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MASS PARAMETERS FOR DINUCLEAR SYSTEM

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1. Introduction

The most important degrees of freedom which are necessary to describe the interaction of two nuclei are the distance between the centers of colliding nuclei R, mass asymmetry degree of freedom $\eta \equiv (A_1 - A_2)/A$ (A_1 and A_2 are mass numbers of nuclei, $A = A_1 + A_2$) and a neck radius or other characteristic of a neck [1–6]. Therefore, the Hamiltonian of dinuclear system should depend on these dynamical variables. The important ingredient of the Hamiltonian is an inertia tensor. There are different approaches to calculate its value. These approaches mainly use the cranking expression and perform calculations in different single particle basis. In refs. [2–4] the calculations have been done using adiabatic two-center shell model basis. The approach based on the dissipative diabatic dynamics has been realized in [7,8] by exploiting the diabatic two-center shell model. In refs. [9,12] the inertia tensor has been found in the framework of the linear response theory using quasi-adiabatic two-center basis. Inertia tensor obtained in terms of the adiabatic representation is given in refs. [13–15].

The values of mass parameters and their dependence on dynamical variables considerably influence dynamics of the dinuclear system. For instance, in refs. [11,12] it was shown that the nondiagonal component of the inertia tensor describing the motion in the $R-\eta$ plane increases significantly with increase of mass asymmetry. Due to the coupling of $R-$ and $\eta-$ modes of motion the part of the kinetic energy of $\eta-$mode transforms into the kinetic energy of the radial motion and system approaches to the radial potential barrier with the increase of η. As a consequence, the stability of the dinuclear system for large mass asymmetry decreases and the production of light isotopes increases.

The nondiagonal component of inertia tensor plays an important role in the evolution of the dinuclear system . However, its dependence on various dynamical variables makes the computations very cumbersome. To obtain the results in a simpler way, we develop in this paper a simple and mainly analytical method to find the components of the mass tensor for a dinuclear system.

The paper is organized as follows. In sect. 2 we obtain the general form for inertia tensor and discuss some useful relations. In sect. 3 the definition of neck is introduced. The simple analytical expressions are obtained to calculate mass parameters both macroscopically and microscopically. The results of calculations are presented in sect. 4.

2. Inertia tensor

It can be shown [16,17]) that the nuclear Hamiltonian of the general type with the two-body forces

$$H = \frac{\hbar^2}{2m}\int d\mathbf{r}\, \nabla\psi^+(\mathbf{r})\nabla\psi(\mathbf{r}) + \int d\mathbf{r}d\mathbf{r}'\, \psi^+(\mathbf{r})\psi(\mathbf{r})u(\mathbf{r}-\mathbf{r}')\psi^+(\mathbf{r}')\psi(\mathbf{r}'), \qquad (1)$$

where ψ^+, ψ are nucleon field operators, can be expressed in terms of current

$$\mathbf{j}(\mathbf{r}) = -\frac{i\hbar}{2m}\left(\psi^+(\mathbf{r})\nabla\psi(\mathbf{r}) - \nabla\psi^+(\mathbf{r})\psi(\mathbf{r})\right) \qquad (2)$$

and density

$$\rho(\mathbf{r}) = \psi^+(\mathbf{r})\psi(\mathbf{r}) \qquad (3)$$

operators as given below:

$$H = \frac{m}{2}\int d\mathbf{r}\, \mathbf{j}(\mathbf{r})\rho^{-1}(\mathbf{r})\mathbf{j}(\mathbf{r}) + ... \qquad (4)$$

The omitted term depends only on ρ and is not necessary for the following considerations. Operators ρ and $\mathbf{j}$ satisfy the commutation relation [16])

$$[\rho(\mathbf{r}), \mathbf{j}(\mathbf{r}')] = -\frac{i\hbar}{m}\nabla_{\mathbf{r}}(\delta(\mathbf{r}-\mathbf{r}')\rho(\mathbf{r})) \qquad (5)$$

and the current operator has the following functional representation

$$\mathbf{j}(\mathbf{r}) = \frac{i\hbar}{m}\nabla\left(\rho(\mathbf{r})\frac{\delta}{\delta\rho(\mathbf{r})}\right). \qquad (6)$$

Using eqs. (4)–(6) we can represent the kinetic energy part T of the total Hamiltonian in terms of the functional derivatives of ρ

$$T = -\frac{\hbar^2}{2m}\int d\mathbf{r}\, \nabla\left(\rho(\mathbf{r})\frac{\delta}{\delta\rho(\mathbf{r})}\right)\frac{1}{\rho(\mathbf{r})}\nabla\left(\rho(\mathbf{r})\frac{\delta}{\delta\rho(\mathbf{r})}\right). \qquad (7)$$

In what follows, we shall use the expression (7) to derive the inertia tensor for the collective motion of two-center system. We assume that the density ρ depends on some number of collective variables q_j which are defined by the relations

$$q_j = \int d\mathbf{r}\, \rho(\mathbf{r})g_j(\mathbf{r}), \qquad (8)$$

where $g_j(\mathbf{r})$ are functions required to derive q_j. Then, with the expression for the functional derivative as given below

$$\frac{\delta}{\delta\rho(\mathbf{r})} = \sum_j g_j(\mathbf{r})\frac{\partial}{\partial q_j} \qquad (9)$$

we obtain the kinetic energy term as:

$$\begin{aligned} T &= -\frac{\hbar^2}{2m}\sum_{j,j'}\frac{\partial}{\partial q_j}\int d\mathbf{r}\, \rho(\mathbf{r})\nabla g_j(\mathbf{r})\nabla g_{j'}(\mathbf{r})\frac{\partial}{\partial q_{j'}} \\ &\equiv -\frac{\hbar^2}{2}\sum_{j,j'}\frac{\partial}{\partial q_j}(B^{-1})_{jj'}\frac{\partial}{\partial q_{j'}}. \end{aligned} \qquad (10)$$

It is evident froni (10) that the components of inverse inertia tensor are:

$$(B^{-1})_{jj'} = \frac{1}{m}\int d\mathbf{r}\, \rho(\mathbf{r})\nabla g_j(\mathbf{r})\nabla g_{j'}(\mathbf{r}). \qquad (11)$$

Using the density of the dinuclear system

$$\rho(\mathbf{r}) = <0|\sum_{k=1}^{A_1+A_2}\delta(\mathbf{r}-\mathbf{r}_k)|0>,$$

we can rewrite the formula (11) in the following way:

$$(B^{-1})_{jj'} = \frac{1}{m} < 0|\sum_k \nabla_k g_j(\mathbf{r}_k)\nabla_k g_{j'}(\mathbf{r}_k)|0> . \qquad (12a)$$

If the nucleon–nucleon forces are velocity independent then

$$\begin{aligned} (B^{-1})_{jj'} &= \frac{1}{\hbar^2} < 0|[q_{j'}, [H, q_j]]|0> && (12b) \\ &= \frac{2}{\hbar^2}\sum_{n\neq0}\frac{< n|[H, q_j]|0 >< 0|[H, q_{j'}]|n >}{E_n - E_0} && (12c) \\ &= \frac{2}{\hbar^2}\sum_{n\neq0}(E_0 - E_n) < n|q_j|0 >< 0|q_{j'}|n >, && (12d) \end{aligned}$$

where $|0>$ and $|n>$ are the ground and excited states of the dinuclear system with corresponding energies E_0 and E_n. Since

$$\frac{\partial}{\partial q_j} = -\frac{1}{\hbar^2}\sum_{j'} B_{jj'}[H, q_{j'}] = -\frac{i}{\hbar}\sum_{j'} B_{jj'}\dot{q}_{j'} \tag{13}$$

and

$$\sum_{j_1} B_{jj_1}(B^{-1})_{j_1 j'} = \delta_{jj'}, \tag{14}$$

we can obtain from (10) the well known expression for the collective kinetic energy

$$T = \frac{1}{2}\sum_{jj'} B_{jj'}\dot{q}_j\dot{q}_{j'}.$$

Usually, the inertia tensor $B_{jj'}$ is calculated with the help of cranking expression

$$B_{jj'} = 2\hbar^2 \sum_{n\neq 0} \frac{< n|\partial/\partial q_j|0 >< 0|\partial/\partial q_{j'}|n >}{E_0 - E_n}, \tag{15}$$

using the two-center shell model basis.

To demonstrate the equivalence of the inertia tensor in (11,12) and the cranking expression (15), we insert (13) into (15) and, using (12c), obtain the following relation

$$B_{jj'} = \sum_{j_1 j_2} B_{jj_1} B_{j_2 j'} (B^{-1})_{j_1 j_2} \tag{16}$$

which is satisfied identically because of (14).

From eq. (16) we can express matrix elements $B_{jj'}$ in terms of matrix elements $(B^{-1})_{jj'}$ and vice versa. For example, if the relative distance between the fragment centers R and mass asymmetry parameter η are taken as the collective variables, then, all the components of inertia tensor are:

$$B_{RR} = \frac{(B^{-1})_{\eta\eta}}{(B^{-1})_{\eta\eta}(B^{-1})_{RR} - [(B^{-1})_{R\eta}]^2}, \tag{17a}$$

$$B_{\eta\eta} = \frac{(B^{-1})_{RR}}{(B^{-1})_{\eta\eta}(B^{-1})_{RR} - [(B^{-1})_{R\eta}]^2}, \tag{17b}$$

$$B_{R\eta} = -\frac{(B^{-1})_{R\eta}}{(B^{-1})_{\eta\eta}(B^{-1})_{RR} - [(B^{-1})_{R\eta}]^2}. \tag{17c}$$

For $(B^{-1})_{R\eta}$ to be small, above components (17) can be expressed as:

$$B_{RR} \approx \frac{1}{(B^{-1})_{RR}}, \qquad B_{\eta\eta} \approx \frac{1}{(B^{-1})_{\eta\eta}}, \qquad B_{R\eta} \approx -\frac{(B^{-1})_{R\eta}}{(B^{-1})_{\eta\eta}(B^{-1})_{RR}}.$$

The solution of eq. (16) as given by eqs. (17), leads to the following useful relations:

$$B_{\eta\eta}(B^{-1})_{\eta\eta} = B_{RR}(B^{-1})_{RR},$$

$$B_{R\eta} = -B_{RR}\frac{(B^{-1})_{R\eta}}{(B^{-1})_{\eta\eta}} = -\sqrt{\frac{B_{RR}B_{\eta\eta}}{(B^{-1})_{RR}(B^{-1})_{\eta\eta}}}(B^{-1})_{R\eta},$$

$$B_{R\eta}^2 = B_{RR}B_{\eta\eta} - \frac{B_{RR}}{(B^{-1})_{\eta\eta}} = B_{RR}B_{\eta\eta} - \frac{B_{\eta\eta}}{(B^{-1})_{RR}},$$

$$B_{R\eta}(B^{-1})_{R\eta} + B_{\eta\eta}(B^{-1})_{\eta\eta} = B_{R\eta}(B^{-1})_{R\eta} + B_{RR}(B^{-1})_{RR} = 1,$$

$$[B_{\eta\eta}B_{RR} - B_{R\eta}^2][(B^{-1})_{\eta\eta}(B^{-1})_{RR} - (B^{-1})_{R\eta}^2] = 1,$$

$$B_{R\eta}^2 < B_{\eta\eta}B_{RR},$$

$$(B^{-1})_{R\eta}^2 < (B^{-1})_{\eta\eta}(B^{-1})_{RR}.$$

3. Macroscopic and microscopic considerations

Let us firstly consider as collective variables the relative distance R between the fragment centers and the mass asymmetry parameter η. For well separated fragments we get the usual definitions of R and η if we substitute in (8) the expressions for g_R and g_η defined by the equations:

$$\frac{dg_R}{dz} = \frac{\theta(z)}{A_1} - \frac{\theta(-z)}{A_2}, \tag{18a}$$

$$g_\eta = \frac{1}{A}(\theta(z) - \theta(-z)). \tag{18b}$$

Here z is the axis connecting fragment centers and θ is the step function. The $z = 0$ is the point where the densities of nuclei are equal to each other. From (18a) it follows that:

$$g_R = z\left(\frac{\theta(z)}{A_1} - \frac{\theta(-z)}{A_2}\right). \tag{18c}$$

Fragment mass numbers are defined by the relation as given below:

$$A_{\binom{1}{2}} = \int d\mathbf{r}\, \rho(\mathbf{r})\theta(\pm z). \tag{19a}$$

stage we shall use as a basis the single particle wave functions of the noninteracting nuclei i.e. projectile (φ_1) and target (φ_2). Thus, taking into account the small overlapping of the colliding nuclei, ρ can be written as

$$\rho(\mathbf{r}) = \sum_{1,1'}\varphi_1^*(\mathbf{r})\varphi_{1'}(\mathbf{r})a_1^+a_{1'} + \sum_{2,2'}\varphi_2^*(\mathbf{r})\varphi_{2'}(\mathbf{r})a_2^+a_{2'} + \sum_{1,2}(\varphi_1^*(\mathbf{r})\varphi_2(\mathbf{r})a_1^+a_2 + h.c.).$$

Assuming the chaoticity of the phases of the nondiagonal matrix elements of ρ and neglecting their contribution we obtain from (26)

$$< 0|\rho(\mathbf{r})|0 >= \sum_{k\in 1,2} n_k|\varphi_k(\mathbf{r})|^2;$$

where n_k are the Fermi occupation numbers. Ultimately, eq. (23) becomes:

$$A_{neck} = \sum_{k\in 1,2} n_k \int d\mathbf{r}\exp\left(-\frac{z^2}{b^2}\right)|\varphi_k(\mathbf{r})|^2. \qquad (26)$$

Because of the approximations used to derive the components of inverse inertia tensor the influence of the interacting nuclei shell structure on the value of $B_{jj'}$ is only partly taken into account. This influence manifests through the microscopical definitions of A_{neck}. In the consistent microscopical calculation we should use directly the cranking expression to determine the inertia tensor. Thus, use of (17), (22), (25) and (27) in numerical calculations allows us to obtain the smoothed parts of the elements of inertia tensor. However, the obtained results give possibility to elucidate the values of mass parameters and its dependences on the collective variables.

4. Results of calculations

To illustrate the results of the previous sections we have considered the systems ^{58}Ni+^{58}Ni$\rightarrow^{116}$Ba and ^{118}Pd+^{118}Pd$\rightarrow^{236}$U. The components of the inertia tensor are calculated exploiting the expressions (17), (22), (25) and (27). Using the single particle wave functions of harmonic oscillator in the cylindrical representation, the matrix elements in (27) can be obtained analytically. For the frequency parameter

of the harmonic oscillator wave functions we have taken the value [19] $\nu = (0.9A^{1/3} + 0.7)^{-1}$ fm^{-2} which reproduces the systematics of nuclear radii. We have used $b = 0.8$ fm in our calculations. The dependences of A_{neck} on η at $R = R_1 + R_2 + d$ ($R_{1,2} = 1.15A_{1,2}^{1/3}$ fm are the nuclear radii and $d = -1, 0, 2$ fm) are presented in figs. 2 and 3. It is seen that the nucleon number in neck decreases with increasing η. The values of A_{neck} are larger at smaller R i.e. A_{neck} is proportional to the overlapping volume of nuclei in contact.

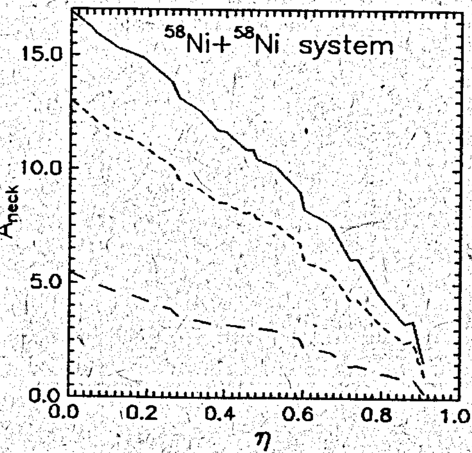

Fig.2. Dependence of A_{neck} on mass asymmetry η at various values of fragment separations $R = R_1 + R_2 + d$ for the system ^{58}Ni+^{58}Ni. The calculated results at $d = -1, 0, 2$ fm are presented by solid, short dashed and long dashed lines, respectively.

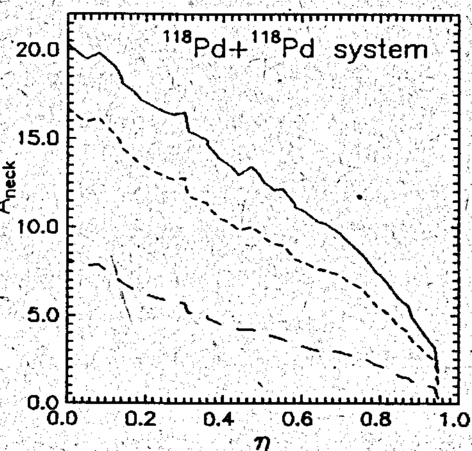

Fig.3. The same as in fig. 2, but for the system ^{118}Pd+^{118}Pd.

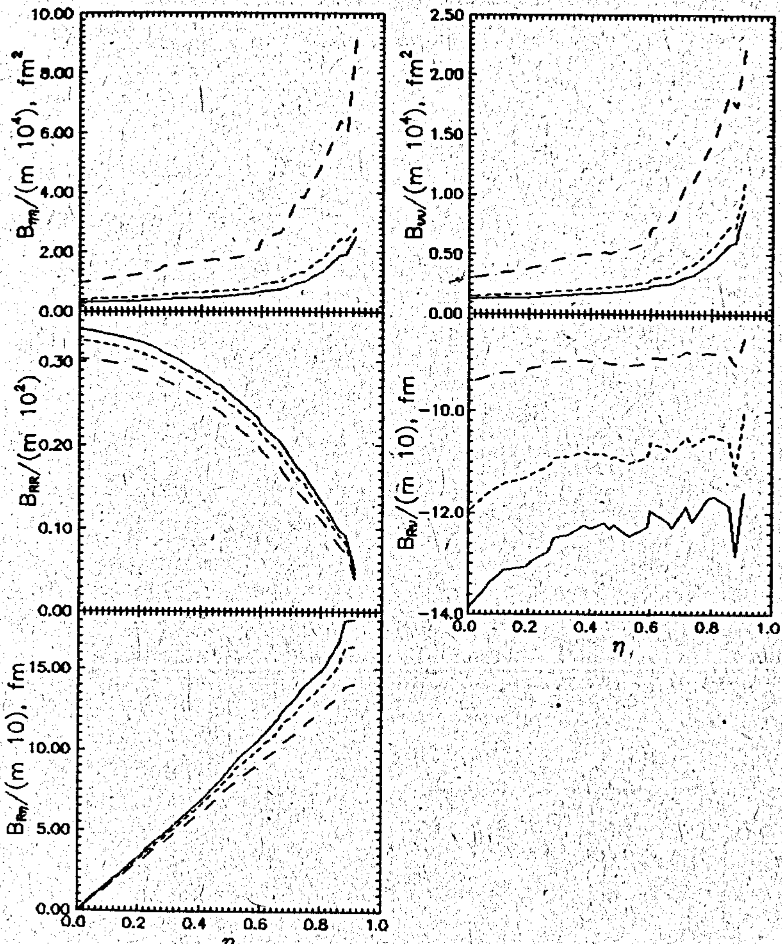

Fig.4. Mass parameters $B_{\eta\eta}$, B_{RR}, $B_{R\eta}$, B_{vv} and B_{Rv} as functions of mass asymmetry η at various values of fragment separations $R = R_1 + R_2 + d$ for the system ^{58}Ni+^{58}Ni. The calculated results at $d = -1, 0, 2$ fm are presented by solid, short dashed and long dashed lines, respectively.

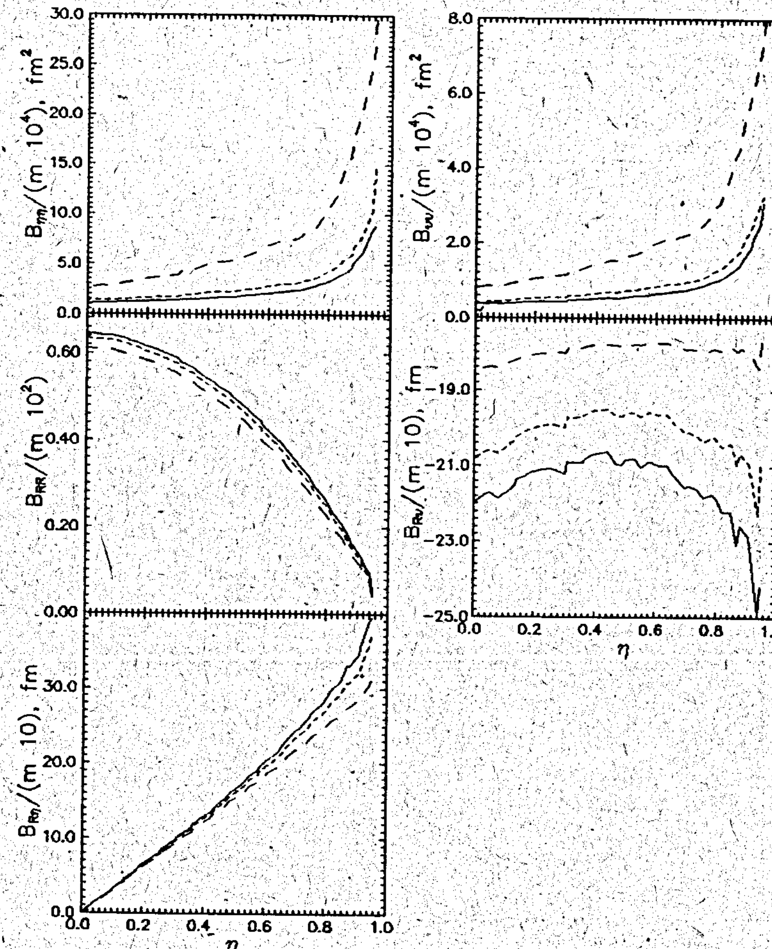

Fig.5. The same as in fig. 4, but for the system ^{118}Pd+^{118}Pd.

The mass parameters $B_{jj'}$ calculated as functions of η for various values of the fragment separation R are presented in figs. 4 and 5. The oscillations of the values of mass parameters which are seen in figs. 4 and 5 are the consequences of the nuclear shell structure. The values of $B_{\eta\eta}$ and B_{vv} increase with increasing η. In the asymptotic limit when $R \to \infty$, the components of inertia tensor go to: $B_{R\eta} \to 0$, $B_{R\upsilon} \to 0$, $B_{\eta\eta} \to \infty$, $B_{\upsilon\upsilon} \to \infty$ and $B_{RR} \to \mu$. The influence of the intrinsic degrees of freedom on the radial motion becomes visible with decrease of R since B_{RR} increases as compared to μ.

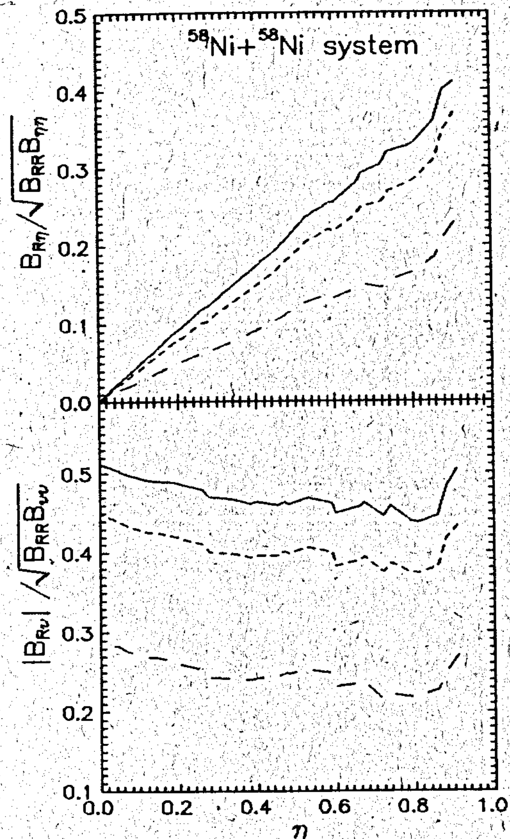

Fig.6. Dependences of ratios $B_{R\eta}/\sqrt{B_{RR}B_{\eta\eta}}$ and $|B_{R\upsilon}|/\sqrt{B_{RR}B_{\upsilon\upsilon}}$ on mass asymmetry η at various values of fragment separations $R = R_1 + R_2 + d$ for the system $^{58}Ni+^{58}Ni$. The calculated results at $d = -1, 0, 2$ fm are presented by solid, short dashed and long dashed lines, respectively.

It is seen that the values of $B_{\eta\eta}$ and $B_{\upsilon\upsilon}$ are not too different. The change of collective variables depends on the initial velocities of these modes of motion, the values of mass parameters and the corresponding gradients of the potential energy surface. So, there is a possibility of a small growth of a neck size during the evolution of the dinuclear system. In this case the nuclei forming the dinuclear system retain their individual properties [20,21]). The results obtained are important

for the description of the nuclear fusion process because the consideration of the neck formation is necessary to investigate the dinuclear system transition to mononucleus.

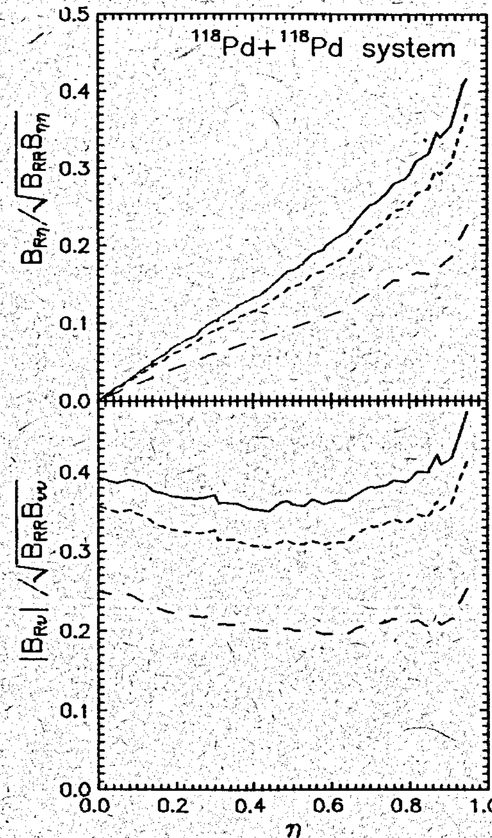

Fig.7. The same as in fig. 6, but for the system $^{118}Pd+^{118}Pd$.

For the symmetric configurations the coupling between $R-$ and $\eta-$ modes of motion vanishes. However, the value of the ratio $B_{R\eta}/\sqrt{B_{RR}B_{\eta\eta}}$ (figs.6,7) increases significantly with increasing mass asymmetry η and can approach to 0.4 in the limit $\eta \to 1$. This behavior of $B_{R\eta}$ is in agreement with the results of refs. [23,24]). The condition $B_{R\eta} \ll \sqrt{B_{RR}B_{\eta\eta}}$ is not correct for strongly asymmetric dinuclear systems and the nondiagonal components of the inertia tensor should be taken into account. The coupling between $R-$ and $\eta-$ modes of motion can be the reason of the enhanced yield of light particles in fusion-type reactions observed in the experiments [20,21,24]).

The role of the coupling between $R-$ and $v-$modes of motion is considerable as well (figs.6,7). So, all components of inertia tensor are necessary to describe the evolution of asymmetric dinuclear system.

5. Summary

On the basis of approach suggested above the general expressions for the diagonal and nondiagonal components of inertia tensor, describing a dinuclear system formed in the dissipative heavy ion collisions, have been obtained. The derivation is based on the nuclear Hamiltonian of a general form. The results obtained confirm the conclusion of refs. [21,22]) that the nondiagonal component of inertia tensor connecting $R-$ and $\eta-$ modes of motion is small for almost symmetric configurations but it increases strongly if mass asymmetry increases. Thus, it is important to take into account the nondiagonal matrix elements of the inertia tensor to consider dynamics of the strongly asymmetric systems. However, for almost symmetric configurations the condition $B_{R\eta} \ll \sqrt{B_{RR}B_{\eta\eta}}$ is justified [4]). The results of this paper can be useful in the consideration of nuclear fusion process.

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