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VARIATIONAL DESCRIPTION OF THE ROTATING NUCLEI

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1. Introduction

The discovery of superdeformed bands in atomic nuclei that extend to very high angular momenta[1,2] opens a possibility for further study of coexistence and competition between various forces acting inside a nucleus. The leading factors that determine the nuclear behaviour in such extreme conditions are: 1^0 – the average, strongly deformed single–particle nuclear field, 2^0 – the Coriolis and centrifugal forces induced by the fast rotation, 3^0 – the pairing forces that generate superfluid–type correlations, 4^0 – other residual forces often imitated by the separable potential with different multipolarity, 5^0 – thermal degrees of freedom. It has been shown by several authors[3,4] that, even if the static pairing correlations disappear, at high spin region there are observable effects which can be connected with the pairing fluctuations. The microscopic model for the description of quasiparticle and collective excitations of rotational nuclei has been proposed by Marshalek[5,6] and by Janssen and Mikhailov[7]. This model, based on cranking plus RPA, was developed and firstly applied independently in two papers (Ref. 8,9) and then in many others. The results of this investigations were described in the review–articles (Refs. 10, 11, 12). In the paper Shimizu et al[12] pairing fluctuations in the rotating nuclei are discussed in detail. In this article a lot of theoretical and experimental data were presented up to 1989. In the paper Bes et al[13] static and dynamic pairing correlations in strongly rotating nuclei are investigated. The comparison between the RPA and the number of particle projection method is given in Ref. 14.

To describe the growing number of more and more detailed experimental data, there are also developed different approaches which aim is to take into account higher order terms of the perturbation theory or/and the anharmonic corrections (the nonlinear terms in equations of motion) as a consequence of the mode–mode coupling (see for example Ref. 15). Such procedure actually complicates the theory already expanded and requires a great deal of computer work. If we limit our consideration to the quasiparticle plus RPA phonon model, we can see, that historically rather step by step method of approximation has been developed. Firstly, the free quasiparticle spectrum and wave functions have been calculated. Secondly the residual interaction has been added and *on the base of this static solution* the collective excitations problem has been solved in RPA. With this connection, in the realistic calculations, to reproduce the experimental data concerning the vibrational excitations, many authors must change "by hands" the single–particle spectrum given by the phenomenological potential (Nillsson or Saxon–Woods). Therefore, we would like to pay attention to one more possibility how to improve the harmonic approximation in the more consistent way. Namely, keeping the algebraic scheme unchanged, we will use more general trial wave function which contains the phonon degrees of freedom too. This idea comes from Hara and from Rowe[16] who took into account the ground state correlations. More recently this problem has been discussed by Klein[17]. In present paper we shall see that the mutual dependence of the quasiparticle and phonon amplitudes will appear as the simple generalization of this idea.

We consider the model of nucleus treated as a system of nucleons moving in a deformed single–particle potential which rotates with a constant frequency Ω around an axis fixed in space. The residual interaction is assumed to be due to the monopole pairing. Our approach differs from the standard BCS+RPA in three points: we assume the ground state of the system to be the RPA phonon vacuum, secondly, we take into account the Pauli principle a little more accurately than standard RPA, and thirdly, we obtain the system of equations for the quasiparticle and phonon amplitudes from one variational principle. It will be shown, that in this way in the harmonic approximation some coupling between the quasiparticle and vibrational degrees of freedom can be found. In the next section the problem will be formulated in details and the quasiparticle and vibrational variables will be defined.

In the section 3 we obtain the basic equations of the problem. There will be considered the limit cases and possible approximations. The comparison of our results with the results of the previous papers will be also given. Conclusions are collected in the section 4.

2. The model Hamiltonian and approximations

The model Hamiltonian is

$$H^{\Omega} = H_0^{\Omega} + H_p \tag{1}$$

where

$$H_0^{\Omega} = \sum_{\alpha\beta} \varepsilon_{\alpha\beta} a_\alpha^+ a_\beta - \lambda N - \Omega j^x, \tag{2}$$

$$H_p = -GP^+P, \qquad P^+ = \sum_{\beta>0} a_\beta^+ a_{\bar{\beta}}^+, \tag{3}$$

j^x – the x–component of the angular momentum,

G – pairing force constant,

λ – chemical potential,

$|\bar{\beta}>$ – single–particle state time–reversed to $|\beta>$,

$\varepsilon_{\alpha\beta}$ – one–particle energy matrix in some shell–model basis.

In the spherical basis

$$|\bar{\beta}> \equiv T|\beta> = T|jm> = (-1)^{j+m}|j,-m> .$$

In the cranking model where a nuclear average field rotates around the x–axis we have to use the base vectors corresponding to the two eigenvalues r of the rotation operator R_x

$$R_x = \exp(-r\pi j^x)$$

and $r = \pm i$, (signature quantum number). Instead of pairs of states $|\beta>, |\bar{\beta}>$ we will employ the positive $|\nu>$ $(r = +i)$ and negative $|\tilde{\nu}>$ $(r = -i)$ signature states. Performing the Goodman transformation (see for example the book Ref. 18).

$$\begin{pmatrix} a_{\nu}^{+} \\ a_{\tilde{\nu}}^{+} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} a_{\beta}^{+} \\ a_{\bar{\beta}}^{+} \end{pmatrix} \tag{4}$$

we can rewrite the Hamiltonian (1) in the new basis:

$$\sum_{\alpha\beta} \varepsilon_{\alpha\beta} a_{\alpha}^{+} a_{\beta} = \sum_{\mu\nu} \varepsilon_{\mu\nu} (a_{\mu}^{+} a_{\nu} + a_{\tilde{\mu}}^{+} a_{\tilde{\nu}}), \tag{5}$$

$$N = \sum_{\alpha} a_{\alpha}^{+} a_{\alpha} = \sum_{\nu} (a_{\nu}^{+} a_{\nu} + a_{\tilde{\nu}}^{+} a_{\tilde{\nu}}), \tag{6}$$

$$P^{+} = \sum_{\nu} a_{\nu}^{+} a_{\tilde{\nu}}^{+} = \frac{1}{2} \sum_{\nu} (a_{\nu}^{+} a_{\tilde{\nu}}^{+} - a_{\tilde{\nu}}^{+} a_{\nu}^{+}), \tag{7}$$

$$j^{x} = \sum_{\mu\nu} j_{\mu\nu}^{x} (a_{\mu}^{+} a_{\nu} - a_{\tilde{\mu}}^{+} a_{\tilde{\nu}}) \tag{8}$$

Now, we define, as usual, the quasiparticles α_i^{+}, α_i by the Bogolubov transformation:

$$\begin{aligned} a_{\mu}^{+} &= \sum_{i} (A_{\mu}^{i} \alpha_{i}^{+} + B_{\mu}^{\bar{i}} \alpha_{\bar{i}}) \\ a_{\tilde{\mu}}^{+} &= \sum_{i} (A_{\tilde{\mu}}^{\bar{i}} \alpha_{\bar{i}}^{+} + B_{\tilde{\mu}}^{i} \alpha_{i}) \end{aligned} \tag{9}$$

The summation over i means also the summation over the states with positive and negative signatures. Asking α_i, α_i^{+} to be fermionic operators we obtain the unitary conditions:

$$\begin{aligned} &\sum_{i} (A_{\mu}^{i} A_{\nu}^{i} + B_{\mu}^{\bar{i}} B_{\nu}^{\bar{i}}) = \delta_{\mu\nu}, \qquad \sum_{i} (A_{\tilde{\mu}}^{\bar{i}} A_{\tilde{\nu}}^{\bar{i}} + B_{\tilde{\mu}}^{i} B_{\tilde{\nu}}^{i}) = \delta_{\mu\nu}, \\ &\sum_{\mu} (A_{\mu}^{i} A_{\mu}^{k} + B_{\tilde{\mu}}^{i} B_{\tilde{\mu}}^{k}) = \delta_{ik}, \qquad \sum_{\mu} (A_{\tilde{\mu}}^{\bar{i}} A_{\tilde{\mu}}^{\bar{k}} + B_{\tilde{\mu}}^{i} B_{\tilde{\mu}}^{k}) = \delta_{ik}. \end{aligned} \tag{10}$$

The hamiltonian (1) can be expressed by the quasiparticles in the following manner:

$$H_0^\Omega = \sum_{\mu\nu ik}(\epsilon_{\mu\nu} - \lambda\delta_{\mu\nu})[(A_\mu^i A_\nu^k - B_{\bar\mu}^k B_{\bar\nu}^i)\rho_{ik} + (A_{\bar\mu}^{\bar i} A_{\bar\nu}^{\bar k} - B_\mu^{\bar k} B_\nu^{\bar i})\rho_{\bar i\bar k}+$$

$$+(A_\mu^i B_\nu^{\bar k} - A_{\bar\mu}^{\bar k} B_{\bar\nu}^i)(b_{i\bar k}^+ + b_{i\bar k})$$

$$+\frac{1}{2}\delta_{ik}(B_\mu^{\bar i} B_\nu^{\bar i} + B_{\bar\mu}^i B_{\bar\nu}^i - A_\mu^i A_\nu^i - A_{\bar\mu}^{\bar i} A_{\bar\nu}^{\bar i} + 2\delta_{\mu\nu})]-$$

$$-\Omega\sum_{\mu i\nu k} j_{\mu\nu}^x[(A_\mu^i A_\nu^k + B_{\bar\mu}^k B_{\bar\nu}^i)\rho_{ik} - (A_{\bar\mu}^{\bar i} A_{\bar\nu}^{\bar k} + B_\mu^{\bar k} B_\nu^{\bar i})\rho_{\bar i\bar k}+ \tag{11}$$

$$+(A_\mu^i B_\nu^{\bar k} + A_{\bar\mu}^{\bar k} B_{\bar\nu}^i)(b_{i\bar k}^+ + b_{i\bar k})$$

$$+\frac{1}{2}\delta_{ik}(B_\mu^{\bar i} B_\nu^{\bar i} - B_{\bar\mu}^i B_{\bar\nu}^i - A_\mu^i A_\nu^i + A_{\bar\mu}^{\bar i} A_{\bar\nu}^{\bar i} + 2\delta_{\mu\nu}]$$

where

$$b_{i\bar k}^+ = \alpha_i^+ \alpha_{\bar k}^+, \qquad b_{i\bar k} = \alpha_{\bar k}\alpha_i,$$

$$\rho_{ik} = \alpha_i^+ \alpha_k, \qquad \rho_{\bar i\bar k} = \alpha_{\bar i}^+ \alpha_{\bar k};$$

$$-GP^+P = -G\left(\sum_{ij}(S_{i\bar j}b_{i\bar j}^+ + L_{i\bar j}b_{i\bar j} + \mathcal{P}_{ij}\rho_{ij} - \mathcal{P}_{\bar i\bar j}\rho_{\bar i\bar j}) + \Delta_{st}\right) \times \tag{12}$$

$$\times\left(\sum_{mn}(S_{m\bar n}b_{m\bar n} + L_{m\bar n}b_{m\bar n}^+ + \mathcal{P}_{nm}\rho_{mn} - \mathcal{P}_{\bar n\bar m}\rho_{\bar m\bar n}) + \Delta_{st}\right).$$

where

$$S_{i\bar{j}} = \sum_\mu A^i_\mu A^{\bar{j}}_{\bar{\mu}}, \qquad L_{i\bar{j}} = \sum_\mu B^i_{\bar{\mu}} B^{\bar{j}}_{\mu},$$

$$\mathcal{P}_{ij} = \sum_\mu A^i_\mu B^j_{\bar{\mu}}, \qquad \mathcal{P}_{\bar{i}\bar{j}} = \sum_\mu A^{\bar{i}}_{\bar{\mu}} B^{\bar{j}}_{\mu}, \tag{12a}$$

and

$$\Delta_{st} = \sum_i \Delta_{\bar{i}} \equiv \sum_i \mathcal{P}_{\bar{i}\bar{i}} = -\sum_i \Delta_i = -\sum_i \mathcal{P}_{ii} \tag{12b}$$

is the static part of the gap (without the pairing vibrations). Now, exploiting the idea of Hara we will leave on the right hand side the commutators

$$[b_{m\bar{n}}, b^+_{i\bar{k}}]$$

more than the first c-number term. Namely, we assume:

$$[b_{m\bar{n}}, b^+_{i\bar{k}}] = \delta_{mi}\delta_{\bar{n}\bar{k}} - \delta_{mi}\rho_{\bar{k}\bar{n}} - \delta_{\bar{n}\bar{k}}\rho_{im} \simeq$$

$$\simeq \delta_{mi}\delta_{\bar{n}\bar{k}}(1 - \rho_i - \rho_{\bar{k}}) \tag{13}$$

where $\rho_i, \rho_{\bar{k}}$ are the ground state averages of the $\rho_{ii}, \rho_{\bar{k}\bar{k}}$ operators. The factors $(1 - \rho_i - \rho_{\bar{k}})$ take into account the blocking effect due to the Pauli principle. This is the first point where our calculations differ from the standard RPA; $\rho_i, \rho_{\bar{k}}$ are the quasiparticle–occupation numbers in the ground state of the system. In case under consideration we solve the problem for the positive signature phonons which satisfies the following conditions[5]:

$$R_x \begin{pmatrix} b^+_{m\bar{n}} \\ b_{m\bar{n}} \end{pmatrix} R_x^{-1} = + \begin{pmatrix} b^+_{m\bar{n}} \\ b_{m\bar{n}} \end{pmatrix}.$$

The generalization for the negative signature phonons is straightforward. To diagonalize the quadratic form in b^+, b part of the Hamiltonian we define the "pure bosons" Q_N, Q^+_N:

$$Q_N^+ = \sum_{ik}(\Psi_{i\bar{k}}^N b_{i\bar{k}}^+ - \Phi_{i\bar{k}}^N b_{i\bar{k}})$$

$$Q_N = \sum_{ik}(\Psi_{i\bar{k}}^N b_{i\bar{k}} - \Phi_{i\bar{k}}^N b_{i\bar{k}}^+) \tag{14}$$

with commutation relations

$$[Q_N, Q_{N'}] = [Q_N^+, Q_{N'}^+] = 0; \qquad [Q_N, Q_{N'}^+] = \delta_{NN'}. \tag{15}$$

As result we have the orthogonality conditions

$$\sum_{ik}(1 - \rho_i - \rho_{\bar{k}})(\Psi_{i\bar{k}}^N \Psi_{i\bar{k}}^M - \Phi_{i\bar{k}}^N \Phi_{i\bar{k}}^M) = \delta_{MN},$$

$$\sum_{N}(1 - \rho_i - \rho_{\bar{k}})(\Psi_{i\bar{k}}^N \Psi_{m\bar{n}}^N - \Phi_{i\bar{k}}^N \Phi_{m\bar{n}}^N) = \delta_{im}\delta_{\bar{k}\bar{n}}, \tag{16}$$

and the transformation (14) can be reversed

$$b_{i\bar{k}}^+ = (1 - \rho_i - \rho_{\bar{k}})\sum_N(\Psi_{i\bar{k}}^N Q_N^+ + \Phi_{i\bar{k}}^N Q_N),$$

$$b_{i\bar{k}} = (1 - \rho_i - \rho_{\bar{k}})\sum_N(\Psi_{i\bar{k}}^N Q_N + \Phi_{i\bar{k}}^N Q_N^+). \tag{17}$$

After the last transformation we obtain the Hamiltonian H^{Ω} as a sum of the following terms: c–number, linear in $\rho_{ik}, \rho_{\bar{i}\bar{k}}$ and linear in Q_N, Q_N^+ terms, the terms of the $Q_N\rho_{ik}, Q_N^+\rho_{ik}, Q_N^+\rho_{\bar{i}\bar{k}}, Q_N\rho_{\bar{i}\bar{k}}$ type, the quadratic form in Q_N, Q_N^+ and the bilinear form in $\rho_{ik}, \rho_{\bar{i}\bar{k}}$. The second basic assumption of this paper is that we approximate the ground state of the system by the vacuum of the Q_N bosons[16,19]. With this assumption, by using the relation between the quasiparticle vacuum $|0>$ and the Q_N bosons vacuum $|0>_Q$:

$$|0>_Q = U|0>, \tag{18}$$

where

$$U = \mathcal{N} \exp(-\mathcal{S}), \tag{19}$$

$\mathcal{N}$ is the normalization constant and

$$\mathcal{S} = \frac{1}{2} \sum_{iki'k'} S_{i\bar{k},i'\bar{k}'} b^+_{i\bar{k}} b^+_{i'\bar{k}'},$$

we can calculate the ground state averages. This averages depend on the forward and backward amplitudes $\Psi^N_{i\bar{k}}, \Phi^N_{i\bar{k}}$ in the following manner:

$$\rho_i = \frac{1}{2} \left[\sum_{N,k} (1 - \rho_i - \rho_{\bar{k}})((\Psi^N_{i\bar{k}})^2 + (\Phi^N_{i\bar{k}})^2) - \sum_k 1 \right],$$

$$\rho_{\bar{i}} = \frac{1}{2} \left[\sum_{N,k} (1 - \rho_{\bar{i}} - \rho_k)((\Psi^N_{k\bar{i}})^2 + (\Phi^N_{k\bar{i}})^2) - \sum_k 1 \right]. \tag{20}$$

Finally, our third basic idea is to treat the ground state average of H^Ω as a functional depending on two sets of parameters: A, B and Ψ, Φ. The equations for these parameters will be obtained in the next section as the necessary conditions

$$_Q < 0|H^\Omega|0 >_Q$$

to be minimum while taking into account the auxiliary conditions (10) and (16). Because we are limited to the positive signature excitations $\mathcal{S}$ will be depend only on the positive signature bosons, and the matrix $\mathcal{S}_{i\bar{k},l\bar{m}}$ possesses the following symmetries:

$$\mathcal{S}_{i\bar{k},l\bar{m}} = -\mathcal{S}_{\bar{k}i,l\bar{m}} = -\mathcal{S}_{i\bar{k},\bar{m}l} = \mathcal{S}_{\bar{k}i,\bar{m}l} \tag{21}$$

3. Variational principle and equations for the amplitudes

The requirement that

$$\delta_Q < 0|H^{\Omega}|0 >_Q = 0$$

under the auxiliary conditions (10) and (16) can be fulfilled by the Lagrange multipliers method. We define the functional

$$\mathcal{L} = _Q< 0|H^{\Omega}|0 >_Q + \frac{1}{2}\sum_{i\mu} E_i \left[(A^i_\mu)^2 + (B^i_{\bar{k}})^2 - 1\right] +$$

$$+\frac{1}{2}\sum_{i\mu} E_{\bar{i}} \left[(A^{\bar{i}}_{\bar{\mu}})^2 + (B^{\bar{i}}_{\mu})^2 - 1\right] - \tag{22}$$

$$-\frac{1}{2}\sum_{N} \omega_N \left\{\sum_{ik}(1-\rho_i-\rho_{\bar{k}})\left[(\Psi^N_{i\bar{k}})^2 - (\Phi^N_{i\bar{k}})^2\right] - 1\right\}$$

and require $\mathcal{L}$ to be minimum. Necessary conditions for this are the equations

$$\frac{\delta\mathcal{L}}{\delta A^p_\sigma} = 0, \qquad \frac{\delta\mathcal{L}}{\delta B^p_{\bar{\sigma}}} = 0,$$

$$\frac{\delta\mathcal{L}}{\delta A^{\bar{p}}_{\bar{\sigma}}} = 0, \qquad \frac{\delta\mathcal{L}}{\delta B^{\bar{p}}_{\sigma}} = 0, \tag{23}$$

$$\frac{\delta\mathcal{L}}{\delta \Psi^M_{p\bar{q}}} = 0, \qquad \frac{\delta\mathcal{L}}{\delta \Phi^M_{p\bar{q}}} = 0.$$

We keep in $\mathcal{L}$ only the terms quadratic in the phonon amplitudes. Also the terms bilinear in $\rho_i\rho_{\bar{i}}$ were not taken into account.

The equations for the quasiparticle amplitudes are the following:

$$\sum_\mu(\varepsilon_{\mu\sigma} - \lambda\delta_{\mu\sigma} - \Omega j^x_{\mu\sigma})(1-2\rho_p)A^p_\mu - \tilde{\Delta}_p B^p_{\bar{\sigma}} + \Pi(\sum_{Nk} A^{\bar{k}}_{\bar{\sigma}}\Phi^N_{p\bar{k}}) = E_p A^p_\sigma, \tag{24a}$$

$$-\tilde{\Delta}_p A^p_\sigma - \sum_\mu (\varepsilon_{\mu\sigma} - \lambda\delta_{\mu\sigma} + \Omega j^x_{\mu\sigma})(1-2\rho_p) B^p_{\bar{\mu}} + \Pi(\sum_{Nk} B^{\bar{k}}_\sigma \Psi^N_{p\bar{k}}) = E_p B^p_{\bar{\sigma}}, \quad (24b)$$

$$\sum_\mu (\varepsilon_{\mu\sigma} - \lambda\delta_{\mu\sigma} + \Omega j^x_{\mu\sigma})(1-2\rho_{\bar{p}}) A^{\bar{p}}_{\bar{\mu}} + \tilde{\tilde{\Delta}}_p B^{\bar{p}}_\sigma + \Pi(\sum_{Nk} A^k_\sigma \Phi^N_{k\bar{p}}) = E_{\bar{p}} A^{\bar{p}}_{\bar{\sigma}}, \quad (25a)$$

$$\tilde{\tilde{\Delta}}_p A^{\bar{p}}_{\bar{\sigma}} - \sum_\mu (\varepsilon_{\mu\sigma} - \lambda\delta_{\mu\sigma} - \Omega j^x_{\mu\sigma})(1-2\rho_{\bar{p}}) B^{\bar{p}}_\mu + \Pi(\sum_{Nk} B^k_{\bar{\sigma}} \Psi^N_{k\bar{p}}) = E_{\bar{p}} B^{\bar{p}}_\sigma, \quad (25b)$$

where

$$\tilde{\Delta}_i = G[\Delta_{s'}(1-2\rho_i) + \Delta_d]$$

$$\Delta_d = \sum_k \Delta_k \rho_k - \sum_k \Delta_{\bar{k}} \rho_{\bar{k}},$$

$$\tilde{\tilde{\Delta}}_i = \tilde{\Delta}_i(\rho_i \to \rho_{\bar{i}}), \quad (26)$$

$$\Pi = 2G \sum_N \Phi^N,$$

$$\Phi^N = \sum_{ik} (S_{i\bar{k}} \Phi^N_{i\bar{k}} + L_{i\bar{k}} \Psi^N_{i\bar{k}}).$$

Equations (24a), (24b) and (25a), (25b) would look the same if we replace $B^{\bar{p}}_\sigma$ by A^p_σ, $A^{\bar{p}}_{\bar{\sigma}}$ by $B^p_{\bar{\sigma}}$ and E_p by $-E_{\bar{p}}$. The presence of the last term on the left hand side of this equations spoils this symmetry. Let us remark that this problem was also considered in the paper[20] based on the Green's functions method. The last term connects the static pairing field with the amplitudes of pairing vibrations. The averaging over phonon vacuum state leads to more general equations for the cranking model. The quasiparticle energies E_i, $E_{\bar{i}}$ depend not only on the single–particle levels with a cranking term Ωj^x and standard gap Δ, but also on the phonon amplitudes. At the same time the definition of the gap is generalized due to the dynamical part of pairing (eqs. 26). The gap depends on the quantities $\rho_i, \rho_{\bar{i}}$, e.g.

on the ground state correlations. Unfortunately, we lose the signature symmetry which has place in the ordinary cranking model. Now we must solve the equations for the negative and positive signature energies together with the equations for the phonon amplitudes (see below). The structure of the equations (25) (26) shows the mutual dependence of the quasiparticle and phonon amplitudes. Using the formulae (20) we find

$$\frac{\partial \rho_i}{\partial \Psi^M_{p\bar{q}}} = \Psi^M_{p\bar{q}} \delta_{ip}, \qquad \frac{\partial \rho_{\bar{i}}}{\partial \Psi^M_{p\bar{q}}} = \Psi^M_{p\bar{q}} \delta_{\bar{i}\bar{q}}$$
$$\frac{\partial \rho_i}{\partial \Phi^M_{p\bar{q}}} = \Phi^M_{p\bar{q}} \delta_{ip}, \qquad \frac{\partial \rho_{\bar{i}}}{\partial \Phi^M_{p\bar{q}}} = \Phi^M_{p\bar{q}} \delta_{\bar{i}\bar{q}} \tag{27}$$

Therefore, the next group of equations reads:

$$\left[(\mathcal{E}_p + \mathcal{E}_{\bar{q}}) - \Omega(J^x_p - J^x_{\bar{q}}) - 2G\Delta_{st}(\Delta_p - \Delta_{\bar{q}})\right] \Psi^M_{p\bar{q}} - \\ -2G\Phi^M L_{p\bar{q}} = \omega^M \Psi^M_{p\bar{q}}, \tag{28a}$$

$$\left[(\mathcal{E}_p + \mathcal{E}_{\bar{q}}) - \Omega(J^x_p - J^x_{\bar{q}}) - 2G\Delta_{st}(\Delta_p - \Delta_{\bar{q}})\right] \Phi^M_{p\bar{q}} \\ -2G\Phi^M S_{p\bar{q}} = -\omega^M \Phi^M_{p\bar{q}} \tag{28b}$$

where

$$\mathcal{E}_p = \sum_{\mu\nu}(A^p_\mu A^p_\nu - B^p_{\bar\nu} B^p_{\bar\mu})(\varepsilon_{\mu\nu} - \lambda\delta_{\mu\nu}),$$

$$\mathcal{E}_{\bar q} = \sum_{\mu\nu}(A^{\bar q}_\mu A^{\bar q}_\nu - B^{\bar q}_\nu B^{\bar q}_\mu)(\varepsilon_{\mu\nu} - \lambda\delta_{\mu\nu}),$$

$$J^x_p = \sum_{\mu\nu}(A^p_\mu A^p_\nu + B^p_{\bar\mu} B^p_{\bar\nu}) j^x_{\mu\nu}, \tag{29}$$

$$J^x_{\bar q} = \sum_{\mu\nu}(A^{\bar q}_\mu A^{\bar q}_\nu + B^{\bar q}_\nu B^{\bar q}_\mu) j^x_{\mu\nu}.$$

Formally we can write the solutions as follows

$$\Psi^M_{p\bar q} = \frac{2G\Phi^M L_{p\bar q}}{D_{p\bar q} - \omega^M}, \tag{30a}$$

$$\Phi^M_{p\bar q} = \frac{2G\Phi^M S_{p\bar q}}{D_{p\bar q} + \omega^M}, \tag{30b}$$

where

$$D_{p\bar q} = \mathcal{E}_p + \mathcal{E}_{\bar q} - \Omega(J^x_p - J^x_{\bar q}) - 2G\Delta_{st}(\Delta_p - \Delta_{\bar q}) \tag{31}$$

are the "new" energies of quasiparticles.

To obtain the secular equation defining the phonon spectrum we multiply eq. (30a) by $L_{p\bar q}$ and eq. (30b) by $S_{p\bar q}$ and next perform a summation over p, q. The result is as follow

$$\Phi^M = 2G\Phi^M(S_1 + S_2) \tag{32}$$

hence

$$\frac{1}{2G} = S_1 + S_2 \tag{33}$$

where

$$S_1 = \sum_{pq} \frac{(L^2_{p\bar q} + S^2_{p\bar q})D_{p\bar q}}{D^2_{p\bar q} - (\omega^M)^2}, \tag{34a}$$

$$S_2 = \sum_{i\bar{i}} \frac{(L^2_{i\bar{i}} - S^2_{i\bar{i}})\omega^M}{D^2_{i\bar{i}} - (\omega^M)^2}. \tag{34b}$$

The solution of eq. (33) define the pairing vibration phonons spectrum of the rotating nuclei. From eqs. (24), (25), (30)–(31) we can see that the phonon solutions and the quasiparticle ones depend each on other in the self–consistent manner, and the final result may be obtained by the iteration procedure. Namely, starting from standard cranking + RPA calculations then we have to solve with some values of $\rho_i, \rho_{\bar{i}}$ the (24)–(25), (30), (33) equations. The procedure have to be iteratively repeated with the new $\rho_i, \rho_{\bar{i}}$ values up to the required degree of accuracy.

4. Summary

The high–spin states of the nuclei has been described successively in the cranking model frame using the free quasiparticle approximation. Description of the excitations built on the rotational states, such as giant resonances, has been done in the RPA. The EUROBALL and GAMMASPHERE programmes promise the new experimental data in the nearest future. Therefore, detailed analysis and further improvement of the existing theoretical methods seems to be important. With this connection we have found some generalization of the quasiparticles plus RPA phonons method treating consistently quasiparticle and phonon branches of excitations. To realize the programme we propose to treat the quasiparticle and phonon amplitudes as unknown parameter of variational problem for the energy functional, assuming the trial wave function to be a vacuum of a phonon state. It is obvious, if we have to minimalize some functional depending on the many different parameters, the best solution is to solve simultaneously the system of equations which are the necessary conditions of extremum. In this way the ground state correlations has been taken into account for the cranking model. In the natural and consistent manner we obtain the dynamical part of the gap which in the self–consistent way depends on the quasiparticle and phonon amplitudes. This result encloses the result obtained in paper Ref. 13. It is necessary to underline that due to our approach the calculation scheme for cranking model needs iteration procedure where the standard cranking model plus RPA can be used as the start point. Moreover, one must solve the nonlinear system of equations for the positive and the negative signature quasiparticle states (eigenvalues and eigenfunctions) simultaneously with the RPA phonon energies and amplitudes. The simple case of the spherical nuclei (when the basis is much more restricted without cranking term) has been solved[21] recently. It was shown that such kind of correlations are important to understand the behaviour of charge transition density in ^{64}Zn.

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