilde{\beta}_\mu, \beta_\nu\} = \{\tilde{\beta}^+_\mu, \beta^+_\nu\} = \{\tilde{\beta}_\mu, \beta^+_\nu\} = \{\tilde{\beta}^+_\mu, \beta_\nu\} = 0$$

The coefficients x_j, y_j are the thermal (Fermi in our case) occupation numbers of the states $|n\rangle$.

$$x_j = \sqrt{1 - n_j} \,, \; y_j = \sqrt{n_j}$$

$$n_j = \frac{1}{1 + exp(\beta E_j)}$$

These relations can be derived from the expression for the particle density in the thermal vacuum $|0(\beta)\rangle$ [7]. In the next section, we shall show the validity of this statement for the system of the Bogolubov quasiparticles at $T \neq 0$.

We use the formalism of the TFD to transform the Hamiltonian of the quasiparticle - phonon nuclear model [5, 6] for $T \neq 0$. The Hamiltonian of the QPM consisits of the average fields for protons and neutrons, the monopole proton - proton and neutron - neutron pairing interactions and the separable multipole and spin - multipole particle - hole interactions with the isoscalar and isovector parts:

$$H = H_{sp} + H_{pair} + H_{ph}$$

We escape here describing thoroughly numerous parameters of the QPM. We have to mention only that all of them are supposed to be temperature independent. It means that the present approach is valid for $T \leq 6$ MeV.

3 The pairing correlations at $T \neq 0$

We organize the following discussion in such a way as to perform the derivation as parallel as possible with the $T = 0$ case.

The first stage of the transformation of the QPM Hamiltonian at $T = 0$ is the canonical Bogolubov transformation from the nucleon creation and annihilation operators to the quasiparticle ones

$$\alpha_{jm} = u_j a_{jm} - v_j a^+_{\overline{jm}}$$

$$a^+_{\overline{jm}} = (-)^{j-m} a^+_{j-m}$$

The coefficients u_j, v_j are chosen to minimize the expectation value of the following part of the Hamiltonian H: $H' = H_{sp} + H_{pair}$ for the quasiparticle vacuum $|0\rangle$. As a result, H' becomes diagonal in the quasiparticle representation:

$$H' = \sum_{jm} \varepsilon_j \alpha^+_{jm} \alpha_{jm}$$

The expression for the quasiparticle energy ε_j can be found, e.g., in ref.[4].

We use the same transformation for $T \neq 0$ as well. But in addition to the u, v- transformation of the usual and tilde nucleon creation and annihilation operators we make the above-mentioned thermal Bogolubov transformation (i.e. $\{x, y\}$ - transformation) to thermal quasiparticles. So the resulting transformation has the form:

$$\begin{pmatrix} a_{jm} \\ a^+_{\overline{jm}} \\ \tilde{a}^+_{jm} \\ \tilde{a}_{\overline{jm}} \end{pmatrix} = \begin{pmatrix} A & B \\ -B & A \end{pmatrix} \begin{pmatrix} \beta_{jm} \\ \beta^+_{\overline{jm}} \\ \tilde{\beta}^+_{jm} \\ \tilde{\beta}_{\overline{jm}} \end{pmatrix} \quad (1)$$

$$A = \sqrt{1-n_j} \begin{pmatrix} u_j & v_j \\ -v_j & u_j \end{pmatrix}$$

$$B = \sqrt{n_j} \begin{pmatrix} u_j & v_j \\ -v_j & u_j \end{pmatrix}$$

To choose the coefficients u_j, v_j, x_j, y_j we use the condition for a nucleus to be in the thermal equilibrium at $T = const$. It means we have to find a minimum of the grand thermodynamic potential $\Omega = E - TS$. In the calculations one has to have in mind that the ground state energy E is equal $\langle 0(\beta)|H'|0(\beta)\rangle$ but not $\langle 0(\beta)|\mathcal{H}'|0(\beta)\rangle$ [7]. The entropy S of the system is

$$S = -\sum_j (2j+1)(n_j \ln n_j + (1-n_j)\ln(1-n_j))$$

The calculation of Ω is straightforword

$$\Omega = \langle 0(\beta)|H_{av}(\tau) + H_{pair}(\tau)|0(\beta)\rangle - TS =$$

$$= \sum_j{}^{\tau} (2j+1)(E_j - \lambda_\tau)(u_j^2 n_j + v_j^2(1-n_j)) -$$

$$-\frac{G_\tau}{4}\left(\sum_j{}^{\tau}(2j+1)u_j v_j (1-2n_j)\right)^2 - TS \quad (2)$$

We use in (2) the following notation: E_j is the single - particle energy; λ_τ is the chemical potential; G_τ is the constant of the pairing interaction. The index τ is an isotopic index and takes two values, $\tau = n, p$. We suppose that τ is included in the set of shell model quantum numbers $[nlj\tau]$ that we usually denote by one index j. The symbol $\sum^\tau$ means that the summation is taken only over neutron or proton single - particle states. Correctly speaking, E_j in eq.(2) is not exactly the single - particle energy because we included in it a renormalization term from the pairing correlations. Now this term depends on T. We suppose these terms are much less than possible variations of the single - particle energies.

After variation of Ω over u_j, v_j, n_j we have

$$(E_j - \lambda_\tau)u_j v_j - \frac{G_\tau}{4}(u_j^2 - v_j^2)\sum_j{}^{\tau}(2j+1)u_j v_j(1-2n_j) = 0$$

$$(E_j - \lambda_\tau)(u_j^2 - v_j^2) + u_j v_j G_\tau \sum_j{}^{\tau}(2j+1)u_j v_j (1-2n_j) +$$

$$+T(\ln n_j - \ln(1-n_j)) = 0$$

For the coefficients u_j, v_j, n_j the following relations are valid:

$$u_j^2 = \frac{1}{2}\left(1 + \frac{E_j - \lambda_\tau}{\varepsilon_j}\right), \; v_j^2 = \frac{1}{2}\left(1 - \frac{E_j - \lambda_\tau}{\varepsilon_j}\right)$$

$$\varepsilon_j = \sqrt{(E_j - \lambda_\tau)^2 + \Delta_\tau^2}, \; n_j = \frac{1}{1 + exp(\beta\varepsilon_j)}$$

In its turn $\Delta_\tau, \lambda_\tau$ are found from the equations:

$$N_\tau = \frac{1}{2}\sum_j{}^{\tau}(2j+1)\left(1 - \frac{E_j - \lambda_\tau}{\sqrt{(E_j-\lambda_\tau)^2 + \Delta_\tau^2}}(1-2n_j)\right)$$

$$\frac{4}{G_\tau} = \sum_j{}^{\tau}(2j+1)\frac{1-2n_j}{\sqrt{(E_j-\lambda_\tau)^2+\Delta_\tau^2}}, \quad (3)$$

N_τ is the number of nucleons of a given type.

Eqs.(3) are the well known equations for pairing at $T \neq 0$ with the Hamiltonian of Bardeen - Cooper - Shrieffer (see e.g. ref.[11]). Using the TFD formalism they were derived also in [10].

With the above-mentioned expressions for u_j, v_j the Hamiltonian $\mathcal{H}' = H' - \tilde{H}'$ takes the form:

$$\mathcal{H}' = \mathcal{H}_{sp} + \mathcal{H}_{pair} = \sum_{jm} \varepsilon_j(\beta^+_{jm}\beta_{jm} - \tilde{\beta}^+_{jm}\tilde{\beta}_{jm}),$$

i.e. $\mathcal{H}'$ becomes diagonal in terms of the thermal quasiparticles.

4 The random phase approximation at $T \neq 0$

Now we turn to derivation of the random phase approximation equation at $T \neq 0$. The term $\mathcal{H}_{ph}$ also has to be written with the creation and annihilation operators of the thermal quasiparticles $\beta, \beta^+, \tilde{\beta}, \tilde{\beta}^+$ but in this case we suppose the coefficients u_j, v_j, n_j to be known. The corresponding expression of $\mathcal{H}(\beta^+, \beta, \tilde{\beta}^+, \tilde{\beta})$ is very long and complicated to be displayed here. The point is that one can join different terms of $\mathcal{H}_{ph}$ into the groups with the given numbers of creation and annihilation operators, as it has been done with H_{ph} for the quasiparticles operators [8, 9]. For example, one has the terms $\beta^+\beta^+\beta\beta$, $\beta^+\tilde{\beta}^+\beta\beta$, $\tilde{\beta}^+\tilde{\beta}^+\beta\beta$, $\beta^+\beta^+\tilde{\beta}\beta$, $\beta^+\beta^+\tilde{\beta}\tilde{\beta}$ etc. instead of $\alpha^+\alpha^+\alpha\alpha$ or $\beta^+\beta^+\beta^+\beta$, $\beta^+\beta^+\beta^+\tilde{\beta}$, $\beta^+\beta^+\tilde{\beta}^+\beta$ etc. instead of $\alpha^+\alpha^+\alpha^+\alpha$. Moreover, the number of thermal quasiparticles in the thermal vacuum vanishes $\langle 0(\beta)|\sum_{jm}\beta^+_{jm}\beta_{jm}|0(\beta)\rangle = 0$ as for the number of the Bogolubov quasiparticles in the quasiparticle vacuum $|0\rangle$. So at this stage, one can see a very close correspondence between the Bogolubov quasiparticles at $T = 0$ and the thermal quasiparticles at $T \neq 0$.

Let us introduce the thermal phonon operator (or the phonon operator constructed from the thermal quasiparticles):

$$Q^+_{\lambda\mu i} = \frac{1}{2}\sum_{jj'}\Big(\psi^{\lambda i}_{jj'}[\beta^+_{jm}\beta^+_{j'm'}]_{\lambda\mu} + \tilde{\psi}^{\lambda i}_{jj'}[\tilde{\beta}^+_{\overline{jm}}\tilde{\beta}^+_{\overline{j'm'}}]_{\lambda\mu} +$$

$$+2\eta^{\lambda i}_{jj'}[\beta^+_{jm}\tilde{\beta}^+_{\overline{j'm'}}]_{\lambda\mu} - (-)^{\lambda-\mu}\phi^{\lambda i}_{jj'}[\beta_{jm}\beta_{j'm'}]_{\lambda-\mu} -$$

$$-(-)^{\lambda-\mu}\tilde{\phi}^{\lambda i}_{jj'}[\tilde{\beta}_{\overline{jm}}\tilde{\beta}_{\overline{j'm'}}]_{\lambda-\mu} - 2(-)^{\lambda-\mu}\tilde{\eta}^{\lambda i}_{jj'}[\beta_{jm}\tilde{\beta}_{\overline{j'm'}}]_{\lambda-\mu}\Big) \tag{4}$$

In eq.(4) the notation $[\ ]_{\lambda\mu}$ means the coupling of single - particle angular momenta j, j' to the sum angular momentum λ.

Next, define the ground state of an even - even nucleus as a vacuum for the thermal phonon operators $|\Psi_0(\beta)\rangle$:

$$Q_{\lambda\mu i}|\Psi_0(\beta)\rangle = 0\ , \tilde{Q}_{\lambda\mu i}|\Psi_0(\beta)\rangle = 0,$$

and assume that the number of thermal quasiparticles in this new vacuum state vanishes:

$$\langle\Psi_0(\beta)|\beta^+_{jm}\beta_{jm}|\Psi_0(\beta)\rangle \approx 0$$

The one-phonon states have to be orthonormalized:

$$\langle\Psi_0(\beta)|[Q_{\lambda\mu i}\ , Q^+_{\lambda'\mu' i'}]|\Psi_0(\beta)\rangle = \delta_{\lambda\lambda'}\delta_{\mu\mu'}\delta_{ii'} \tag{5}$$

$$\langle\Psi_0(\beta)|[Q_{\lambda\mu i}, \tilde{Q}^+_{\lambda'\mu' i'}]|\Psi_0(\beta)\rangle = 0$$

From eq.(5) one gets the following constraint on the amplitudes $\psi, \phi, \eta, \tilde{\psi}, \tilde{\phi}, \tilde{\eta}$:

$$\frac{1}{2}\sum_{jj'}(\psi^{\lambda i}_{jj'})^2 - (\phi^{\lambda i}_{jj'})^2 + (\tilde{\psi}^{\lambda i}_{jj'})^2 - (\tilde{\phi}^{\lambda i}_{jj'})^2 + 2(\eta^{\lambda i}_{jj'})^2 - 2(\tilde{\eta}^{\lambda i}_{jj'})^2 = \delta_{\lambda\lambda'}\delta_{ii'} \tag{6}$$

Then one can express the bifermionic operators like $[\beta^+_{jm}\ , \beta^+_{j'm'}]$ etc. through the phonon operators $Q^+_{\lambda\mu i}\ , Q_{\lambda\mu i}$ by the transformation which is inverse to (4) and write $\mathcal{H}$ in terms of the phonons. The part of $\mathcal{H}$ which gives the nonvanishing contribution to the expectation value of the thermal Hamiltonian over one-phonon state, has the form:

$$\mathcal{H}_{RPA} = \sum_{jm}\varepsilon_j(\beta^+_{jm}\beta_{jm} - \tilde{\beta}^+_{jm}\tilde{\beta}_{jm}) -$$

$$-\frac{1}{8}\sum_{\lambda\mu ii'}\sum_{\tau,\rho=\pm 1}\frac{k_0^{(\lambda)} + \rho k_1^{(\lambda)}}{2\lambda+1}D^{\lambda i}_{\tau}D^{\lambda i'}_{\rho\tau}\{(Q^+_{\lambda\mu i} + (-)^{\lambda-\mu}Q_{\lambda-\mu i})\times$$

$$\times(Q_{\lambda\mu i'} + (-)^{\lambda-\mu}Q^+_{\lambda-\mu i'}) -$$

$$-(\tilde{Q}^+_{\lambda\mu i} + (-)^{\lambda-\mu}\tilde{Q}_{\lambda-\mu i})(\tilde{Q}_{\lambda\mu i'} + (-)^{\lambda-\mu}\tilde{Q}^+_{\lambda-\mu i'})\}$$

$$D^{\lambda i}_{\tau} = \sum_{jj'}{}^{\tau} f^{\lambda}_{jj'}[u_{jj'}(\sqrt{1-n_j}\sqrt{1-n_{j'}}(\psi^{\lambda i}_{jj'} + \phi^{\lambda i}_{jj'}) - \sqrt{n_j}\sqrt{n_{j'}}(\tilde{\psi}^{\lambda i}_{jj'} + \tilde{\phi}^{\lambda i}_{jj'})) -$$

$$-2v_{jj'}\sqrt{1-n_j}\sqrt{n_{j'}}(\eta^{\lambda i}_{jj'} + \tilde{\eta}^{\lambda i}_{jj'})]$$

We write these formulas for the multipole - multipole particle - hole interaction $\mathcal{H}_{ph}$ and introduce the following notations: $f^{\lambda}_{jj'}$ is the reduced single - particle matrix element of the multipole operator; $k_0^{(\lambda)}\ , k_1^{(\lambda)}$ are the coupling constants of the isoscalar and the isovector multipole - multipole interactions, respectively; $u_{jj'} = u_j v_{j'} + u_{j'} v_j\ ,\ v_{jj'} = u_j u_{j'} - v_{j'} v_j$. Changing the sign of $\tau \to -\tau$ means changing $n \leftrightarrow p$.

In the expression of $\mathcal{H}$ in terms of the thermal phonons we omit the so-called quasiparticle - phonon interaction $\sim Q\beta^+\beta$ and the term like $\sim \beta^+\beta\beta^+\beta$. The former term doesn't give a contribution to the expectation value of $\mathcal{H}$ over one-phonon state, the contribution of the last term is supposed to be small in the RPA as for the analogous term at $T = 0$.

The expectation value of $\mathcal{H}_{RPA}$ over the one-phonon state $Q^+_{\lambda\mu i}|\Psi_0(\beta)\rangle$ has the form:

$$\langle\Psi_0(\beta)|Q_{\lambda\mu i}\mathcal{H}_{RPA}Q^+_{\lambda\mu i}|\Psi_0(\beta)\rangle =$$

$$= \frac{1}{2}\sum_{jj'}\Big\{(\varepsilon_j + \varepsilon_{j'})[(\psi^{\lambda i}_{jj'})^2 + (\phi^{\lambda i}_{jj'})^2 - (\tilde{\psi}^{\lambda i}_{jj'})^2 - (\tilde{\phi}^{\lambda i}_{jj'})^2] +$$

(7)

$$+2(\varepsilon_j-\varepsilon_{j'})[(\eta^{\lambda i}_{jj'})^2+(\tilde{\eta}^{\lambda i}_{jj'})^2]\Big\}-\frac{1}{4}\frac{1}{2\lambda+1}\sum_{\tau,\rho=\pm1}(k_0^{(\lambda)}+\rho k_1^{(\lambda)})D_\tau^{\lambda i}D_{\rho\tau}^{\lambda i}$$

After variation of (7) at the constraint (6) over $\psi,\phi,\eta,\tilde{\psi},\tilde{\phi},\tilde{\eta}$, one gets the homogeneous system of linear equations. It can be resolved if the energy of the one-phonon state $\omega_{\lambda i}$ is the root of the following secular equation:

$$[X_\tau^{\lambda i}(\omega)+X_{-\tau}^{\lambda i}(\omega)](k_0^{(\lambda)}+k_1^{(\lambda)})-4k_0^{(\lambda)}k_1^{(\lambda)}X_\tau^{\lambda i}(\omega)X_{-\tau}^{\lambda i}(\omega)=1 \qquad (8)$$

$$X_\tau^{\lambda i}(\omega)=\frac{1}{2\lambda+1}\sum_{jj'}{}^{\tau}(f_{jj'}^{\lambda})^2\left(\frac{u_{jj'}^2(1-n_j-n_{j'})(\varepsilon_j+\varepsilon_{j'})}{(\varepsilon_j+\varepsilon_{j'})^2-\omega^2}-\right.$$
$$\left.-\frac{v_{jj'}^2(n_j-n_{j'})(\varepsilon_j-\varepsilon_{j'})}{(\varepsilon_j-\varepsilon_{j'})^2-\omega^2}\right)$$

For the bifermionic amplitudes of the one-phonon wave function one gets:

$$\psi_{jj'}^{\lambda i}=\sqrt{\frac{1}{2\mathcal{N}_\tau^{\lambda i}}}\frac{f_{jj'}^{\lambda}u_{jj'}\sqrt{1-n_j}\sqrt{1-n_{j'}}}{(\varepsilon_j+\varepsilon_{j'})-\omega_{\lambda i}} \qquad \tilde{\psi}_{jj'}^{\lambda i}=\sqrt{\frac{1}{2\mathcal{N}_\tau^{\lambda i}}}\frac{f_{jj'}^{\lambda}u_{jj'}\sqrt{n_j}\sqrt{n_{j'}}}{(\varepsilon_j+\varepsilon_{j'})+\omega_{\lambda i}}$$

$$\phi_{jj'}^{\lambda i}=\sqrt{\frac{1}{2\mathcal{N}_\tau^{\lambda i}}}\frac{f_{jj'}^{\lambda}u_{jj'}\sqrt{1-n_j}\sqrt{1-n_{j'}}}{(\varepsilon_j+\varepsilon_{j'})+\omega_{\lambda i}} \qquad \tilde{\phi}_{jj'}^{\lambda i}=\sqrt{\frac{1}{2\mathcal{N}_\tau^{\lambda i}}}\frac{f_{jj'}^{\lambda}u_{jj'}\sqrt{n_j}\sqrt{n_{j'}}}{(\varepsilon_j+\varepsilon_{j'})-\omega_{\lambda i}}$$

$$\eta_{jj'}^{\lambda i}=-\sqrt{\frac{1}{2\mathcal{N}_\tau^{\lambda i}}}\frac{f_{jj'}^{\lambda}v_{jj'}\sqrt{1-n_j}\sqrt{n_{j'}}}{(\varepsilon_j-\varepsilon_{j'})-\omega_{\lambda i}} \qquad \tilde{\eta}_{jj'}^{\lambda i}=-\sqrt{\frac{1}{2\mathcal{N}_\tau^{\lambda i}}}\frac{f_{jj'}^{\lambda}v_{jj'}\sqrt{1-n_j}\sqrt{n_{j'}}}{(\varepsilon_j-\varepsilon_{j'})+\omega_{\lambda i}}$$

$$\mathcal{N}_\tau^{\lambda i}=N_\tau^{\lambda i}(\omega_{\lambda i})+\left(\frac{1-X_\tau^{\lambda i}(\omega_{\lambda i})(k_0^{(\lambda)}+k_1^{(\lambda)})}{X_{-\tau}^{\lambda i}(\omega_{\lambda i})(k_0^{(\lambda)}-k_1^{(\lambda)})}\right)^2N_{-\tau}^{\lambda i}(\omega_{\lambda i})$$

$$N_\tau^{\lambda i}(\omega)=\frac{2\lambda+1}{2}\frac{\partial}{\partial\omega}X_\tau^{\lambda i}(\omega)$$

The secular eq.(8) is the same as in [12] where it has been derived using the Green function method. This is true for the expressions for the amplitudes ψ,ϕ,η as well. The last expressions can be found, e.g., in ref.[13]. One should keep in mind that we define the phonon operator in terms of the thermal quasiparticles thus giving rise to additional factors proportional to n_j and $(1-n_j)$ in our expressions.

5 Conclusions

Prior to closing it is necessary to dwell on the paper of N.D.Dang [14] where the attempt to extend the QPM to $T\neq0$ using the TFD approach has been proclaimed also. Unfortunately, the author didn't understand the meaning of the dual tilde states of the TFD and confused them with the time-reversed states. So the results of [14], which are in strong contradiction with the results of other authors, are wrong. The same mistake has been made by N.D.Dang in the paper [15].

In conclusion, with the formalism of the TFD we formulate the consistent procedure to extend the QPM to $T\neq0$ case. Along this way, we have got the equation for pairing as well as the RPA equations at $T\neq0$. Our results are in agreement with the earlier results of other authors derived by other methods. So we conclude that the TFD is a quite transparent and effective approach in the theory of hot nuclei. Our next step will be to extend this approach beyond the RPA, i.e., to study the interaction among the thermal phonons and quasiparticles.

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Вдовин А.И., Косов Д.С. E4-93-417
Термополевое преобразование
в квазичастично-фононной модели ядра

Предложен способ обобщения квазичастично-фононной модели ядра для описания нагретых ядер. Для этого использован формализм термополевой динамики, следуя которому гамильтониан КФМ записан в терминах тепловых квазичастиц. Коэффициенты преобразования, связывающего исходный гамильтониан КФМ с тепловым гамильтонианом, определены из условия минимума большого термодинамического потенциала в состоянии теплового вакуума. Затем в тепловом гамильтониане КФМ выделена часть, описывающая возбуждения нагретого ядра в приближении случайной фазы. Выведены уравнения ПСФ, которые совпадают с полученными ранее методами функций Грина и линеаризации уравнений движения.

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The Thermofield Transformation
in the Quasiparticle — Phonon Nuclear Model

The method of extension of the quasiparticle — phonon nuclear model to describe hot nuclei is proposed. For this aim the formalism of the thermofield dynamics is used. Following the main principles of the TFD we express the Hamiltonian of the QPM in terms of thermal quasiparticles. The coefficients of the corresponding transformation are determined by minimizing the grand thermodynamical potential of a hot nucleus in the thermal vacuum state. Then the RPA part of the thermal QPM Hamiltonian is extracted and the RPA equations are derived. They are in agreement with the RPA equations derived by the Green function method and the equation-of-motion method.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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