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A.A.Chumbalov, S.S.Kamalov, R.A.Eramzhyan*

POLARIZATION OBSERVABLES IN NEGATIVE PION PHOTOPRODUCTION OFF NUCLEI. *T*- AND *P*-ASYMMETRIES

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*Institute for Nuclear Research, Moscow 117312, Russia

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1. Introduction

At present, polarization observables attract considerable attention as a powerful tool for investigating the nuclear reactions' mechanisms. Single polarization experiments at fixed kinematics enrich knowledge, which comes from the unpolarized differential cross-sections, with three more observables, namely, photon asymmetry (Σ), target asymmetry (T) and recoil asymmetry (P). They are given by an interference between the non-spin-flip and the spin-flip transitions, thus enhancing the effect of the small parts of the reaction amplitude. So far, the properties hidden in the unpolarized characteristics can be revealed.

Recently, the first measurements of pion scattering and single charge exchange on polarized ^{13}C and ^{15}N nuclei from PSI, LAMPF and TRIUMF have been reported [1,2]). The data provide a sensitive test of nuclear model input in different theoretical models elaborated for the description of these reactions. For example, the theoretical predictions of A_y for the charged pion-nuclear scattering from ref.[3]) with two different phenomenological sets of the nuclear transition densities have opposite signs. This shows the sensitivity of the polarization observables to the nuclear structure. It is natural to extend the researches like that to the $A(\gamma,\pi)A'$ reaction. The first theoretical attempts to study the polarization observables for the photopion production have been undertaken, for instance, in refs.[4–8]).

In some special cases there exists a good possibility to measure the P-asymmetry. As has been discussed in ref.[8]), if the residual nucleus is a γ emitter, then the recoil polarization can be determined completely by measuring the intensity and the circular asymmetry of the emitted photons. Unfortunately, the theoretical analysis has been performed in this paper in terms of the Plane Wave Impulse Approximation (PWIA) which is insufficient for the calculation of the T- and P-asymmetries, as will be shown below.

The T and P asymmetries are proportional to the imaginary parts of the bilinear combinations of the reaction amplitudes. In the elementary operator, which we used in our calculations [refs.[9,10])], the imaginary parts come from the Δ-isobar contribution to the elementary process $\gamma + N \to \pi + N$. That is why there is a good possibility to study the properties of the Δ in the nuclear medium. This topic becomes more attractive in view of recent results obtained by Koch *et al.*[11]). In these papers a modification of the Δ-isobar propagator in the reaction $\gamma + {}^{13}C \to \pi^- + {}^{13}N$ has been studied. For this reaction an influence of the nuclear medium on the elementary operator is more essential for the longitudinal part connected with the $E0$ nuclear transition. Therefore, the T and P asymmetries are expected to be extremely sensitive to the medium effects as they are proportional to the imaginary part of the product of the $E0$ and $M1$ transitions.

The imaginary part of the nuclear photoproduction amplitude comes also from the effects of the pion-nuclear interaction in the final state. This topic is discussed in details below.

There is also a pure nuclear aspect in the presented investigation. For the spin-$\frac{1}{2}$-nuclei, we can study the interplay of the magnetic-type $M1$ transition and the electric-type $E0$ one. In other words, one can obtain a good piece of information about the isovector transition $E0$, whose contribution to the elastic electron-nuclear scattering is small in comparison with the large isoscalar $C0$ transition. The investigation of the polarization observables can provide also additional restrictions on the strength of the monopole ($L=0$) and quadrupole ($L=2$) parts of the magnetic transition operator.

The structure of this paper is as follows. In section 2, we start with the qualitative analysis in terms of the PWIA, we discuss some general properties of the T- and P-asymmetries and deduce useful relations between the observables for the free neutron, s- and p-shell nuclei. The Distorted Wave Impulse Approximation (DWIA) is briefly outlined in section 3. The sensitivity of the polarization observables to different ingredients of the model are analyzed in section 4. The main topic of this section is the study of the effects which come from different nuclear model input. The obtained results are summarized in Conclusions.

2. General Properties of T and P

Following the Madison Convention [12]) we directed the photon momentum $\vec{k}$ along the z axis and the vector $[\vec{k}\times\vec{q}]$ along the y axis. The polarization observables are written so far in the following manner:

target asymmetry

$$T(\theta)=\frac{\sigma^t_+(\theta)-\sigma^t_-(\theta)}{\sigma^t_+(\theta)+\sigma^t_-(\theta)}, \tag{1}$$

where σ^t_+ (σ^t_-) is the differential cross-section for the target polarization parallel (antiparallel) to the vector $[\vec{k}\times\vec{q}]$;

recoil asymmetry

$$P(\theta)=\frac{\sigma^r_+(\theta)-\sigma^r_-(\theta)}{\sigma^r_+(\theta)+\sigma^r_-(\theta)}, \tag{2}$$

where σ^r_+ (σ^r_-) is the differential cross-section for the recoil polarization parallel (antiparallel) to the vector $[\vec{k}\times\vec{q}]$.

It is convenient to calculate the polarization observables in terms of the spherical spin-tensors $\tau_{k\kappa}$. Their matrix elements in the case of spin-$\frac{1}{2}$-nuclei are

$$\langle m'|\tau_{k\kappa}|m\rangle\equiv\tau^{m'm}_{k\kappa}=\hat{k}\langle\frac{1}{2}m,\,k\kappa|\frac{1}{2}m'\rangle, \tag{3}$$

where $\hat{k}\equiv\sqrt{2k+1}$. Then, one can write

$$\frac{|\vec{k}|}{|\vec{q}|}\frac{d\sigma}{d\Omega}T=\imath(T_{11}+T_{1-1})/\sqrt{2},\qquad \frac{|\vec{k}|}{|\vec{q}|}\frac{d\sigma}{d\Omega}P=\imath(t_{11}+t_{1-1})/\sqrt{2}, \tag{4}$$

where

$$T_{k\kappa}=\sum_{ii'f}M^{fi'}_{\pi\gamma}\tau^{i'i}_{k\kappa}M^{fi^*}_{\pi\gamma},\qquad t_{k\kappa}=\sum_{iff'}M^{fi}_{\pi\gamma}M^{f'i^*}_{\pi\gamma}\tau^{f'f}_{k\kappa}. \tag{5}$$

The standard expansion of the elementary operator reads [13])

$$F^{(\lambda)}_{\pi\gamma}=iF_1(\vec{\sigma}\cdot\vec{\epsilon}_\lambda)+F_2(\vec{\sigma}\cdot\hat{\vec{q}})(\vec{\sigma}[\hat{\vec{k}}\times\vec{\epsilon}_\lambda])+iF_3(\vec{\sigma}\cdot\hat{\vec{k}})(\hat{\vec{q}}\cdot\vec{\epsilon}_\lambda)+iF_4(\vec{\sigma}\cdot\hat{\vec{q}})(\hat{\vec{q}}\cdot\vec{\epsilon}_\lambda), \tag{6}$$

where we introduce the unit vectors $\hat{\vec{q}}=\vec{q}/|\vec{q}|$ and $\hat{\vec{k}}=\vec{k}/|\vec{k}|$ and the photon polarization vector $\vec{\epsilon}_\lambda$ with $\lambda=\pm1$.

Let's now write down the elementary amplitude for the photopion production in the cyclic basis [14]):

$$F^{(\lambda)}_{\pi\gamma}=\Big(G_1\sigma_\lambda+\lambda G_2\sigma_0+G_3\sigma_{-\lambda}+G_4\Big)\tau^{(+)}\equiv\sum_{\beta=1}^{4}C_\beta G_\beta\sigma^{(S_\beta)}_{\mu_\beta}\tau^{(+)}, \tag{7}$$

where $C_\beta=1$ for $\beta=1,3,4$, $C_2=\lambda$; $S_\beta=1$ for $\beta=1,2,3$ corresponds to the spin-flip and $S_4=0$ to the non spin-flip transitions; $\mu_1=-\mu_3=\lambda$ and $\mu_2=\mu_4=0$.

For the qualitative understanding of the results we make at this point two approximations. First, in nuclear matrix elements we neglect the quadrupole $L=2$ transitions and adopt the factorization approximation [14]) while averaging over the nucleon Fermi-motion. Then the nuclear photoproduction amplitude in terms of the PWIA can be written in the following form:

$$M_{m'm}=-f_{c.m.}(Q)W_AR^{(n'l',nl)}_0\sum_{\beta=1}^{4}\hat{S}_\beta C_\beta\langle\frac{1}{2}m,S_\beta\mu_\beta|\frac{1}{2}m'\rangle(-1)^l\hat{l}G_\beta\psi^{(n'l',nl)}_{J(LS_\beta)1}\delta_{L0}\delta_{JS_\beta}, \tag{8}$$

where the determination of the coefficient $\psi^{(n'l',nl)}_{J(LS_\beta)1}$ is given in Appendix A. This amplitude describes the pion photoproduction off $|\frac{1}{2}m>$ to $|\frac{1}{2}m'>$ nuclear states, $\vec{Q}=\vec{k}-\vec{q}$ is the transferred momentum, $f_{c.m.}(Q)=exp(Q^2b^2/4A)$ eliminetes the center of mass motion, b is an oscillator parameter and W_A is a kinematical factor. In the given above expression we define the nuclear radial integrals

$$R^{(n'l',nl)}_0(Q)=\int_0^\infty R_{n'l'}(r)j_0(Qr)R_{nl}(r)r^2dr, \tag{9}$$

where $j_0(Qr)$ is the spherical Bessel function and $R_{nl}(r)$ are the radial parts of the bound nucleon wave functions.

To bring the general properties of the T and P asymmetries into better focus, let us consider in this section the very simple extreme single particle nuclear models. It means that we put the nuclear transition densities in the jj-coupling to be

$$\begin{aligned}\psi_{S_\beta T}(0s_{1/2},0s_{1/2})&=(-1)^{S_\beta+T}\quad\text{for } {}^3H,\\ \psi_{S_\beta T}(1p_{1/2},1p_{1/2})&=1\quad\text{for } {}^{13}C,\\ \psi_{S_\beta T}(1p_{1/2},1p_{1/2})&=(-1)^{S_\beta+T}\quad\text{for } {}^{15}N,\end{aligned}$$

thus assuming ${}^3H$ (${}^{15}N$) to have a pure $0s_{1/2}$ ($1p_{1/2}$) proton hole in a closed-shell ${}^4He$ (${}^{16}O$), and the ${}^{13}C$ nucleus as a pure $1p_{1/2}$ neutron outside a closed $1p_{3/2}$ shell.

After some straightforward manipulations one finds:

- $n(\gamma,\pi^-)p$:

$$M^{\lambda}_{m'm} = -\sum_{\beta} C_\beta G_\beta \hat{S}_\beta \langle \frac{1}{2}m, S_\beta\mu_\beta | \frac{1}{2}m' \rangle, \quad (10)$$

- ${}^3H(\gamma,\pi^-){}^3He$:

$$M^{\lambda}_{m'm} = -R_0^{(0s,0s)}(Q) W_A f_{c.m.} \sum_{\beta} (-1)^{S_\beta} C_\beta G_\beta \hat{S}_\beta \langle \frac{1}{2}m, S_\beta\mu_\beta | \frac{1}{2}m' \rangle, \quad (11)$$

- ${}^{13}C(\gamma,\pi^-){}^{13}N$:

$$M^{\lambda}_{m'm} = R_0^{(1p,1p)}(Q) W_A f_{c.m.} \sum_{\beta} (-1)^{S_\beta} C_\beta G_\beta \frac{1}{\hat{S}_\beta} \langle \frac{1}{2}m, S_\beta\mu_\beta | \frac{1}{2}m' \rangle, \quad (12)$$

- ${}^{15}N(\gamma,\pi^-){}^{15}O$:

$$M^{\lambda}_{m'm} = R_0^{(1p,1p)}(Q) W_A f_{c.m.} \sum_{\beta} C_\beta G_\beta \frac{1}{\hat{S}_\beta} \langle \frac{1}{2}m, S_\beta\mu_\beta | \frac{1}{2}m' \rangle. \quad (13)$$

Looking at the matrix elements we easily observe that the relative weights of the spin-flip and non-spin-flip transitions are identical in the first two cases: $\hat{S}_\beta \equiv \sqrt{2S_\beta+1}$. The only difference (to say nothing of the form factor R_S which cancels in the expressions for the polarization observables) is in the phase factor $(-1)^{S_\beta}$ which appears in the expression for the ${}^3H$ matrix element. For the p-shell nuclei the relative weights of the spin-flip and non spin-flip transitions change by factor $1/\hat{S}_\beta$ in comparison with the free neutron case. The interpretation of this effect is clear in view of additional possibilities for the spin-flip and non spin-flip transitions for the nucleons with the orbital momentum $l=1$.

We proceed by writing down the expressions for the polarization observables in terms of the elementary amplitudes

$$\frac{d\sigma(n)}{d\Omega}T(n) \sim sin\theta\, Im(F_3F_1^* + F_2F_4^* + (F_4F_1^* - F_3F_2^*)cos\theta + F_3F_4^* sin^2\theta),$$
$$\frac{d\sigma(n)}{d\Omega}P(n) \sim -sin\theta\, Im(2F_2F_1^* + F_3F_1^* + F_2F_4^* + (F_4F_1^* - F_3F_2^*)cos\theta + F_3F_4^* sin^2\theta),$$

$$\frac{d\sigma({}^3H)}{d\Omega}T({}^3H) = -\frac{d\sigma(n)}{d\Omega}P(n),$$
$$\frac{d\sigma({}^3H)}{d\Omega}P({}^3H) = -\frac{d\sigma(n)}{d\Omega}T(n),$$
$$\frac{d\sigma({}^{13}C)}{d\Omega}T({}^{13}C) = -\frac{d\sigma({}^{15}N)}{d\Omega}P({}^{15}N) \sim sin\theta(4F_2F_1^* + F_3F_1^* + F_2F_4^* + (F_4F_1^* - F_3F_2^*)cos\theta + F_3F_4^* sin^2\theta),$$
$$\frac{d\sigma({}^{13}C)}{d\Omega}P({}^{13}C) = -\frac{d\sigma({}^{15}N)}{d\Omega}T({}^{15}N) \sim -sin\theta(-2F_2F_1^* + F_3F_1^* + F_2F_4^* + (F_4F_1^* - F_3F_2^*)cos\theta + F_3F_4^* sin^2\theta). \quad (14)$$

The relations between T and P in the case of the free neutron and ${}^3H$, which have been obtained in ref.[6]), can be directly interpreted. The polarization of the initial ${}^3H$ is given by the spin-state of the proton. Then, due to the Pauli principle the reaction $\gamma + n \rightarrow \pi^- p$ is going with the recoil proton in the ${}^3He$ nucleus in the well defined spin-state as if it is polarized. That is why one has the written above relation between $T({}^3H)$ and $P(n)$.

There is no any simple expression which connects the observables on the ${}^{13}C$ and ${}^{15}N$ nuclei with that for the neutron. It reflects the change of the relative strengths of the spin-flip and spin-independent parts of the reaction amplitude when the neutron is on the p-shell.

The given above expressions provide the general rules for T and P. Due to the $sin\theta$ dependence they are identically zero at $\theta = 0°$ and 180°. Besides, at the threshold region they are very small because of vanishing imaginary parts of the elementary amplitudes.

Let us summarize the results of this section. Using simple one-particle (one-hole) nuclear functions, switching off the rescattering effects and neglecting the $L=2$ term in the transition operator, we got the relations which connect the polarization observables for the free neutron, ${}^3H$, ${}^{13}C$ and ${}^{15}N$. We have found out that the nucleon, being on the p-shell, changes the relative weights of the spin-flip and non spin-flip parts of the reaction's amplitude. It leads to significant difference between the polarization observables for the free nucleon and p-shell nuclei. In addition we have found the simple relations between the polarization observables for ${}^{13}C$ and ${}^{15}N$.

3. DWIA Ingredients

The expression for the full pion photoproduction matrix $T_{\pi\gamma}$, which includes rescattering effects (final state interaction – FSI), is given in refs.[5,14]). For further discussion it is useful to enlist the ingredients of the DWIA. The pion photoproduction matrix $T_{\pi\gamma}$ is given as

$$T_{\pi\gamma}(\vec{q}_0, \vec{k}\lambda) = U_{\pi\gamma}(\vec{q}_0, \vec{k}\lambda) + \int \frac{d\vec{q}}{(2\pi)^3} \frac{T'(\vec{q},\vec{q}_0) U_{\pi\gamma}(\vec{q},\vec{k}\lambda)}{E(q_0) - E(q) + i\epsilon}, \quad (15)$$

where $U_{\pi\gamma}$ is the plane- wave photoproduction amplitude and $E(q) = E\pi(q) + E_A(q)$ is the total energy of the pion-nuclear system. We have introduced an auxiliary matrix T' which is related with the true pion-nuclear scattering T-matrix by

$$T'(\vec{q}, \vec{q}_0) = \frac{A-1}{A} T(\vec{q}, \vec{q}_0). \tag{16}$$

The factor $(A-1)/A$ eliminates double counting of the pion-nucleon interaction as it is included in the plane-wave part of the operator. We construct the T' matrix as a solution of the Lippmann-Schwinger equation with the optical potential taken from [15]. The standard DWIA is strongly based on the assumption that the dominant pion wave distortion effects come from the coherent rescattering of the pion in the spherically symmetrical part of the pion-nuclear optical potential ("coherent approximation"). We have found that the extended version of the DWIA, where the non coherent (spin-flip) rescattering is included, does not lead to any significant changes in the final result for the p-shell nuclei.

The optical potential, used in our analysis of the p-shell nuclei, is splitted into the first order term and a phenomenological ρ^2 term, which stands for true pion absorption and higher order processes [15]. For the ^3He nucleus the second order effects are very small and the first order optical potential provides a satisfactory description of the π-^3He elastic scattering [5].

Using the Impulse Approximation, one can express the matrix elements of $U_{\pi\gamma}$ in terms of the elementary $(\gamma\pi)$ amplitude (see Appendix A.). We have used in our calculations the Blomqvist-Laget model [9,10]. This model is unitary in the p_{33}-channel where the corresponding pion-nucleon phase-shift is large.

The few-body structure formalism, which has been applied to investigate the polarization observables for the $\gamma + ^3H \rightarrow \pi^- + ^3He$, is summarized in ref.[6]. The trinucleon wave function can be calculated "exactly" by Faddeev's techniques, thus the nuclear input here is under control. As in ref.[6], we used in this paper the wave functions by Brandenburg *et al.* [16] which have been obtained by solving the Faddeev equations in momentum space for the Reid soft-core potential. Using this functions, Maize and Kim [17] have predicted the three-body magnetic form factor quite close to the data.

The complexities of the nuclear many-body input for the p-shell nuclei are isolated in the one-body density-matrix elements (the nuclear transition densities) traditionally denoted as $\psi^{(1p,1p)}_{J(LS),T}$ [14,18]. These values accompany the reduced matrix elements of the single-particle multipole operators:

$$j_L(Qr) \quad [Y_L \otimes \sigma^S]_J \tau^T, \tag{17}$$

where $S = 0, 1$, $T = 0, 1$, $\sigma^1 = \vec{\sigma}$ and $\tau^1 = \vec{\tau}$ are the Pauli matrices, respectively, $\sigma^0 = \tau^0 = 1$, Y_L is the spherical function.

As the same set of the transition operators appears in the electron-nuclear scattering, this reaction is a traditional source of information about the values of the corresponding nuclear transition densities. However, the analysis like that retains two uncertainties for the spin-$\frac{1}{2}$-targets. The first one is that it does not provide a value for the $E0$-type isovector transition density ($L = S = J = 0, T = 1$). The second uncertainty is that there is no possibility to separate the contributions of the monopole $L = 0$ and quadrupole $L = 2$ parts of the $M1$-type transition (S and $J = 1$).

At present, the ^{13}C magnetic form factor is not satisfactorily understood, despite a rather intensive efforts. For example, the extreme single-particle model, used in the previous section, fails to explain the data on the magnetic scattering in particular in the second maximum. An improvement of the extreme single-particle model results can be achieved by allowing the configuration mixing between $1p_{1/2}$ and $1p_{3/2}$ orbitals within the $0\hbar\omega$ shell model space (the Cohen-Kurath model [19]).

On the other hand, instead of microscopical calculations it is possible to extract nuclear transition densities from the electromagnetic form factors data by allowing them for a free variation with constraints that keep them close to the calculated ones. This pure phenomenological procedure considerably improves of the description of the magnetic form factors [18,20], but it is not sufficient to fix the unique value of the $L = 2$ transition. For example, two independent analyses performed in the $0\hbar\omega$ model space give: $\psi^{1p,1p}_{1(21),1} = .34$ [18] and .58 [20].

Recent analysis of the experimental data for the $^{13}C(\gamma, \pi^-)^{13}N$ reaction shows the necessity of an essential suppression of the isovector electric transition $E0$. As has been shown in ref.[21], this suppression can be achieved by adding a few-percent admixture of the $2p$-shell configurations. Recently, this fact has been verified within microscopical constraints which take into account the contributions of the $2\hbar\omega$ configurations in the ground state of the ^{13}C nucleus.

The comparison of experimental data and theoretical predictions for the ^{15}N nucleus shows the same features, as disscused in detail for ^{13}C. The magnetic form factor, for example, is too large in the maximum, and falls too quickly at large Q. As for ^{13}C, the analysis of the pion photoproduction and pion scattering reactions on ^{15}N also reveals the importance of the $2\hbar\omega$ configurations [22].

Taking into account all these circumstances we expect the influence of higher configurations on the polarization observables in pion photoproduction to be much larger and informative.

Below in our analysis we are going to use two models. The first has been elaborated phenomenologically in the $0\hbar\omega$ model space by Tiator and Wright (TW) [18], where the $M1$ transition density has been extracted from the magnetic form factors while the $E0$ transition density is the same as in the Cohen-Kurath model. In the second microscopical model developed by van Hees with collaborators [23], the transition densities have been obtained within a full $(0+2)\hbar\omega$ shell model space.

As follows from Figure 1, where we represent the charge form factor for ^{13}C and ^{15}N, these two models give different strengths of the isovector electric transition. The effect of this difference on the polarization observables, which are determined by the interference of these two types of transitions, is substantial as will be shown below.

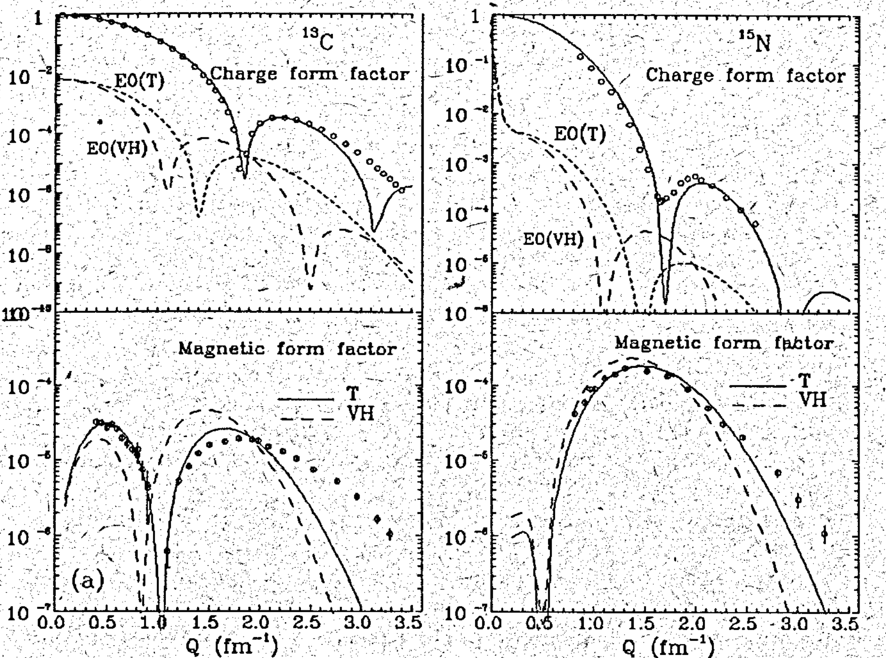

Fig. 1. Charge and magnetic form factor for: (a) ^{13}C and (b) ^{15}N nuclei calculated in the $0\hbar\omega$ (TW model[18]) and $(0+2)\hbar\omega$ (VH model[23]) spaces.

4. Results and Discussion

4.1 PHOTOPRODUCTION ON ^3H.

We start our discussion with the results for the $\gamma +^3 H \to \pi^- +^3 He$ reaction, keeping in mind a possibility to study it through the inverse reaction $\pi^3 + \vec{H}e \to \gamma +^3 H$. As goes from Figure 2a, the values of the polarization observables are small at low energies of photons. Actually, in the threshold region the dominant part of the elementary operator is the Kroll-Rudermann term, which is real (s-wave pions). Therefore, T and P are small because they are proportional to the imaginary parts of bilinear combinations of the elementary amplitudes.

As the energy of the incident photons increases, the role of the Δ-isobar excitations increases. That is why in the resonance region the T and P asymmetries are about -0.5 and -1.0, respectively.

An additional contribution in the imaginary part of the reaction amplitude comes due to the rescattering of the outgoing pion on the nucleus in the final state. In Figure 2, we separate the effect of the final state interaction (FSI) by plotting the results of the DWIA calculations with the Born part of the elementary amplitude only. Note

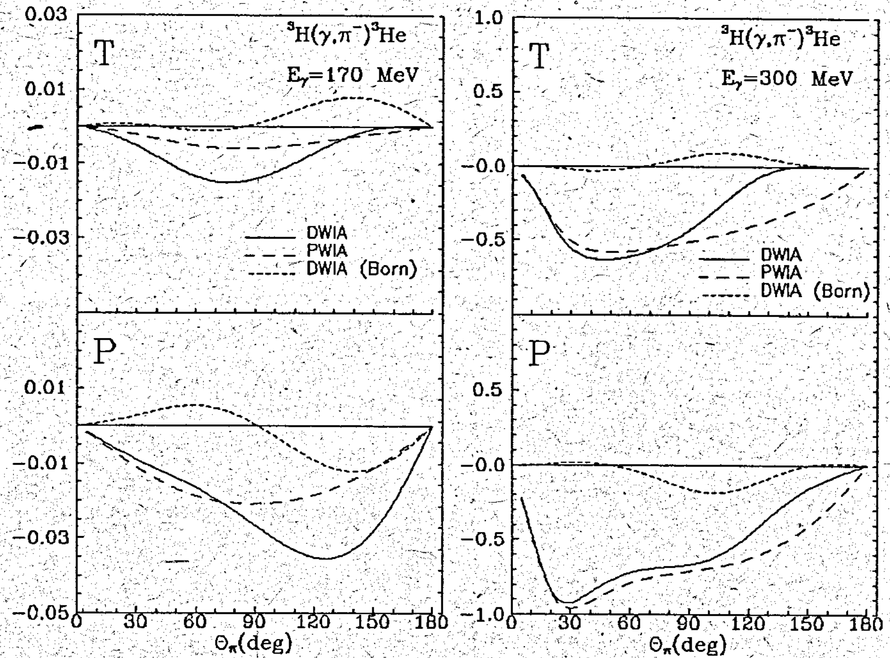

Fig. 2. T- and P-asymmetries for $\gamma +^3 H \to \pi^- +^3 He$ reaction calculated with Faddeev wave functions at $E_\gamma = 170$ and 300 MeV.

that in terms of the PWIA the pure Born approximation leads to zero values of the asymmetries. Thus, all deviations from zero are referred to the FSI contribution. As goes from Figure 2, these contributions are important at large angles. For the forward angles, the role of the FSI is small. This fact is clearly seen especially in the resonance region. As has been shown in ref.[6]), in this region the simple relations between the polarization characteristics for the elementary process and that for the three-nucleon system (like those presented in eqn.(14)) are valid with a high degree of accuracy. Hence, one has a good possibility to study the properties of the Δ-isobar in the nuclear medium. Any experimental deviation from these relations could serve as an evidence of the Δ-isobar's characteristics change inside a nucleus.

To simulate this phenomenon, we analyse a small ($\sim 3\%$) variation of the Δ-mass in a spirit of ref.[24]). The corresponding results are plotted in Figure 3 with $M_\Delta \to \beta M_\Delta$, $\beta = 0.97$, 1.00 and 1.03. The caused differences are very impressive. However, a more thorough investigation is needed to study this effect, keeping the unitarity of the elementary amplitude. In the end of this part of the discussion we note that in our opinion a role of the imaginary parts of the elementary E_{0+} and other multipoles, different from the resonant M_{1+} and E_{1+}, is not significant. As these imaginarities are originated by the unitarity constraint, they are small because of smallness of the corresponding pion-nucleon phase shifts in the energy region of interest.

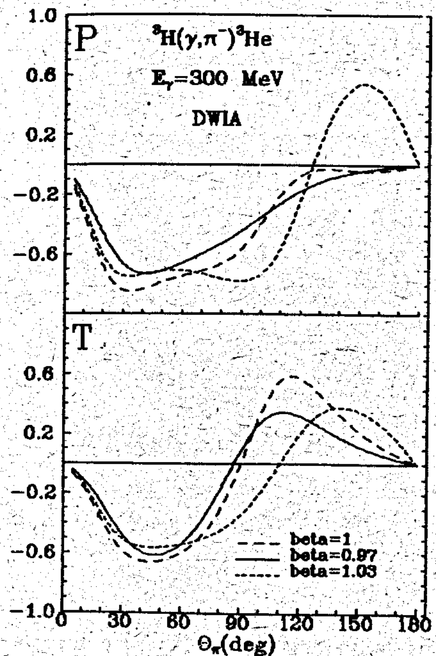

Fig. 3. The sesitivity of the P and T-asymmetries to the changes of the Δ-isobar mass (see text).

4.2 PHOTOPRODUCTION ON THE P-SHELL NUCLEI.

Let us return to the p-shell nuclei. Here T and P are expected large because of the large influence of the distortion effects (or the FSI). Thus, the FSI almost determines the values of the T and P asymmetries at low energies where the contribution of the Δ-isobar is small.

However, at large energies the role of the Δ-isobar is significant and leads to the relative diminution of the FSI effects at small angles ($\theta < 50°$). In this region, where the eikonal approximation is relevant, the FSI can be included as a multiplicative factor and thus cancels in the expressions for the polarization observables. That is why it doesn't play any significant role at $\theta < 50°$ as is reflected in the plots: the PWIA and DWIA results are close to each other. But at large angles the discrepancy between these two cases is clearly seen: the rescattering effects cause an additional anomaly in the angular dependence of the polarization observables at $\theta < 100°$ (see Figure 4). Its origin is in the second minimum for the isovector $E0$ type transition.

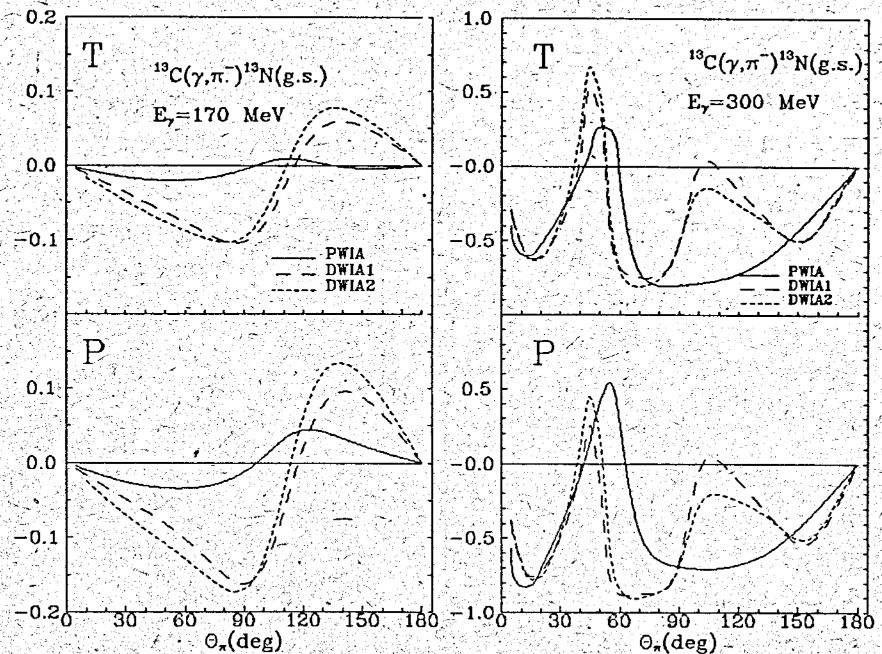

Fig. 4. The sesitivity of the P and T-asymmetries for $\gamma + ^{13}C \rightarrow \pi^- + ^{13}N$ reaction to the FSI. Transition densities are from Tiator and Wright[18]). DWIA1 - calculations with factorization approximation, DWIA2 - calculations with numerical averaging over the nucleon momentum inside nucleus.

Because of the strong (attractive) πA interaction this minimum is effectively shifted in the region of small transferred momentum. Like for the electron scattering, in order to take into account the pion wave distortion one can introduce the "effective" momentum

$$\vec{q}^{\,2} \rightarrow \vec{q}_{eff}^{\,2} = \vec{q}^{\,2} - 2E_\pi \bar{U}_{\pi A}, \tag{18}$$

where the mean optical potential $\bar{U}_{\pi A} < 0$ and, consequently, $|\vec{q}_{eff}| > |\vec{q}|$.

In Figure 4, we also present the analysis of the nucleon Fermi-motion effect. The calculations have been performed in terms of the factorization approximation (see eqn.(A5)) and with numerical averaging over the nucleon momentum distribution inside a nucleus. The differences are small at low energies. The "exact" treatment of the nucleon Fermi motion provides some effect at low energies as it brings into play the gradient terms which change the relative contribution of the spin-flip and non spin-flip transitions. At high energies, the effect is small. The general conclusion is that the factorization approximation is valid for the quantitative analysis of the T and P asymmetries.

We finish our discussion with the analysis of the sensitivity of the polarization observables to the nuclear structure. As has been mentioned, we choose for this

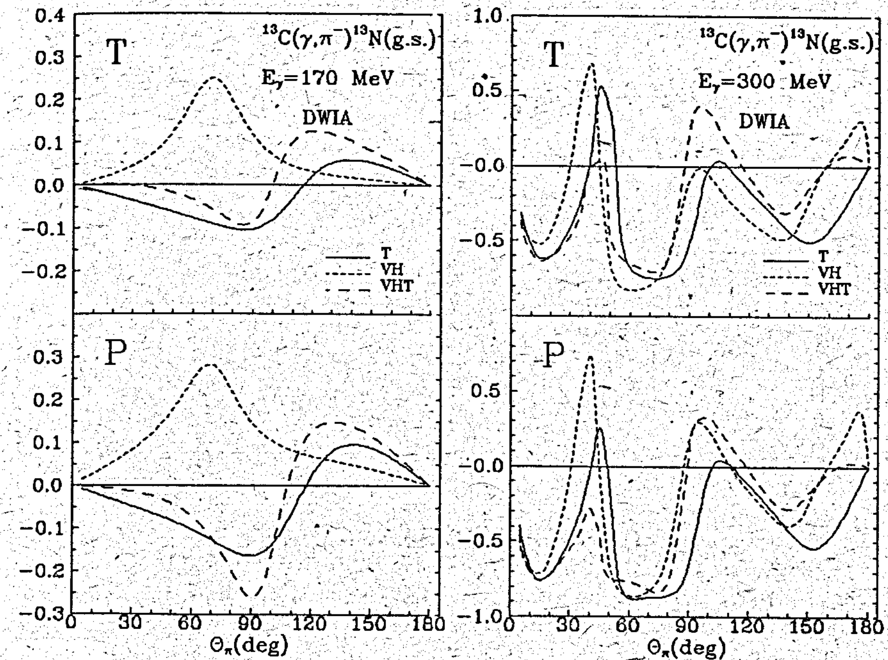

Fig. 5. The sensitivity of the P and T-asymmetries for $\gamma + ^{13}C \rightarrow \pi^- + ^{13}N$ reaction to the nuclear model input. DWIA calculations at E_γ = 170 and 300 MeV.

aim the two nuclear models: *i*) the phenomenological model by Tiator and Wright[18]) with the $0\hbar\omega$-restricted space and *ii*) microscopical model with the $2\hbar\omega$ configurations included. As goes from Figures 5 and 6, these two models give even opposite sings for the T and P asymmetries. The results with a hybrid nuclear model - VHT - where the value of the $M1$ type transition density is taken from ref.[18]) indicate that the main effect is that from the $M1$ type transition. The detailed analysis shows that the results are very sensitive to the strength of the quadrupole part of this transition ($J = 1$, $L = 2$) which dominates at large angles [4,7]). Thus, the study of the polarization observables provide additional restrictions on the strength of the quadrupole part of the $M1$ transition.

We would like to finish with one more interesting circumstance. As goes from

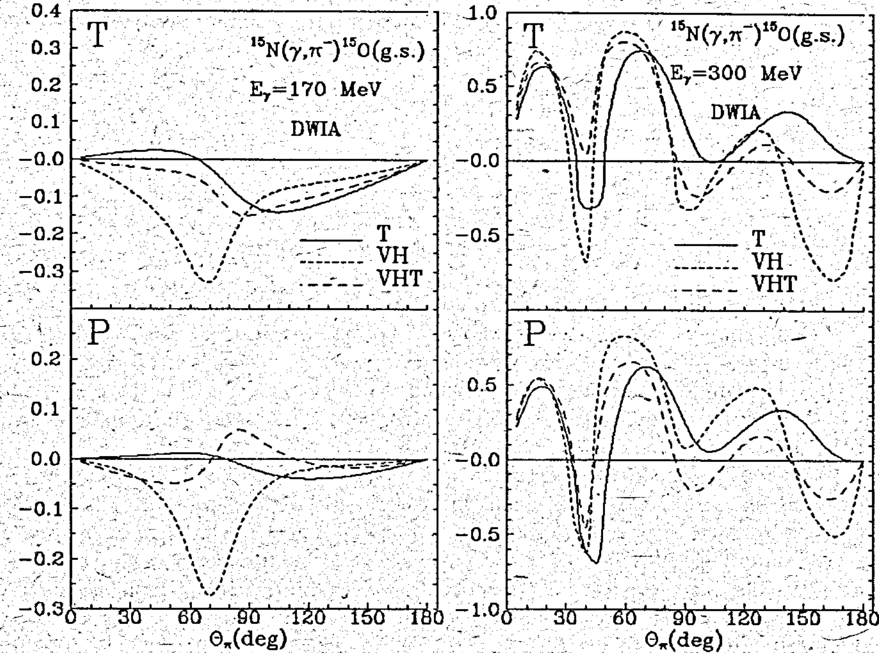

Fig. 6. The sensitivity of the P and T-asymmetries for $\gamma + ^{15}N \rightarrow \pi^- + ^{15}O$ reaction to the nuclear model input. DWIA calculations at E_γ = 170 and 300 MeV.

our plane-wave analysis

$$T(^{13}C) \sim -P(^{15}N), \tag{19}$$

$$T(^{15}N) \sim -P(^{13}C). \tag{20}$$

These relations are reliable at all energies and do not depend on the nuclear model and the FSI effects.

5. Conclusion

We have analyzed the target and recoil polarization asymmetries for negative pion photoproduction off the neutron and spin-$\frac{1}{2}$-nuclei and have found out some very interesting effects which necessitate experimental confirmation. At low energies, the asymmetries for the ^3H nucleus are extremely small due to the smallness of the Δ-isobar contribution and the FSI effects. In the resonance region, the polarization observables are large enough for the experimental investigation and are determined mainly by the Δ-isobar contribution. It is a good basis for the investigation of the influence of the nuclear surrounding on the Δ-isobar characteristics. The preferable kinematical condition for such an investigation is the region of small pion angles as the values of the asymmetries are largest here and the effects of the pion rescattering are small. We have also found a dramatic effect which comes from the Δ-isobar mass change. Once more we want to point out that a more physically founded approach is necessary for the quantitative predictions.

For the p-shell nuclei ^{13}C and ^{15}N the T and P asymmetries are large even at low energies. Their values are determined by the FSI effects, the role of which increases with atomic number. At the same time, the influence of the nuclear structure increases as well. Different nuclear models (phenomenological and microscopical) provide different signs of the asymmetries. The main reason for such an effect is in different strengths of the quadrupole part of the $M1$ transitions ($J=1$, $L=2$, $S=1$). The similar result has been obtained in the resonance region at small angles ($\theta < 50°$). At large angles ($\theta > 90°$), the rescattering leads to an additional anomaly of the polarizations behaviour.

For the p-shell nuclei there are no simple relations which connect the polarization observables in the nuclear case with that for the elementary process, as has been obtained for the s-shell nuclei. However, we have found the relation between the T and P asymmetries for the ^{13}C and ^{15}N nuclei. These relations have been verified in our DWIA analysis. It has been shown that they hold independently on the nuclear model input and the FSI effects at all energies. We have also found a strong effect connected with the changes of the Δ-mass.

The factorization approximation has been shown to be reliable not only for the unpolarized cross-sections but for the polarization observables as well.

The results presented in this paper seem to be useful for the forthcoming next generation experiments with the polarized nuclear targets. The simultaneous analyses of the pion scattering and pion photoproduction reactions on the same polarized targets give an opportunity to check the model ingredients and, in more practical sense, to resolve a number of problems which have been revealed in the first polarization measurements for the (π,π') reaction.

Appendix A.

PLANE WAVE AMPLITUDE

Performing the numerical integration over the nucleon momentum distribution inside the nucleus in terms of the Impulse Approximation one comes to the following expression for the plane-wave amplitude $M_{m'm}$

$$M_{m'm}(\vec{q},\vec{k}\lambda) = \tfrac{1}{2}\textstyle\sum_J \langle \tfrac{1}{2}m, JM_J|\tfrac{1}{2}m'\rangle\langle \tfrac{1}{2}N, 1N_T|\tfrac{1}{2}N'\rangle U_{JT} f_{c.m.}(\vec{q}-\vec{k})$$

where $\hat{J}=(2J+1)^{1/2}$, $f_{c.m.}(Q)=exp(Q^2b^2/4A)$.

We have defined new amplitudes U_{JT}

$$U_{JT} = \sqrt{6}\hat{J}\sum_L \langle LM_L, 1\mu_\beta|JM_J\rangle \psi^{1p,1p}_{J(L1),1} I^\beta_{LM_L}(1p,1p) \qquad (A.1)$$

$$+\delta_{LJ}\delta_{M_L M}\psi^{1p,1p}_{J(L1),0} I^4_{LM_L}(1p,1p). \qquad (A.2)$$

The dynamics of the process is contained in the following function

$$I^\beta_{LM_L}(n'l'nl) = (-1)^{l+M_L}\int d^3p R_{n'l'}(p')G_\beta(\vec{q},\vec{k})R_{nl}(p)[Y_{l'}(p')\otimes Y_l(p)]_{L,-M_L}. \qquad (A.3)$$

Here R_{nl} are the radial parts of a bound nucleon wave-function, G_β are directly related to the elementary photoproduction operator.

The coefficients $\psi^{1p,1p}_{J(L1),1}$ represent the nuclear many-body input. They can be easily connected with the spin- and isospin-reduced matrix elements as

$$\psi^{n'l'nl}_{J(L1),1} = \sum_{j,j'} \hat{S}\hat{L}\hat{j}\hat{j}' \begin{Bmatrix} 1/2 & 1/2 & S \\ j' & j & J \\ l' & l & L \end{Bmatrix} \psi_{JT}(\alpha',\alpha), \qquad (A.4)$$

where $\alpha=\{n,l,j\}$ and

$$\psi_{JT}(\alpha',\alpha) = (\hat{J}\hat{T})^{-1}\langle J_f T_f|||[c^+_{\alpha'}\otimes c_\alpha]_{JT}||J_i T_i\rangle.$$

There is one more way to handle the nucleon Fermi-motion known as the factorization approximation. In terms of it one adopts the following substitution:

$$\vec{p}\rightarrow \vec{p}_{eff.} = -\frac{\vec{k}}{A} - \frac{A-1}{2A}(\vec{k}-\vec{q}), \quad \vec{p}'\rightarrow \vec{p}_{eff.}+\vec{q}-\vec{k}. \qquad (A.5)$$

and comes to evaluation of $I^\alpha_{LM_L}$ in the form

$$I^\alpha_{LM_L}(n'l'nl) = \sqrt{4\pi}\frac{\hat{l}'\hat{l}}{\hat{L}} Y^*_{LM_L}(\vec{Q})\imath^L\langle l0, l'0|L0\rangle \times \qquad (A.6)$$

$$G_\beta(\vec{q},\vec{k}\lambda,\vec{p}_{eff})\int_0^\infty R_{n'l'}(r)j_L(Qr)R_{nl}(r)r^2dr. \qquad (A.7)$$

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