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RADIATIVE AND RARE μ AND π DECAYS

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The radiative decay of polarized muon is investigated in order to search for possible deviations from the Standard Model (SM) predictions.

Assuming the $\mu\nu_\mu W$ vertex as a linear combination of vector and axial currents and a pure V-A interaction for the electron as a minimal expansion of SM

$$\mathcal{L}^{\mu}_{c.c.} = \frac{g}{2\sqrt{2}} W^{+}_{\alpha} \bar{\Psi}_{\nu_\mu} \gamma^{\alpha} (g_V - \gamma^5 g_A) \Psi_\mu + h.c., \quad (1)$$

one may obtain in the case of polarized muon

$$u_\mu(p,s)\bar{u}_\mu(p,s) = (\hat{p} + m)(1 - \gamma^5 \hat{s}), \qquad (ps) = 0 \quad (2)$$

the differential width:

$$\frac{\pi d\Gamma^{\mu\to\nu\bar{\nu}e}}{\Gamma_0 dy d\Omega} = 3y^2(1-y) + 2\rho y^2(\frac{4}{3}y - 1) \quad (3)$$
$$-\vec{n}\vec{s}y^2[3(1-y) + \frac{2}{3}\rho(8y-7)],$$

$$y = \frac{2\varepsilon_e}{m_\mu}, \qquad \vec{n} = \frac{\vec{p}_e}{|\vec{p}_e|}, \qquad \Gamma_0 = \frac{G^2 m_\mu^5}{192\pi^3}(g_{LL}^2 + g_{LR}^2),$$

where ρ is the Michel parameter,

$$\rho = \frac{3}{4}\left(1 + \frac{g_{LR}^2}{g_{LL}^2}\right)^{-1}, \qquad g_{LL} = \frac{g_V - g_A}{2}, \qquad g_{LR} = \frac{g_V + g_A}{2}. \quad (4)$$

In SM we have $g_{LR} = 1$ and $g_{LR} = 0$. Our result differs from the one presented in the review by Pich [1], and for $\rho = 3/4$ coincides with the result given in the book by Okun [2].

Traditionally in experiments in order to search for right currents the kinematical region $\vec{n}\vec{s} \approx 1$ is considered. To extract their influence one has to take into account radiative corrections. The first part of this paper is devoted to their calculation in the case of polarized muon. We present the contributions from an emission of virtual and real (soft and hard) photons. Some distributions are given. We note a disagreement with the results of T. Kinoshita and A. Sirlin [3].

The second part is devoted to calculations of radiative corrections of the pion β-decay. The width of this channel in the Born approximation

$$\Gamma_0 = \frac{G^2\Delta^5}{30\pi^3}|V_{ud}|^2(1 - \frac{3}{2}\frac{\Delta}{m_\pi} - \frac{5m_e^2}{\Delta^2}), \quad (5)$$
$$\Delta = m_{\pi^+} - m_{\pi^0} \approx 4.5 Mev$$

is planned to extract the Kobayashi-Moskava matrix element $|V_{ud}|$. The knowledge of $|V_{ud}|$ with the accuracy $\sim 0.1\%$ is essential for the solution of the question about a possible fourth generation of quarks. We note that in the theory with point-like propagator of W-boson some ultraviolet divergencies appear. The cut-off parameter Λ has to be replaced by the mass of W-boson ($\Lambda = m_W$) (because of the fact that SM is a renormalizable theory), or by the mass of a hadron ($\Lambda = m_\rho$), which looks more preferable. Some uncertainties due to the choice of the scale parameter of the strong interaction were estimated to be no more than $O(\alpha/\pi) \le 0.2\%$. Large electroweak corrections arising from the W-boson self-energy, we note, are the same as for the muon decay width. The ratio of the π decay width and the muon decay one is free from the electroweak corrections:

$$\frac{\Gamma(\pi^+ \to \pi^0 e^+ \nu_e)}{\Gamma(\mu^+ \to \bar{\nu}_\mu e^+ \nu_e)} = \frac{|V_{ud}|^2 192}{30}(\frac{\Delta}{m_\mu})^5(1 + \frac{\alpha}{2\pi}(\pi^2 - \frac{25}{4})) \quad (6)$$

$$\times(1+\delta_\pi)(1-\frac{3\Delta}{2m_\pi}-\frac{5m_e^2}{\Delta^2}).$$

1 Radiative Polarized μ Decay

So far as our results differ from the ones obtained in [3] we will put here some intermediate formulae. The second reason for this: one can use them in a precise experiment.

After standard [2] procedures — a) the integration over neutrino momenta; b) the unification of the denominators in the Feynman loop amplitude; c) the integration over the loop momentum — one obtains for the summed over polarizations of final particles the doubled product of the Born and the loop amplitude:

$$I=2\sum M_0^*M_1=\frac{16G^2\alpha}{3}\int_0^1 2zdz\int_0^1 dx\{\frac{1}{d}(A+B\vec{n}\vec{s})$$
$$+\frac{1}{2}y(3-2y+\vec{n}\vec{s}(1-2y))(\ln\frac{\Lambda^2}{dm_\mu^2}-\frac{3}{2})\},\qquad(7)$$
$$A=-\frac{1}{2}y^2(3-2y)+\frac{1}{2}zy(y(3-2y)+3x(1-y))$$
$$+\frac{1}{2}z^2xy(1-y)(-2y+x(2y-3)),$$
$$B=-\frac{1}{2}y^2(1-2y)+\frac{1}{2}zy(y(1-2y)+x(1-y))$$
$$-\frac{1}{2}z^2xy(1-y)(2-x),$$
$$d=z^2(x-x(1-y)(1-x)+\frac{m_e^2}{m_\mu^2}(1-x))+(1-z)\frac{\lambda^2}{m_\mu^2},$$

where λ is the "photon mass", $y=2\varepsilon_e/m_\mu$ is the energy fraction of the final electron, Λ is the ultraviolet cut-off parameter.

The regularization of the expression (7) includes the renormalization constants of the free muon and electron. It results in the replacement

$$\ln\frac{\Lambda^2}{m_\mu^2}\longrightarrow-\frac{9}{2}-\frac{3}{2}\ln\frac{m_\mu^2}{m_e^2}-2\ln\frac{\lambda^2}{m_\mu^2}.\qquad(8)$$

The emission of soft photons is described by the known formula ($\Delta\varepsilon$ is the photon energy, $\Delta\varepsilon\ll m_\mu$):

$$\frac{d\Gamma^{soft}}{dyd\Omega}=\Gamma_0\frac{\alpha}{4\pi^2}(3-2y+\vec{n}\vec{s}(1-2y))\qquad(9)$$
$$[(\eta-2)2\ln\frac{2\Delta\varepsilon}{\lambda}+2+\eta$$
$$-\frac{1}{2}\eta^2-\frac{\pi^2}{3}],$$
$$\eta=L+2\ln y,\qquad L=\ln\frac{m_\mu^2}{m_e^2}.$$

In the sum of the corrections due to the emission of virtual and soft photons the "photon mass" parameter disappears:

$$\frac{d\Gamma^{soft+virt}}{dyd\Omega}=\Gamma_0\frac{\alpha}{2\pi^2}y^2\Big\{(3-2y+\vec{n}\vec{s}(1-2y))\qquad(10)$$
$$\times[(\eta-2)\ln\sigma+\frac{3}{4}\eta-\eta\ln y-1$$
$$-\frac{\pi^2}{3}+\frac{3}{2}\ln y+\ln y\ln(1-y)-\int_0^y\frac{dx}{x}\ln(1-x)],$$
$$+(y-3)\ln y+\vec{n}\vec{s}(y\ln y-y+1)\Big\},\qquad\sigma=\frac{2\Delta\varepsilon}{m_\mu}\ll 1.$$

Straightforward but slightly tedious calculation gives the distribution on the electron energy fraction in the total sum ((10)

plus hard photon emission contribution):

$$\frac{d\Gamma^{soft+virt+hard}}{dyd\Omega}=\Gamma_0\frac{\alpha}{2\pi^2}(A+B\vec{n}\vec{s}), \qquad (11)$$

$$A=y^2(3-2y)[-2+(\eta-2)\ln(1-y)-\frac{1-y}{y}\ln(1-y)$$
$$+\eta(\frac{3}{4}-\ln y)-\frac{1}{2}\ln y+\ln y\ln(1-y)-\frac{\pi^2}{3}$$
$$-2\int_0^y\frac{dx}{x}\ln(1-x)]+3y^2(1-y)\ln y$$
$$+\frac{1}{12}(1-y)[\eta(5+17y-34y^2)-44y+68y^2],$$
$$B=y^2(1-2y)[(\eta-2)\ln(1-y)+\eta(\frac{3}{4}-\ln y)-\frac{\pi^2}{3}$$
$$-2\int_0^y\frac{dx}{x}\ln(1-x)+\ln y\ln(1-y)-1+\frac{3}{2}\ln y]$$
$$-\frac{1}{12}(1-y)(1+y+34y^2)\eta+y^2(y\ln y+1-y)$$
$$-\frac{1-y}{3y}(2-4y+5y^2-6y^3)\ln(1-y)$$
$$-\frac{1}{2}+\frac{5}{3}y+\frac{19}{6}y^2-\frac{10}{3}y^3.$$

In the limit $y\to 1$ we obtain

$$\left.\frac{d\Gamma^{soft+virt+hard}}{dyd\Omega}\right|_{y\to 1}=\Gamma_0\frac{\alpha}{2\pi^2}A_0(1-\vec{n}\vec{s}), \qquad (12)$$
$$A_0=(L-2)\ln(1-y)+\frac{3}{4}L-2.$$

We compare this result with the Born one (3), with right currents involved by the Michel parameter, in the same limit

$$\left.\frac{d\Gamma^{\mu\to\nu\bar{\nu}e}}{dyd\Omega}\right|_{y\to 1}=\frac{2\rho}{3\pi}(1-\vec{n}\vec{s}). \qquad (13)$$

In this region, as we see, the radiative corrections are very important if one wants to extract a deviation of the Michel parameter ρ from 3/4.

We show here also the integrated on the energy fraction contribution from the hard photon emission region:

$$\omega>\Delta\varepsilon, \qquad \sigma=\frac{2\Delta\varepsilon}{m_\mu}\ll 1, \qquad L=\ln\frac{m_\mu^2}{m_e^2}\approx 11.33,$$
$$\frac{d\Gamma^{hard}}{d\Omega}=\Gamma_0\frac{\alpha}{2\pi^2}\{\frac{1}{2}L\ln\frac{1}{\sigma}-\frac{17}{12}\ln\frac{1}{\sigma}-\frac{7}{12}L-\frac{\pi^2}{12}+\frac{601}{288}$$
$$+\vec{n}\vec{s}(-\frac{1}{6}L\ln\frac{1}{\sigma}+\frac{13}{36}\ln\frac{1}{\sigma}+\frac{5}{36}L+\frac{5}{36}\pi^2-\frac{1213}{864})\}, \qquad (14)$$

as well as the total angular distribution:

$$\frac{d\Gamma^{virt+soft+hard}}{d\Omega}=\Gamma_0\frac{\alpha}{2\pi^2}\{-\frac{\pi^2}{4}+\frac{25}{16} \qquad (15)$$
$$+\vec{n}\vec{s}(\frac{7}{36}\pi^2-\frac{629}{432})\}.$$

For completeness we present here the distribution on the electron energy fraction in the region of the hard photon spectrum:

$$\frac{d\Gamma^{soft+virt+hard}}{dyd\Omega}=\Gamma_0\frac{\alpha}{2\pi^2}y^2\{a(y,\sigma)+b(y,\sigma)\vec{n}\vec{s}\}, \qquad (16)$$
$$\sigma=\frac{2\Delta\varepsilon}{m_\mu}\ll 1, \qquad \omega>\Delta\varepsilon$$

$$a=(3-2y)[-1+\frac{1-y}{y}\ln\frac{1}{1-y}+(\eta-2)\ln\frac{1-y}{\sigma}$$
$$-\int_0^y\frac{dx}{x}\ln(1-x)]+\frac{(1-y)}{12y^2}(5+17y-34y^2)\eta$$
$$+\frac{(1-y)}{3y}(-11+17y),$$

$$b=(1-2y)\left[(\eta-2)\ln\frac{1-y}{\sigma}-\int_0^y\frac{dx}{x}\ln(1-x)\right]$$
$$-\frac{(1-y)}{12y^2}(1+y+34y^2)\eta+\frac{(1-y)}{3y}\left(\frac{2}{y^2}-\frac{4}{y}+5\right.$$
$$\left.-6y\right)\ln\frac{1}{1-y}-\frac{1}{2y^2}+\frac{5}{3y}+\frac{19}{6}-\frac{10}{3}y.$$

We put here also the hard photon emission spectra for the unpolarized case [4]

$$\frac{d\Gamma(\mu\to e\nu\bar{\nu}\gamma)}{dz}=\frac{G^2m_\mu^5\alpha}{9\cdot512\pi^4}(1-z)\{-\frac{17}{z}+\frac{23}{3}-\frac{101}{6}z \quad (17)$$
$$+\frac{55}{6}z^2+(\frac{6}{z}-4(1-z^2))\ln\frac{m_\mu^2(1-z)}{m_e^2}\},\qquad z=\frac{2\omega}{m_\mu}.$$

2 Radiative π β-Decay

Consider now the radiative corrections (RC) for the width of the β-decay of π^+

$$\pi^+(p_1)\to\pi_0(p_2)+e_+(p_e)+\nu(p_\nu)+(\gamma(k)). \qquad (18)$$

The width in the Born approximation has the form [5]

$$\Gamma^0=\frac{G^2\Delta^5|V_{ud}|^2}{\pi^3}(1-\frac{\Delta}{2m_+})^3I(\mu), \qquad (19)$$

$$I(\mu)=\int_\mu^1 dx\,x^2(1-x)^2\beta=\frac{1}{30}[(1-\frac{9}{2}\mu^2-4\mu^4)\sqrt{1-\mu^2}$$
$$+\frac{15}{2}\mu^4\ln\frac{1+\sqrt{1-\mu^2}}{2}], \qquad (20)$$
$$\beta=\sqrt{1-\frac{\mu^2}{x^2}},\qquad \mu=\frac{m_e}{\Delta},\qquad \Delta=m_+-m_0\approx4.59MeV.$$

The differential width of the radiative π^+ β-decay has the form:

$$d\Gamma=-\frac{\alpha}{4\pi^2}\frac{d^3k}{\omega}d\Gamma_0(2\varepsilon_\nu\varepsilon_e-p_\nu p_e)^{-1} \qquad (21)$$
$$\times\{(2\varepsilon_\nu\varepsilon_e-p_\nu p_e)[\frac{1}{\omega^2}+\frac{m_e^2}{(p_ek)^2}-\frac{2\varepsilon_e}{\omega(p_ek)}]$$
$$+(2\omega\varepsilon_\nu-kp_\nu)[\frac{m_e^2}{(p_ek)^2}-\frac{\varepsilon_e+\omega}{\omega(p_ek)}]$$
$$+\frac{\varepsilon_\nu}{\omega}-\frac{2\varepsilon_\nu\varepsilon_e-p_ep_\nu}{(p_ek)}\},$$

where $d\Gamma_0$ is the differential width in the Born approximation, $\varepsilon_\nu,\varepsilon_e,\omega$ are the energies of electron, neutrino and photon respectively. We rearrange the phase space volume in (21) in the following way:

$$\int\frac{d^3k}{\omega}\frac{d^3p_e}{\varepsilon_e}\frac{d^3p_\nu}{\varepsilon_e}\frac{d^3p_e}{\varepsilon_\nu}\frac{d^3p_2}{\varepsilon_2}\delta(p_1-p_2-p_e-p_\nu-k) \qquad (22)$$
$$=\frac{1}{m}\int p_ed\varepsilon_e\varepsilon_\nu d\varepsilon_\nu kd\omega 2(2\pi)^3dc_\nu dc_\gamma\delta(\Delta-\varepsilon_e-\varepsilon_\nu-\omega)$$
$$=16\pi^3m_\pi^{-1}\Delta^5\int_\mu^1dx\int_0^{1-x}dzxz\beta\beta_zdc_\nu dc_\gamma,$$
$$\beta=\sqrt{1-\frac{\mu^2}{x^2}},\qquad \beta_z=\sqrt{1-\frac{\nu^2}{x^2}},$$
$$z=\frac{\omega}{\Delta},\qquad x=\frac{\varepsilon_\nu}{\Delta}\qquad \nu=\frac{\lambda}{\Delta}.$$

We omit in (22) the terms of order Δ/m_π compared with the terms of order unity. The following error has the magnitude

$$\frac{\alpha}{\pi}\frac{\Delta}{m_\pi}\sim10^{-4}. \qquad (23)$$

Performing the angular integration over c_ν and c_γ one obtains

$$\int d\Gamma^{\pi^+\to\pi^0 e^+\nu\gamma} = \frac{G^2\Delta^5\cos^2\Theta_c\alpha}{\pi^4}\int_\mu^1 dx\beta x^2(1-x)^2 \tag{24}$$
$$\times\Big\{\big(-2+\frac{1}{\beta}\ln\frac{1+\beta}{1-\beta}\big)\left[-2+\ln 2+\ln(1-x)+\frac{1-x}{3x}\right.$$
$$\left.+\ln\frac{\Delta}{\lambda}\right]+\frac{(1-x)^2}{24\beta x}\ln\frac{1+\beta}{1-\beta}+\int_0^1\frac{dt}{1-t^2\beta^2}\ln(1-t^2)\Big\}.$$

In order to calculate the correction due to a "virtual photon emission" we have to consider three one-loop Feynman diagrams. Two of them, which contain "contact" vertices, have the forms of a pion and a positron self-energy loop. The doubled interference of their amplitudes with the Born one (summed over final spin states) has the form:

$$2\sum M_0^*(M_a^V+M_b^V) = 24m_\pi^2(2G\cos\Theta_c)^2\left(-\frac{\alpha}{4\pi}\right)\varepsilon_e\varepsilon_\nu$$
$$\times\left[\ln\frac{\Lambda^2}{m_\pi^2}+\frac{3}{2}+\frac{m_e^2}{\varepsilon_e m_\pi}\left(\ln\frac{\Lambda^2}{m_e^2}+\frac{3}{2}\right)\right]. \tag{25}$$

The contributions from the terms in this equation which are proportional to m_e/m_π have an order of (23) and we will omit them further. The parameter Λ here is the well-known cut-off momentum.

Applying the standard procedure of denominators joining and performing a loop-momentum integration, an analogous to (25) expression for the third diagram may be obtained:

$$2\sum M_0^* M_c^V = 16m_\pi^2(2G\cos\Theta_c)^2\left(-\frac{\alpha}{4\pi}\right)\varepsilon_e\varepsilon_\nu \tag{26}$$
$$\times\left[-\frac{7}{2}\ln\frac{\Lambda^2}{m_\pi^2}-\frac{25}{6}+2\left(\ln\frac{m_\pi^2}{m_e^2}-\frac{1}{\beta}\ln\frac{1+\beta}{1-\beta}\right)\right.$$

$$+\frac{2}{\beta}\ln\frac{1+\beta}{1-\beta}\left(\ln\frac{m_\pi^2}{\lambda^2}+\ln\frac{\beta^2\Delta^2x^2}{m_\pi^2}\right) \tag{27}$$
$$\left.+\frac{4}{\beta}\int_0^\beta\frac{dt}{1-t^2}\ln\frac{1-t^2}{t^2}\right].$$

To consider the ultraviolet behavior of the amplitudes correctly, it is necessary to take into account the renormalization constants Z_e and Z_π for the pion and positron wave functions:

$$\frac{1}{\hat{p}-m}\to\frac{Z_e}{\hat{p}-m},\qquad \frac{1}{p_1^2-m_+^2}\to\frac{Z_\pi}{p_1^2-m_+^2}, \tag{28}$$

where

$$Z_e = 1+\frac{\alpha}{2\pi}\left(\ln\frac{m_e^2}{\lambda^2}-\frac{1}{2}\ln\frac{\Lambda^2}{m_e^2}-\frac{9}{4}\right); \tag{29}$$
$$Z_\pi = 1+\frac{\alpha}{2\pi}\left(\ln\frac{\Lambda^2}{m_\pi^2}+\ln\frac{m_\pi^2}{\lambda^2}-\frac{3}{4}\right).$$

The total sum of the radiative π^+ β-decay width which included the loop corrections does not contain the parameter of the "photon mass" λ:

$$\Gamma_0+\Gamma^{(1)} = \Gamma_0(1+\delta_\pi),$$
$$\Gamma_0 = (G\cos\theta_c)^2\left(1-\frac{3\Delta}{2m_\pi}\right)\frac{\Delta^5}{\pi^3}I(\mu), \tag{30}$$

$$\delta_\pi = \frac{\alpha}{\pi}\left(\int_\mu^1 dx x^2(1-x)^2\beta\right)^{-1}\int_\mu^1 dx x^2(1-x)^2\beta \tag{31}$$
$$\times\Big\{\frac{1}{\beta}\int_0^\beta\frac{dt}{1-t^2}\ln\frac{(\beta^2-t^2)t^2}{1-t^2}+\frac{1}{\beta}$$
$$\left(-\frac{3}{2}+\ln 2+\ln\frac{(1-x)}{\beta\sqrt{x}}\right.$$

$$+\frac{(1-x)}{24x}(9-x))\ln\frac{1+\beta}{1-\beta}-\frac{2(1-x)}{3x}+\frac{143}{48}-2\ln 2$$
$$-2\ln(1-x)+\ln\mu^2-\frac{1}{4}\ln\frac{m_\pi^2}{m_e^2}+\frac{3}{4}\ln\frac{\Lambda^2}{m_\pi^2}\Big\}.$$

The fact that $\Gamma^{(1)}$ contains the cut-off parameter Λ is not surprising one, due to nonrenormalizability of the old-fashioned weak interaction theory used here. A similar phenomenon was obtained in paper of Kinoshita in calculation of RC for pion decays [6]. As we noted above, the real cut-off parameter is the scale of strong interactions $\Lambda = m_\rho = 770\ Mev$. The numerical estimation δ_π with such a choice of Λ gives

$$\delta_\pi = -0.023. \qquad (32)$$

3 Background Process for π β-Decay

As a background process for π β-decay we consider the double bremsstrahlung decay of pion:

$$\pi^+(p) \to e(p_e) + \nu(p_\nu) + \gamma(k_1) + \gamma(k_2). \qquad (33)$$

As far as one measures $2\gamma e^+\nu$ final state in π β-decay

$$\pi^+ \longrightarrow e^+\ \nu_e\ \pi^0, \quad \pi^0 \hookrightarrow 2\gamma \qquad (34)$$

the process (33) may imitate the π β-decay in the case that the invariant mass 2γ is near to m_{π^0}.

The differential width of the process (33) has the form (we omit here the structure-dependent contributions compared with the inner bremsstrahlung ones):

$$d\Gamma^{\pi\to e\nu\gamma\gamma} = \frac{1}{2}\frac{(G\alpha m_e f_\pi \cos\vartheta_c)^2}{m_\pi(2\pi)^6}\frac{1}{4}Sp\ p_\nu O_{\lambda\sigma} p_e \tilde{O}_{\lambda\sigma} d\Phi, \qquad (35)$$
$$d\Phi = \frac{d^3p_e}{\epsilon_e}\frac{d^3p_\nu}{\epsilon_\nu}\frac{d^3k_1}{\omega_1}\frac{d^3k_2}{\omega_2}\delta^{(4)}(p-p_e-p_\nu-k_1-k_2),$$

$$O_{\lambda\sigma}e_1^\lambda e_2^\sigma = ((p-k_1-k_2)^2-m_\pi^2)^{-1} \qquad (36)$$
$$\times\{-\frac{2(pe_1)((pe_2)-(k_1e_2))}{(pk_1)}-\frac{2(pe_2)((pe_1)-(k_2e_1))}{(pk_2)}$$
$$-2(e_1e_2)\}+\frac{\hat{p}_e+\hat{k}_1+\hat{k}_2-m_e}{(p_e+k_1+k_2)^2-m_e^2}\{\hat{e}_1\frac{-\hat{p}_e-\hat{k}_2+m_e}{2(p_ek_2)}\hat{e}_2$$
$$+\hat{e}_2\frac{-\hat{p}_e-\hat{k}_1+m_e}{2(p_ek_1)}\hat{e}_1\}-\frac{(e_1p)}{(k_1p)}\frac{(\hat{p}_e+\hat{k}_2-m_e)e_2}{2(p_ek_2)}$$
$$\times-\frac{(e_2p)}{(k_2p)}\frac{(\hat{p}_e+\hat{k}_1-m_e)\hat{e}_1}{2(p_ek_1)}.$$

Rearranging the phase volume element as

$$\int d\Phi = \frac{4\pi^3}{m_\pi}\int_{m_\pi^2-\delta m^2}^{m_\pi^2+\delta m^2} dm^2\int_0^\Delta \varepsilon_e d\varepsilon_e\int_{-1}^{1} dc_e\int_{\frac{m}{2}-\Delta}^{\frac{m}{2}+\Delta} .d\omega_1 \qquad (37)$$

and estimating the trace in (14) as

$$\frac{1}{4}Sp\ p_\nu O p_e \widetilde{O} \approx \frac{\Delta^2}{2p_ek_12p_ek_2}, \qquad (38)$$

one obtains for the ratio R :

$$R = (\Gamma_{\beta\pi^+})^{-1}\int_{m_0^2-\delta m^2}^{m_0+\delta m^2}\frac{d\Gamma^{\pi\to e\nu\gamma\gamma}}{dm^2}dm^2$$
$$\approx\left(\frac{m_e f_\pi\alpha\ln\frac{2\Delta}{m_e}}{m_\pi\Delta}\right)^2\frac{\delta m^2}{m^2}\approx 10^{-4}\frac{\delta m^2}{m^2}. \qquad (39)$$

We see from this estimation that the background from the process (39) is negligible.

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