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VIBRATIONAL STATES IN DEFORMED NUCLEI. CHAOS, ORDER AND INDIVIDUAL NATURE OF NUCLEI

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1. Low-Lying Vibrational States in Doubly Even Well-Deformed Nuclei

1.1. Description of the Vibrational States in the Quasiparticle-Phonon Nuclear Model

The vibrational states and gamma-ray transition rates in well-deformed double even nuclei have been calculated within the Quasiparticle-Phonon Nuclear Model ($QPNM$)[1–6]. The QPNM is used for a microscopic description of the low-spin, small-amplitude vibrational states in spherical nuclei not far from closed shells and well-deformed nuclei. The QPNM calculations were performed in nuclei with small ground state correlations. The ground state correlations increase with the collectivity of the first one-phonon states. A particle-particle interaction reduces the ground state correlation. Therefore, the energies and wave functions of many well-deformed nuclei have been calculated in the QPNM.

The QPNM Hamiltonian contains the average field of a neutron and a proton systems in a form of the axial-symmetric Woods-Saxon potential, monopole pairing, isoscalar and isovector particle-hole (ph) and particle-particle (pp) multipole interaction between quasiparticles. The procedure of calculation is the following. A canonical Bogolubov transformation is used in order to replace the particle operators by the quasiparticle ones. Then, the phonon operators $Q_{\lambda\mu i\sigma}$ are introduced and the RPA equations are solved. The phonon space is used as a QPNM basis. The RPA phonons for the $K^\pi = 0^-$ and 1^- states have been calculated in [5] with ph and pp isoscalar and isovector octupole and ph isovector dipole interactions. The RPA equation for the $K^\pi = 0^+$ states is given in [3] and for $K^\pi \neq 0^+$, 0^- and 1^- states in ref. [2,4].

The QPNM wave functions consist of one- and two- phonon terms, namely,

$$\Psi_\nu(K_0^{\pi_0}\sigma_0) = \left\{ \sum_{i_0} R_{i_0}^\nu Q^+_{\lambda_0\mu_0 i_0\sigma_0} + \sum_{\substack{\lambda_1\mu_1 i_1\sigma_1 \\ \lambda_2\mu_2 i_2\sigma_2}} \frac{(1+\delta_{\lambda_1\mu_1 i_1,\lambda_2\mu_2 i_2})^{1/2}\delta_{\sigma_1\mu_1+\sigma_2\mu_2,\sigma_0 K_0}}{2[1+\delta_{K_0,0}(1-\delta_{\mu_1,0})]^{1/2}} \right.$$
$$\left. P^\nu_{\lambda_1\mu_1 i_1,\lambda_2\mu_2 i_2} Q^+_{\lambda_1\mu_1 i_1\sigma_1} Q^+_{\lambda_2\mu_2 i_2\sigma_2} \right\} \Psi_0, \qquad (1)$$

where $\mu_0 = K_0$. The secular equation for energies E_ν has the form

$$det \parallel (\omega_{\lambda_0\mu_0 i_0} - E_\nu)\delta_{i_0,i_0'} - \sum_{(\lambda_1\mu_1 i_1)\geq(\lambda_2\mu_2 i_2)} \frac{1+\mathcal{K}^{K_0}(\lambda_1\mu_1 i_1,\lambda_2\mu_2 i_2)}{(1+\delta_{\lambda_1\mu_1 i_1,\lambda_2\mu_2 i_2})(1+\delta_{K_0,0}(1-\delta_{\mu_1,0}))}$$
$$\frac{U^{\lambda_0\mu_0 i_0}_{\lambda_1\mu_1 i_1\lambda_2\mu_2 i_2} U^{\lambda_0\mu_0 i_0'}_{\lambda_1\mu_1 i_1\lambda_2\mu_2 i_2}}{\omega_{\lambda_1\mu_1 i_1} + \omega_{\lambda_2\mu_2 i_2} + \Delta\omega(\lambda_1\mu_1 i_1\lambda_2\mu_2 i_2) - \Delta(\lambda_1\mu_1 i_1\lambda_2\mu_2 i_2) - E} \parallel = 0 \qquad (2)$$

Here $\omega_{\lambda\mu i}$ is the RPA energy, the function $\mathcal{K}^{K_0}(\lambda_1\mu_1 i_1, \lambda_2, \mu_2 i_2)$ is responsible for the effect of the Pauli principle in two-phonon terms in (1), the function $U^{\lambda_0\mu_0 i_0}_{\lambda_1\mu_1 i_1,\lambda_2\mu_2 i_2}$ describes the coupling of one- and two-phonon terms in (1); $\Delta\omega(\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2)$ is the shift of the two-phonon pole due to the Pauli principle, $\Delta(\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2)$ represents the effect of three-phonon terms added to the wave function (1) and approximately equals $-0.2\Delta(\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2)$.

The double even deformed nuclei calculated with the parameters of the Woods-Saxon potential fixed earlier. The quantum numbers of a single particle state are denoted by $q\sigma$, $\sigma = \pm 1$, q equals K^π and asymptotic quantum numbers $Nn_z\Lambda \uparrow$ at $K = \Lambda + \frac{1}{2}$ and $Nn_z\Lambda \downarrow$ at $K = \Lambda - \frac{1}{2}$.The isoscalar constants $\kappa_0^{\lambda\mu}$ of ph interactions are fixed so as to reproduce experimental energies of the first $K^\pi_{\nu=1}$ nonrotational states described by (1). The calculations were performed with the isovector constant $\kappa_1^{\lambda\mu} = -1.5\kappa_0^{\lambda\mu}$ for ph interactions and the constant $G^{\lambda\mu} = \kappa_0^{\lambda\mu}$ for pp interactions. The monopole pairing constants were fixed by pairing energies at $G^{20} = \kappa_0^{20}$. The radial dependence of the multipole interactions has the form $dV(r)/dr$, where V(r) is the central part of the Woods-Saxon potential. The phonon basis consists of ten ($i_0 = 1, 2, ...10$) phonons of each multipolarity: quadrupole $\lambda\mu = 20, 21, 22$, octupole $\lambda\mu = 30, 31, 32, 33$, hexadecapole $\lambda\mu = 43, 44$ and $\lambda\mu = 54, 55$.

1.2. General Properties of the Vibrational States in Double Even Well-Deformed Nuclei

The energy and wave function of low-lying nonrotational states in doubly even well-deformed nuclei are mainly determined by the single-particle energies and wave functions of the Woods-Saxon potential, monopole pairing and isoscalar ph multipole interactions. A role of the isovector ph multipole interaction is small. The inclusion of the pp multipole interactions improves the description of collective vibrational states, especially 0^+ states. The energies of several first 0^+ pole of the RPA secular equation change with G^{20}. At $G^{20} = \kappa_0^{20}$ the B(E2) values for excitation of the $I^\pi K_\nu = 2^+0_1$ state and the energies of the 0_2^+ and 0_3^+ states decrease and the wave functions of the 0_1^+, 0_2^+ and 0_3^+ states change in comparison with $G^{20} = 0$. The influence of a quadrupole pairing is insignificant. A role of the isovector dipole ph interaction in description of the $K^\pi = 0^-$ and 1^- states is very important. With inclusion of the isovector ph spin-multipole magnetic interactions the energies, $B(E\lambda)$ values and the largest two-quasiparticle components of the wave function of one-phonon states change slightly [6].

General properties of the nonrotational states in well-deformed nuclei are the following:

1) The anharmonicity of vibrational states with energy below 2 MeV is small. The contribution of a one-phonon component to normalization of the wave function exceeds 90%. Small anharmonicity of the low-lying vibrational states is due to two factors: first, numerical values of the $U^{\lambda_0\mu_0 i_0}_{\lambda_1\mu_1 i_1,\lambda_2\mu_2 i_2}$ range from 0.01 to 0.30 MeV, i.e., one or two orders of magnitude smaller than in spherical open-shell nuclei, and second, the shift $\Delta\omega(\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2)$ results in that energies of two-phonon poles in eq.(2) become larger than 2.3 MeV.

2) The contribution of the two-phonon configuration to the wave function of states with excitation energy below 2 MeV equals (1-30)%. In our previous calculations [2], the shift $\Delta\omega(\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2)$ at $\lambda_1 = \lambda_2, \mu_1 = \mu_2$ and $i_1 = i_2$ for the $K^\pi = 2^+$ and 4^+ states was twice larger by mistake. The wave functions of the $K^\pi = 4^+$ states change

Table 1: Energies and Structure of the first $K^{\pi}_{\nu} = 4^{+}_{1}$ states and $B(E2; 4^{+}4_{1} \to 2^{+}2_{1})$ values.

Nuclei	$K^{\pi}_{\nu} = 4^{+}_{1}$ state exp. E_{ν_1} MeV	calc. E_{ν_1} MeV	Structure	$B(E2; 4^{+}4_{1} \to 2^{+}2_{1}), e^{2}, fm^{4}$ exp.	[ref.]	calc.
^{168}Er	2.055	2.0	441 60% { 221, 221}30%	280 ± 140	[8]	
				90 ± 60	[9]	175
				315	[10]	
^{162}Dy	1.536	1.5	441 97% {221,221} 2.3%	17	[11]	23
^{158}Gd	1.381	1.4	441 96% {221,221} 2%	-		50
^{156}Gd	1.511	1.5	441 96% {221,221} 2%	-		64

strongly. The contribution of the hexadecapole 441 one-phonon and double-gamma vibrational {221,221} components to the normalization of the wave function of the 4^{+}_{1} state and $B(E2; 4^{+}4_{1} \to 2^{+}2_{1})$ values are given in Table 1.

The largest {221,221} components in the 4^{+}_{1} state should be in ^{164}Dy and ^{168}Er. According to our definition, a state is the two-phonon one if a contribution of a two-phonon component to the normalization of the wave function exceeds 50%. The available experimental data do not contradict the conclusion on the absence of collective two-phonon states in well-deformed nuclei. The existence of two-phonon states is possibly expected in transitional nuclei and in nuclei lying close to the boundary *of the region of deformed nuclei.*

3) The structure of the 0^{+} states is very complex. The RPA wave function is a superposition of a great number of two-quasiparticle configurations. The first 0^{+}_{1} states in several rare-earth nuclei cannot be interpreted as beta-vibrational states due a small B(E2) value of transition to the ground state band. The dominance of the E2 reduced transition probability from 0^{+}_{1} states to the $K^{\pi}_{\nu} = 2^{+}_{1}$ state over that to the ground

Table 2: Comparison between experimental and theoretical $B(E\lambda)$ and $B(M\lambda)$ values for gamma-ray transitions between excited states.

Nuclei	Initial state $I^\pi K_\nu$	E_ν MeV	$E\lambda$ or $M\lambda$	Final state $I^\pi K_\nu$	E_ν MeV	$B(E\lambda)$, e^2fm^2 or $B(M\lambda)$, $\mu_N^2 fm^{2\lambda-2}$ exp.	[ref.]	calc.
^{168}Er	4^-4_1	1.094	M2	2^+2_1	0.821	0.42	[12]	0.6
	3^-3_1	1.542	E1	2^+2_1	0.821	$4\cdot10^{-5}$	[12]	$6\cdot10^{-5}$
	3^-3_1	1.542	M1	4^-4_1	1.094	$3\cdot10^{-2}$	[12]	$\cdot10^{-3}$
	3^-3_3	1.999	M1	3^-3_1	1.542	$6\cdot10^{-4}$	[12]	$8\cdot10^{-5}$
	3^-3_3	1.999	M1	2^-2_1	1.569	$2\cdot10^{-4}$	[12]	$2\cdot10^{-2}$
	4^+4_1	2.055	E1	4^-4_1	1.094	$6\cdot10^{-4}$	[13]	$8\cdot10^{-4}$
^{162}Dy	2^-2_1	1.148	E1	2^+2_1	0.888	$1\cdot10^{-4}$	[14]	$3\cdot10^{-3}$
^{158}Gd	2^+0_2	1.517	E2	$0^+0_{g.s.}$	0	18.7	[15]	10
	2^+0_2	1.517	E1	1^-1_1	0.977	$5\cdot10^{-4}$	[15]	$2\cdot10^{-5}$
	2^+0_2	1.517	E1	1^-0_1	1.263	$3\cdot10^{-4}$	[15]	$5\cdot10^{-5}$
^{156}Gd	2^+0_3	1.771	E1	1^-1_1	1.242	$5\cdot10^{-5}$	[16]	$4\cdot10^{-5}$
	2^+0_3	1.771	E1	1^-0_1	1.366	$1.6\cdot10^{-4}$	[16]	$\cdot10^{-4}$
	2^-2_1	1.780	E1	2^+2_1	1.154	$18\cdot10^{-4}$	[16]	$16\cdot10^{-4}$
	2^-2_1	1.780	M1	2^-1_1	1.320	$8\cdot10^{-3}$	[16]	$20\cdot10^{-3}$

state band in $^{16'}Dy$ and ^{168}Er was described within QPNM.

4) The gamma-ray transition rates between different bands gives the new and very important information on the nuclear structure in addition to that from the inelastic scattering, Coulomb excitation, one- and two-nucleon transfer reactions and β-decays. The results of the calculation of the gamma-ray transition rates between excited states are demonstrated in Table 2.

5) Collective vibrational states are not limited by quadrupole and octupole ones.

Table 3: Total sum of the $B(E1; 0^+0_{g.s} \to 1^- K)$ values (in $e^2 fm^2 10^{-3}$) of transition to 1^- states in ^{168}Er with energies up to 4 MeV.

	$\kappa_0^{3K} = 0$	$\kappa_0^{3K} = 0.0167$ fm^{2}MeV^{-1}		
	$\kappa_1^{1K} = 0$	$\kappa_1^{1K} = 0$	$\kappa_1^{1K} = -1.5\kappa_0^{3K}$	$\kappa_1^{1K} = -2\kappa_0^{3K}$
$\sum_i B(E1; 0^+0_{g.s} \to 1^-0_i)$	43	5090	288	167
$\sum_i B(E1; 0^+0_{g.s} \to 1^-1_i)$	12	1180	92	53
$\sum_{K=0,1} \sum_i B(E1; 0^+0_{g.s} \to 1^-K_i)$	55	6270	380	220

There are hexadecapole collective states. In some cases [17], the multipole interactions with $\lambda = 5 - 9$ lead to the mixing of the two-quasiproton and two-quasineutron configurations in the states with large K.

6) The fragmentation of one-phonon states increases with excitation energy. In the energy range from 2 to 4 MeV the fragmentation of one-phonon states is not strong. Therefore, the states with large E1, M1, E2, M2, E3 and M3 strength can be observed experimentally [4–7].

1.3. The Origin and Distribution of the E1 Strength over Low-Lying States in Well-Deformed Nuclei

The origin of the E1 strength in the low-energy region has been investigated in [5]. The isoscalar and isovector ph and pp octupole and isovector ph dipole interactions between quasiparticles were taken into account. It is known that there are no one-phonon 1^- states below the particle threshold in spherical nuclei. The quadrupole deformation is responsible for splitting the subshells of the spherical basis into twice-degenerate single-particle states. Due to this splitting, a part of the E1 strength is shifted to low-lying states. The total sum of the B(E1) values for transition to the 1^- states in ^{168}Er with energies below 4MeV is demonstrated in Table 3. The E1 strength rapidly increases at energies above 5MeV.

The octupole isoscalar interaction between quasiparticles led to formation of the collective octupole states. Due to an octupole interaction, the sum of the B(E1) values for the transitions to the $K^\pi = 0^-$ and 1^- states in the (0-4) MeV energy region increases from $55 \cdot 10^{-3} e^2 fm^2$ to $6270 \cdot 10^{-3} e^2 fm^2$. The isovector dipole ph interaction shifts the largest part of the E1 strength from the low-lying states to the region of the isovector GDR. According to the calculation in [5], the E1 strength in the energy region (0-4) MeV decreases from $6270 \cdot 10^{-3} e^2 fm^2$ at $\kappa_1^{1K} = 0$ to $380 \cdot 10^{-3} e^2 fm^2$ at $\kappa_1^{1K} = -1.5\kappa_0^{3K}$ and to $220 \cdot 10^{-3} e^2 fm^2$ at $\kappa_1^{1K} = -2\kappa_0^{3K}$. The calculation with $\kappa_1^{1K} = -1.5\kappa_0^{3K}$ correctly describes the GDR.

It is possible to state that the origin of the E1 strength in deformed nuclei is connected with the quadrupole equilibrium deformation and octupole isoscalar interaction between quasiparticles. The isovector dipole interaction is responsible for the shift of the most part of the E1 strength from the low-energy region to the GDR region. The quasiparticle-phonon interactions do not lead to the shift of the E1 strength from the GDR to the 1^- states below 4 MeV. Our interpretation of the origin of the E1 strength in low-lying states in the well-deformed doubly even nuclei is different from ones discussed in [18] in terms of the admixture of the GDR to the low-lying 1^- octupole states. Our interpretation is also different from the interpretation due to reflection asymmetric shapes like octupole deformation or cluster configuration discussed in ref [19].

The B(E1) values for the excitation of the first $K_\nu^\pi = 0_1^-$ in ^{156}Gd, $^{162,164}Dy$, ^{168}Er and ^{172}Yb, calculated with the constant $\kappa_1^{10} = -1.5\kappa_0^{30}$ and effective charge $e_{eff}^{(1)}(p) = N/A$ and $e_{eff}^{(1)}(n) = -Z/A$, are 3-5 times as large as experimental ones [19,20]. The total E1 strength for the excitation of the $K^\pi = 0^-$ states is 3-4 times as large as that for excitation of $K^\pi = 1^-$ states in the low-energy region. The strong correlation takes place between the B(E1) and B(E3) values for transitions to the same band. No correlation was observed between the B(E1) and B(M2) values.

One-phonon states with energies below 2.5 MeV are slightly fragmented due to quasiparticle-phonon interaction. The fragmentation of the one-phonon states is not so strong in the energy range 2.5-4.0 MeV. The B(E1) values for the excitation of several $1^- K$ states are relatively large and they can be observed experimentally. In ref [5], the concentration of the E1 strength in the $K^\pi = 0^-$ states at energies 2.6-3.5 MeV in ^{168}Er and 3.6-3.9 MeV in ^{164}Dy has been predicted.

2. Order, Chaos and Individual Nature of Nuclei

2.1. Order and Chaos in Terms of Nuclear Wave Functions

A nuclear excited state is characterized by angular momentum I, parity π, other quantum numbers, energy and a wave function. Much attention has been paid to an interplay between order and chaos in nuclei [21]. Studies concerning the nearest-neighbour level spacing distribution in nuclei have usually identified chaos via agreement with Gaussian Orthogonal Ensemble (GOE) statistics [21]. The nuclear wave function of an excited state with energy more than 2-4 MeV has many components with a different number of quasiparticles, with different K quantum numbers, with isospin quantum numbers T_0 and $T_0 + 1$ and so on. Such wave functions are superpositions of several interacting GOE spectra. Therefore, the GOE of level-spacing distribution cannot prove that the nuclear structure is chaotic. In ref [22], this has been demonstrated using a simple soluble model in which the appearance of a GOE-type distribution function for the nearest-neighbour level spacing does not directly correspond to a dissolution of the quantum numbers associated with the model. Therefore, it is necessary to investigate an order and a chaos in nuclei and the order-to-chaos transition in terms of properties of the nuclear wave functions [23].

The purpose of this part of the paper is to discuss the regularity in atomic nuclei and the order-to-chaos transitions is terms of nuclear wave functions.

The nuclear mean field is responsible for the order. The residual interaction plays a two-fold role: 1) The superconducting pairing interaction stabilizes the regularity of the nuclear mean field. The coherent interaction between quasiparticles leads to the formation of low-lying vibrational states and giant resonances generating regularity in the nuclei. 2) The quasiparticle-phonon interaction leads to the fragmentation of quasiparticle and phonon states. It generates the chaos in the nuclei.

To simplify the problem as much as possible, we confine ourselves to the low-spin bound and quasibound stationary nonrotational states of rigid nuclei. We treat all nonrotational states as small-amplitude collective or weakly collective vibrational or quasiparticle states. The ground, low-lying and high-lying nuclear states are very complex. We find a representation in which several states are described in the simplest way, though the wave functions of other states are very complex. If the density matrix is diagonal in the Hartree-Fock-Bogolubov approximation, then the average nuclear field and superconducting pairing interactions can be separated [24]. A representation is usually used in which the density matrix is diagonal for the ground states of the doubly-closed shell or well-deformed nuclei. This is the mean field representation. In the mean field representation, the wave function of an excited state can be written as an expansion of a number of many-quasiparticle and many-phonon operators. In this representation, there is a hierarchy of the components of the wave function with different numbers of quasiparticles. According to [25,26], the wave function of an excited state with a fixed angular momentum and the parity of a doubly even-mass nucleus has the following form:

$$\Psi_n(J^\pi) = \{\sum_{12} b^n_{12}\alpha^+_1\alpha^+_2 + \sum_a b^n_a Q^+_a + \sum_{1234} b^n_{1234}\alpha^+_1\alpha^+_2\alpha^+_3\alpha^+_4 + \sum_{12a} b^n_{12a}\alpha^+_1\alpha^+_2 Q^+_a + \sum_{aa'} b^n_{aa'}Q^+_a Q^+_{a'} + \sum_{123456} b^n_{123456}\alpha^+_1\alpha^+_2\alpha^+_3\alpha^+_4\alpha^+_5\alpha^+_6 + \cdots\}\Psi_0 \quad (3)$$

Here $| b^n |^2$ defines the contribution of the corresponding quasiparticle or phonon or quasiparticle-phonon component to the normalization of the wave function (3). A highly excited state can be occupied through one component of the wave function (3) and decays through another component of the wave function. In this case, the excitation of a state is independent of its decay. In this consideration, there is no sense in investigating how a complex state is formed from a simple one.

The wave function (3) consists of the phonon operators. Several high-spin isomers cannot be treated as pure many-quasiparticle states. The decays of an isomeric state with a mean life of $4.5\mu s$ and spin $65/2^-$ in ^{213}Fr [27] and of on isomeric state with $34\mu s$ and a spin 34^+ in ^{212}Fr [28] demonstrate a very large E3 transition strength, namely $B(E3) = 20 - 60$ s.p.u. Therefore, several excited states in ^{213}Fr and ^{212}Fr should be treated as many-quasiparticle- octupole phonon states. The two-quasiparticle and two-phonon octupole excitation 12^+ at 3.981 MeV in ^{148}Gd has been observed in [29]. Due to the Pauli principle the phonon operators are destroyed by many-quasiparticle operators in the wave functions. These experimental data have shown that phonons survived among many-quasiparticle configurations. Therefore, the wave function (3) should consist of the phonon operators. This wave function can be used for a treatment of the excited states whose life time is much longer than the internal equilibration time.

In ref [23], it is stated that there is order in the large and chaos in the small quasiparticle or phonon or quasiparticle-phonon components of the nuclear wave functions. The available experimental data on the large components of the wave function of low-lying, isobaric analog states as well as high-spin many-quasiparticle isomers have demonstrated a regularity in nuclei.

Practically, there are no experimental data on the small components of the wave functions of the low-lying states. The experimental values of the reduced neutron and partial radiative widths were used in [25,26] to estimate the average values of the one- and two-quasiparticle components of the wave functions of the neutron resonances. For nuclei in the region $50 < A < 250$ they were found to be $|\bar{b}|^2 = 10^{-5} - 10^{-8}$. The small components of the wave function manifest themselves in the distribution function of partial widths for the transition from a neutron resonance to few-quasiparticle components of the wave function of the low-lying state. The distribution of the partial radiative widths of the neutron resonances is in good agreement with the GOE statistics. This shows that the one- or two-quasiparticle components of the wave functions of a neutron resonance have a chaotic character. This distribution, however, does not contain any information concerning the entire wave function.

We consider the transition from order to chaos as a transition from large to small components of the nuclear wave function. It is important to analyze how the nuclear wave function changes with increasing excitation energy. Therefore, it is needed to investigate the fragmentation (strength distribution) of the few- and many-quasiparticle and quasiparticle-phonon configurations. In our consideration, if all quasiparticle and phonon components of a wave function are small, it means chaos. Fluctuation properties, generic to all systems that show chaos, are independent of the specific properties of the system. In this case, one does not need to study such excited states. It is highly desirable to establish the excitation energy limit for the order-to-chaos transition as a function of the nuclear mass. Our treatment is different from ones which used a statement that if the classical system is nonintegrable, its quantum correspondent shows chaos. It is possible to state that none of physical problems can be solved mathematically rigorously. Only simple models are integrable. From this point of view, all physical problems are chaotic.

2.2. Fragmentation of Few and Many-Quasiparticles and Quasiparticle-Phonon States

There is experimental information that the wave functions of the low-lying states have one dominating one-quasiparticle or one-phonon component. They demonstrate the regularity in nuclei. A reasonably good description for the low-lying states has been obtained by means of the dominant component alone. The low-lying states show individuality.

With increasing excitation energy, the structure of the states becomes more complex and the wave function (3) has several relatively large components; the domination of the single component decreases. The fragmentation of the one-quasiparticle component increases with the excitation energy. Experimental investigations on the fragmentation of the one-quasiparticle states in spherical nuclei have shown [39] that pronounced maxima of the strength distribution take place up to an excitation energy of 10 MeV. This means that one-quasiparticle states with a large angular momentum lying in a region rather far from the Fermi surface are not fully fragmented. The fragmentation of one-quasiparticle states in spherical nuclei has been described within the QPNM with the wave function containing quasiparticle, quasiparticle-phonon and quasiparticle-two-phonon components [2,30–32]. The role of the quasiparticle-phonon interaction increases with the excitation energy. The structure of the nuclear states becomes more complex and the contribution of few-quasiparticle components to the wave function strongly decreases with the excitation energy. The wave function of the states with energies greater than 3-4 MeV are superpositions of many terms with different numbers of quasiparticles and phonons.

In [33], it is stated that the broad peaks in the one-nucleon transfer reactions on the targets $^{207,208}Pb$, ^{209}Bi, ^{89}Y and $^{90,91}Zr$ at excitation energies of the giant quadrupole resonances are due to the excitation of one-quasiparticle states. It is highly desirable to establish the excitation energy limit for complete damping of one-quasiparticle states as a function of the target mass. Usually [34], such a damping of one-quasiparticle states is considered as a transition to chaos. It is not true. The many-quasiparticle configurations can give a large contribution to its wave function.

The study of the fragmentation of the three-five-quasiparticle states and quasiparticle-phonon and two-phonon states is the next step in investigating an order-to-chaos transition. The fragmentation of three-quasiparticle states in ^{175}Yb, $^{175,177}Lu$, ^{179}Hf and $^{179,181}Ta$ can be investigated in (d,p), (d,t) and (t,α) reactions on the doubly-odd targets ^{176}Lu and ^{180}Ta. The fragmentation of the five-quasiparticle states in ^{177}Lu and $^{177,179}Hf$ can be studied in a one-nucleon transfer reaction on the long-lived isomer [35] $^{178m2}Hf$ with $K^{\pi} = 16^{+}$. The energy centroids of the three- and five-quasiparticle states in these nuclei have been calculated in [36]. Experimental data on the three-quasiparticle states and the energy centroids of the three and five-quasiparticle states excited in ^{176}Lu (d,p) and $^{178m2}Hf$ (t,α) reactions are given in Fig 1. Using these reactions, it is possible to study the fragmentation in ^{177}Lu of the three-quasiparticle states in the energy range 2-4 MeV and the five-quasiparticle states in the energy range 4-6 MeV.

Information on the fragmentation of the few-quasiparticles-phonon configurations can be obtained from gamma-decays of high-spin isomers. For example, the gamma-ray de-excitations of $65/2^{-}$ isomer in ^{213}Fr [27] and 34^{-} isomer in ^{212}Fr [28] are demonstrated as fragmentation of several quasiparticle- octupole-phonon states. Several states arising from three- and five-quasiparticle configurations have been indentified in ^{143}Nd via $^{130}Te(^{18}O,5n)$ reaction [37]. Partly fragmented two-quasiparticle-two-phonon states in ^{148}Gd have been observed in [29].

2.3. On the K-Dependence in the Gamma-Decay of Neutron Resonances

It is interesting to study how far in the excitation energy K can be considered as a good quantum number. For this aim, it has been proposed in ref [26] to analyse both K-allowed and K-forbidden gamma-ray transition rates from the neutron resonance region to the low-lying states in well-deformed nuclei. In that paper, the K mixing has been studied by considering the decay of the $I^{\pi} = 13/2^{-}$ and $11/2^{-}$ neutron resonances in compound nuclei ^{177}Lu. The decay properties of the neutron resonances in ^{168}Er and ^{178}Hf have been investigated in [38]. It was shown that the intensities of gamma ray transitions from neutron-resonance states depend on the K values of the final states. In paper [39], it has been claimed that the results in [37] are in conflict with the statistical model. The detailed analysis, provided in ref [40], confirms the conclusion of ref [38].

In [23], it has been stated that the investigation of gamma transitions from a neutron resonance states to low-lying states gives information concerning K-mixing of the two- and four-quasiparticle components of the wave function of the neutron resonances but do not give information on the whole wave function. Using the thermal-neutron capture data it is possible to get information on the K-mixing for the tails of the two- and four-quasiparticle components of the neutron resonance wave function at the energy outside the levels.

For investigation of the K-mixing it is necessary to take into account the quasiparticle selection rules for matrix elements of the Eλ and Mλ K-allowed transitions. We demonstrate how important is the quasiparticle selection rule for the analysis of the K-mixing of the transition rates for gamma-decay of the neutron capture states in ^{168}Er. The ground state of the target nucleus ^{167}Er has $K^{\pi} = 7/2 + 633\uparrow$. Hence, a large number of resonances with $I^{\pi} = 3^{+}$ and 4^{+} is populated in ^{168}Er due to

Table 4: The centroids of the distribution of x-value for K-forbidden transitions $< x >_F$ and for K-allowed $< x >_A$ and K-and quasiparticle selection rules allowed transitions $< x >_{A(q)}$ from thermal capture and ARC-2 keV neutron capture states in ^{168}Er.

	$< x >_F$	$< x >_A$	$< x >_{A(q)}$
Thermal capture	0.76	1.10	1.63
ARC-2 keV	0.97	1.03	1.13

the dominance of s-wave neutron capture. From the relatively large $S_0 = 1.8 \pm 0.2$ value, it follows that the wave functions of the neutron resonances in ^{168}Er have the two-quasineutron component nn 633 ↑ ±651 ↓. We consider E1- and M1- transitions from the component nn 633 ↑ ±651 ↓ of the thermal and ARC 2 keV neutron capture states to the two-quasineutron components of the wave function of the final low-lying states. Only the two-quasineutron components contributing, according to [7], more than 20% to the normalization of the wave function of the final state are taken into account.

We have used averaged energy-corrected transition intensities in ^{168}Er, presented in Table 1 in [40], for investigation of the K-values mixing. Following [40], the energy-corrected intensity of each individual transition is represented by the ratio

$$x_i = \frac{I_i(I^\pi)}{\bar{I}(I^\pi)}, \bar{I}(I^\pi) = N^{-1}\sum_i I_i(I^\pi), I_i(I^\pi) = I_{\gamma i}(I^\pi)E_{\gamma i}^{-5},$$

where the sum is taken over all transitions $I_i(I^\pi)$ in the ensemble. The distribution of x-values has been obtained in [40] for K-forbidden $< x >_F$ and K-allowed $< x >_A$ transitions. We select from Table 1 in [40] only such K-allowed transitions which satisfy the quasiparticle selection rule. We obtained the distribution of x-values for the quasiparticle selection rule allowed $< x >_{A_{(q)}}$ transitions. The $< x >_F$ and $< x >_A$ values from [40] and the $< x >_{A_{(q)}}$ values from our consideration are presented in Table 4.

For transitions from the thermal and 2 keV ARC neutron capture states to both parity low-lying bands we obtain $< x >_{A_{(q)}} = 1.63$ and $< x >_{A_{(q)}} = 1.13$, respectively. The difference between these transitions is mostly due to that the M1 transitions from 2 keV ARC neutron capture states to the $K^{\pi}_{\nu} = 2^{+}_{5}$ band are not observed. As it is shown in Table 4, these values are much larger than $< x >_F = 0.76$ and $< x >_F = 0.97$ for K-forbidden transitions from the thermal and 2 keV ARC neutron capture states. The K-allowed transitions satisfying quasiparticle selection rule are much faster than K-forbidden transitions. It means that the incomplete mixing of the two-quasiparticle components of the wave function with different K quantum number takes place in the neutron resonance region.

The quasiparticle selection rule allowed gamma transition from the neutron resonance states occupiés the same component of the wave function of the low-lying state which is excited in the (d,p) reaction. Therefore, for investigation of the K quantum number in a neutron resonance region it is needed to measure the intensities of the gamma-ray transition from the neutron capture and the cross section (d,p) reaction. The (n,γ) and (d,p) reactions in ^{176}Lu for example should be studied.

The nuclear resonance fluorescence measurements of well-deformed nuclei [41] show that the experimental branching ratio $B(J = 1 \rightarrow 2^{+}0_{g.s.})/B(J = 1 \rightarrow 0^{+}0_{g.s.})$ is in good agreement with the Alaga rules for most of the excited J=1 levels. It means that K is a rather good quantum number for many states with energy below 4 MeV in well-deformed nuclei.

It is well known that the GDR in deformed nuclei has the lower and higher parts. In agreement with the calculations, a summed strength of the lower part is twice smaller than the higher part. According to the calculations, the lower part has K=0 and higher part K=1. The gamma-decay modes of the GDR in ^{150}Nd have been investigated in [42] via inelastic phonon scattering. It was shown that $d\sigma/d\Omega$ practically equals zero for K-forbidden transitions from the lower part of the GDR and $d\sigma/d\Omega \approx (5 - 10)$ μb/str for K-allowed transitions from the higher part of the GDR to the $K^{\pi} = 2^{+}$ final state. The $d\sigma/d\Omega$ for K-allowed transitions to the $K^{\pi} = 0^{+}$ excited state from the higher part of the GDR is twice larger than from the lower part of the GDR. It means that the K mixing is incomplete in the particle-hole configuration in the GDR region.

2.4. Neutron Resonances as a Key for Studying Order-to-Chaos Transitions

The order-to-chaos transition in terms of nuclear wave functions was formulated in [26,43] in 1972 as "are there relatively large many-quasiparticle components in the wave function of neutron resonances?" The one-quasiparticle configuration with a relatively large angular momentum of spherical nuclei at an excitation energy of 6-8 MeV is not fully fragmented. The one- and three- or two- and four-quasiparticle configurations at excitation energies close to the neutron binding energy are strongly fragmented. At these energies, the five- and seven- or six- and eight-quasiparticle configurations start to fragment. We can expect that the wave function of the neutron resonance states contains large components of many-quasiparticle configurations.

Practically, no experimental data exist concerning the many-quasiparticle components of the wave function of the highly excited low-spin states. What experiments should be performed to answer the question concerning the existence of the large many-quasiparticle components of the wave function of the highly excited states? In [43], it has been suggested that the most favourable way to observe the many-quasiparticle components of the wave function is to study the gamma-transition from the neutron resonance states to the states lying $1 \div 2$ MeV below them.

Some information concerning the values of the many-quasiparticle components can be obtained by studying the E1 and M1 transition probabilities from the neutron resonance states to the levels with energies lower than the neutron resonance energy by $1 \div 2$ MeV. A large contribution of the many-quasiparticle configuration to the normalization of the neutron resonance wave function would enhance in E1 and M1 transitions. If the contribution of the many-quasiparticle component to the normalization of the neutron resonance wave function is equal to 20%, the corresponding reduced gamma-transition probabilities are $3 \div 4$ orders of magnitude larger than the reduced gamma-transition probabilities from the neutron resonance states to the low-lying states. The enhancement of E2 and E3 transition rates between excited states means that there are large few-quasiparticle-quadrupole or octupole terms in the wave function of the initial state.

Coincidence measurements of gamma ray emitted after thermal neutron capture in ^{155}Gd have been performed in [44]. A pronounced local maximum of intensity of the primary gamma ray at 2.5 MeV has been observed. This maximum can be treated as an enhanced gamma ray transition between the many-quasiparticle components of a capture state and excited states with energy 5.5-6.5 MeV in ^{156}Gd. In this case, the neutron resonance state is very close to the neutron binding energy. Therefore, it is possible to use the (n_{th}, γ) reaction for detecting the large many-quasiparticle components of the wave function of states with energy 5.5-8.5 MeV.

The s- and p- wave neutron strength functions demonstrate the individuality of nuclei in the energy region of the neutron resonance states. These average values of the one- or two-quasiparticle components of the wave function of neutron resonances reflect a regularity of the nuclear mean field. The thermal and average resonance neutron capture cross sections manifest individuality of nuclei. But these experimental data involve only the 10^{-6} part of the neutron-resonance wave functions. It is possible to say that the state, whose largest component is described by a single many-quasiparticle configuration being more than 10-20%, has its own individual characteristic feature.

2.5. Conclusions

The above consideration allowed us to derive the following conclusions:

1. The order is governed by the large components of the wave function of the excited states.

2. Chaos takes place in the small components of the wave function of the nuclear excited states. The excited state is chaotic if its wave function is composed of only small components of few and many-quasiparticle or few and many-phonon configurations.

3. It is possible to consider the order-to-chaos transition as a transition from the large to the small components of the nuclear wave function.

4. The experimental investigation of a fragmentation of the many-quasiparticle and quasiparticle-phonon states play a decisive role in studying the order-to-chaos transitions.

5. It is important to continue an experimental study of the K-dependence in the gamma-decay of neutron resonance states. The quasiparticle selection rule should be taken into account.

6. An experimental investigation of the many-quasiparticle components of the wave function of the neutron resonance states may be carried out using the new generation of gamma-ray detectors which could observe the enhanced gamma-transition from neutron resonances to the levels lying $(1 \div 2)$ MeV below them.

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