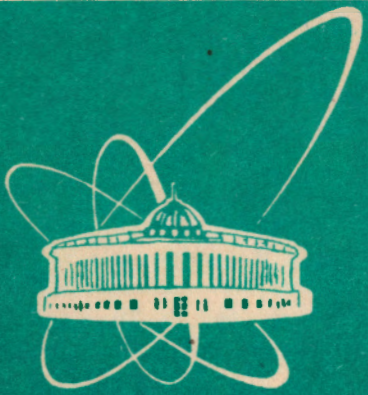

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ELASTIC FORM FACTORS OF HYDROGENLIKE ATOMS IN *nS*-STATES

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1. Introduction

Our interest to the hydrogenlike atoms was caused by papers [1]–[3] where production of the Coulomb bound states of various elementary particles (elementary atoms) was predicted for high energy processes. Till now three such atoms were observed: e^+e^- atom from decay of π^0 [4]–[6]; $\pi\mu$ atom from decay of K_L^0 [7]–[9]; $\pi^+\pi^-$ atom from p Ta interaction at 70 GeV [10].

Interaction of such atoms with ordinary atoms is an essential part of those experiments. Especially for the $\pi^+\pi^-$ atoms whose observation and lifetime measurement is based on accurate calculation of the ionization (breaking up) probability at passing through the target where they are produced [3],[10]. To do so a great number of elementary atom cross sections for various initial states is required.

As shown in Ref.[11],[12] elementary atoms interact with ordinary atoms predominantly via Coulomb potential. So this interaction may be treated in terms of atomic form factors.

The general solution of the classical problem of hydrogenlike atom form factor was found in the paper [13]. Using the group-theoretical method the authors gave the exact analytic formulas of form factors for all transitions. However those formulas have a sufficiently complicated form, long calculation should be done for each value of a transfer momentum.

Here we give the simple exact formulas for the elastic form factor of hydrogenlike atoms in the nS–states obtained using special function algebra. General formulas of form factors for discrete-discrete transitions should be considered later.

2. Formulas

Form factor of hydrogenlike atoms is written as

$$F_n(\vec{q}) = \int |\Psi_n(\vec{r})|^2 e^{i\vec{q}\vec{r}} d\vec{r}, \qquad (1)$$

here $\vec{q}$ is the transferred momentum and $\Psi_n(\vec{r})$ is the atomic wave function. For the nS–states (principal quantum number n and zero numbers l, m) the wave function is

$$\Psi_n(\vec{r}) = \frac{1}{\sqrt{4\pi}} \frac{2(\alpha\mu)^{3/2}}{n^{5/2}} \exp(-\alpha\mu r/n) L^1_{n-1}(2\alpha\mu r/n). \qquad (2)$$

Here L^1_{n-1} is the Laguerre polynomial, α is the fine structure constant, μ is the atomic reduced mass ($\hbar = c = 1$). Then the form factor is written as

$$F_n(q) = \frac{4\alpha\mu^3}{n^5} \int_0^\infty \exp(-2\alpha\mu r/n) \left(L^1_{n-1}(2\alpha\mu r/n)\right)^2 \frac{\sin qr}{qr} r^2 dr. \qquad (3)$$

For S–states form factor dose not depend on a choice of the quatization axis. Using the following substitutions

$$2\alpha\mu r/n = x \qquad \Delta = qn/2\alpha\mu \qquad qr = \Delta x, \qquad (4)$$

one has the expression

$$F_n(q) = \frac{1}{2n^2} \int_0^\infty \exp(-x) \left(L^1_{n-1}(x)\right)^2 \frac{\sin \Delta x}{\Delta x} x^2 dx. \qquad (5)$$

To evaluate the improper integral one should use the following relations for the Laguerre polynomials (see for instance [14]).

$$\left(L^1_{n-1}(x)\right)^2 = \frac{n}{\pi} \sum_{k=0}^{n-1} \frac{\Gamma(k+1/2)\Gamma(n-k-1/2)}{\Gamma(k+2)\Gamma(n-k)} L^2_{2k}(2x) \qquad (6)$$

$$\int_0^\infty \exp(-x) L^2_{2k}(x) \frac{\sin \Delta x}{\Delta x} x^2 dx = \frac{\cos^2(\phi/2)}{\sin\phi} \times$$
$$\times \left[(k+1)\sin\left((2k+1)\phi\right) + (k+1/2)\sin\left((2k+2)\phi\right)\right]. \qquad (7)$$

Here $\tan(\phi/2) = \Delta$.
After substitution of (6) and (7) into (5) the form factor has the form

$$F_n(q) = \frac{\cos^2(\phi/2)}{2n\pi\sin\phi} \times$$
$$\times \left[\sum_{k=0}^{n-1} \frac{\Gamma(k+1/2)\Gamma(n-k-1/2)}{\Gamma(k+1)\Gamma(n-k)} \sin\left((2k+1)\phi\right) + \right.$$
$$\left. + \sum_{k=0}^{n-1} \frac{\Gamma(k+3/2)\Gamma(n-k-1/2)}{\Gamma(k+2)\Gamma(n-k)} \sin\left((2k+2)\phi\right)\right]. \qquad (8)$$

Nothing is changed if one begins the second sum from -1 and then increases index k by unity. After replacing the sin function by the exponential one has

$$F_n(q) = \frac{\cos^2(\phi/2)}{2n\pi\sin\phi} \times$$
$$\times \mathrm{Im}\left[\sum_{k=0}^{n-1} \frac{\Gamma(k+1/2)\Gamma(n-k-1/2)}{\Gamma(k+1)\Gamma(n-k)} \exp\left(i(2k+1)\phi\right) + \right.$$
$$\left. + \sum_{k=0}^{n} \frac{\Gamma(k+1/2)\Gamma(n-k+1/2)}{\Gamma(k+1)\Gamma(n-k+1)} \exp\left(2ik\phi\right)\right]. \qquad (9)$$

The sums in (9) may by expressed via hypergeometric function ${}_2F_1$.

$$F_n(q) = \frac{\cos^2(\phi/2)}{2n\pi\sin\phi} \times$$
$$\times \mathrm{Im}\left[\frac{\Gamma(1/2)\Gamma(n-1/2)}{\Gamma(n)} e^{i\phi} {}_2F_1(1/2, 1-n; 3/2-n; e^{2i\phi}) + \right.$$
$$\left. + \frac{\Gamma(1/2)\Gamma(n+1/2)}{\Gamma(n+1)} {}_2F_1(1/2, -n; 1/2-n; e^{2i\phi})\right]. \qquad (10)$$

After using the following relation for the Legendre polynomial

$$P_n(\cos\phi) = \frac{\Gamma(n+1/2)}{\Gamma(1/2)\Gamma(n+1)} e^{-in\phi} {}_2F_1(1/2, -n; 1/2-n; e^{2i\phi}), \qquad (11)$$

one has the expression

$$F_n(q) = \frac{\cos^2(\phi/2)}{2n\sin\phi} \mathrm{Im}\left[e^{-in\phi} P_n(\cos\phi) + e^{-in\phi} P_{n-1}(\cos\phi)\right]. \qquad (12)$$

Then one replaces back the exponential by the sin function. Finally for the elastic form factor of the hydrogenlike atoms in nS–states one has

$$F_n(q) = \frac{\cos^2(\phi/2)}{2n} \frac{\sin n\phi}{\sin\phi} \left[P_n(\cos\phi) + P_{n-1}(\cos\phi)\right]. \qquad (13)$$

Or it may be expressed via the Jacobi polynomial $P^{(0,1)}_{n-1}$

$$F_n(q) = \frac{\cos^4(\phi/2)}{n} \frac{\sin n\phi}{\sin\phi} P^{(0,1)}_{n-1}(\cos\phi). \qquad (14)$$

For computation one may use well-known recurrence relations for the Jacobi polynomial or for the Legendre polynomials in (13) (see for instance [15]). Using obtained formulas one can investigate n dependence of total cross sections for S–states of the hydrogenlike atoms.

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