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DAMPING OF NUCLEAR COLLECTIVE MOTIONS
WITHIN THE METHOD OF PHASE-SPACE
MOMENTS OF THE WIGNER
DISTRIBUTION FUNCTION

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1. Introduction

The damping of the collective excitations in atomic nuclei is of greatest importance for the understanding of the energy dissipation in Fermi systems. Two damping mechanisms are known to be responsible for the conversion of the energy of the collective motion into excitation energy of intrinsic degrees of freedom. The first one is of quantum mechanical origin and has to do with the fragmentation of the transition strength among the infinite number of excitations of more simple structure which act as a basis for the collective mode. This is the "Landau" or "meanfield" damping¹⁾ and is a well established phenomenon in macroscopic Fermi liquids like ^3He or electron gas²⁾. In the RPA ("collisionless dynamics") Landau damping is combined with nucleon emission to the continuum ("escape width") [ref.³⁾].

The second damping mechanism which reflects in the so-called spreading widths arises from the coupling of the simple states to more complex configurations [refs.^{2,4)}]. In the language of the microscopic theory it means that $1p-1h$ states are coupled to more complex $2p-2h$ configurations which in turn can couple to $3p-3h$ configurations or to the continuum. The coupling to more complex states can continue till finally the energy is equally divided among the nucleons and the equilibrium is reached. Classically it looks like if the quantum particles tend to equalize their velocities by frequent collisions ("collisional dynamics") [ref.³⁾]. To describe the collisional damping it is necessary to go beyond the RPA and to take into account not only $1p-1h$ but also $2p-2h$ excitations of the static ground state (second RPA) [refs.⁴⁻⁷⁾]. The corresponding schemes are very complicated and it is difficult to obtain numerical solutions of the SRPA equations, especially in heavy nuclei. That's why several approximate procedures have been developed [refs.²⁻¹¹⁾].

In this paper we will follow a semiclassical framework to go beyond the one-body theory introducing the effect of two-body collision through a relaxation time approach used in an extended form which accounts for all the conservation laws. We will apply the procedure to the study of monopole giant resonances in ^{208}Pb .

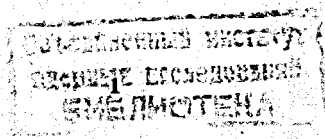
In sect.2 the semiclassical approach is shortly discussed, with particular emphasis on the extended τ -approximation.

In sect.3 the dispersion equations for monopole frequencies are derived.

Results are presented in sect.4 and finally in sect.5 some conclusions are drawn.

2. Semiclassical Collective Dynamics: the method of phase-space moments

A natural procedure to work out a semiclassical theory of nuclear collective motions is to study the Wigner transform of the density matrix. An RPA equation in phase space can be



easily derived, e.g. [ref.¹⁰] and refs. therein, and various approximate approaches have been introduced to get solutions, which in general retain many transparent classical interpretations.

The method of phase-space moments of Wigner distribution function (WDFM), developed in Dubna ^{12,13}, is quite convenient to go beyond the RPA scheme for analyzing both damping mechanisms and the interplay between them.

To take into account the spreading widths of the collective excitations we want to describe in our scheme, it is necessary to add to the right hand side of the basic equation for the density matrix the correlation term ¹³). It's Wigner transform describes the changes in the distribution function $f(\vec{r}, \vec{p}, t)$ due to the coupling to more complex states and is known as a "collision" integral having in mind the analogy with the kinetic theory ¹⁴). Following further the same analogy we impose on the collision integral to conserve particle density, linear momentum and kinetic energy, i.e. $I(f)$ obeys:

$$\begin{aligned} \int I(f) d\vec{p} &= 0 \\ \int I(f) p_i d\vec{p} &= 0 \quad (i = 1, 2, 3) \\ \sum_{i=1}^3 \int I(f) p_i^2 d\vec{p} &= 0 \end{aligned} \quad (1)$$

and the equation for the Wigner distribution function becomes

$$\frac{\partial f(\vec{r}, \vec{p}, t)}{\partial t} = \frac{2}{\hbar} \sin \left[\frac{\hbar}{2} (\vec{\nabla}_{\vec{r}} \cdot \vec{\nabla}_{\vec{p}} - \vec{\nabla}_{\vec{p}} \cdot \vec{\nabla}_{\vec{r}}) \right] H_W f + I(f), \quad (2)$$

where H_W is the Wigner transform of the self-consistent Hamiltonian derived with realistic Skyrme type interaction ¹⁵):

$$H_W = \frac{p^2}{2m} + U(\vec{r}, t) + W(\vec{r}, \vec{p}, t). \quad (3)$$

$U(\vec{r}, t)$ and $W(\vec{r}, \vec{p}, t)$ are the local and non local parts of the mean field potential ³):

$$\begin{aligned} U(\vec{r}, t) &= \frac{3}{4} t_0 n(\vec{r}, t) + \frac{2 + \sigma}{16} t_3 n(\vec{r}, t)^{\sigma+1} \\ W(\vec{r}, \vec{p}, t) &= \frac{3t_1 + 5t_2}{16\hbar^2} [n(\vec{r}, t)p^2 - 2mj(\vec{r}, t)\vec{p} + mP(\vec{r}, t)] \\ &\quad + \frac{5t_2 - 9t_1}{32} \nabla^2 n(\vec{r}, t) \end{aligned}$$

and $t_0, t_1, t_2, t_3, \sigma$ are the Skyrme force parameters. Here and in the following we make use of the moments of the Wigner distribution function:

$$n(\vec{r}, t) = \int f(\vec{r}, \vec{p}, t) d\vec{p} = \frac{\rho(\vec{r}, t)}{m}$$

$$\vec{u}(\vec{r}, t) = \frac{\int f(\vec{r}, \vec{p}, t) \vec{p} d\vec{p}}{\rho(\vec{r}, t)} = \frac{\vec{j}(\vec{r}, t)}{n(\vec{r}, t)}$$

$$P_{ij}(\vec{r}, t) = \frac{1}{m} \int (p_i - mu_i)(p_j - mu_j) f(\vec{r}, \vec{p}, t) d\vec{p} \quad (4)$$

$$P_{ijk}(\vec{r}, t) = \frac{1}{m^2} \int (p_i - mu_i)(p_j - mu_j)(p_k - mu_k) f(\vec{r}, \vec{p}, t) d\vec{p}$$

$$P_{i_1, \dots, i_n}(\vec{r}, t) = m^{1-n} \int (p_{i_1} - mu_{i_1}) \dots (p_{i_n} - mu_{i_n}) f(\vec{r}, \vec{p}, t) d\vec{p}$$

The physical meaning of the first three moments is quite clear: $n(\vec{r}, t)$ and $\rho(\vec{r}, t)$ are particle and mass densities, respectively; $\vec{u}(\vec{r}, t)$ is the collective velocity field and P_{ij} are the components of the pressure tensor $P(\vec{r}, t) = \sum_{i,j=1}^3 P_{ij}(\vec{r}, t)$. P_{i_1, \dots, i_n} are the components of the generalized pressure tensor of rank n which appears in the theory.

The method as described in previous papers [refs.^{12,13}] consists in taking moments of the Wigner distribution function in the whole phase space in order to obtain a set of differential equations (virial theorems) for the time derivatives of a number of chosen integral characteristics such as electric and magnetic multipole moments. Following the usual procedure but now with collision terms it is convenient to write for the variation of the collision integral $\delta I(f)$

$$\delta I(f) = a(f - f^{(0)})$$

$$+ \beta \left[b_0 f^{(0)}(\vec{r}, \vec{p}) + \sum_{i=1}^3 b_i \frac{\partial f^{(0)}(\vec{r}, \vec{p})}{\partial p_i} + \sum_{i,j=1}^3 b_{ij} \frac{\partial^2 f^{(0)}(\vec{r}, \vec{p})}{\partial p_i \partial p_j} + \dots + \sum_{[n]} b_{[n]} \frac{\partial^n f^{(0)}(\vec{r}, \vec{p})}{\partial p_{i_1} \dots \partial p_{i_n}} \right], \quad (5)$$

where $f^{(0)}(\vec{r}, \vec{p})$ is the distribution function of the stationary (ground) state. Hereafter we use "(0)" to denote stationary values. The sense of the parameter β will be explained latter. The sum in (5) runs over all possible sets of indices. In classical case the coefficients a and $b_{[n]}$ depend on coordinates and on time. In quantum mechanics they are functionals of the variations $\delta f = f - f^{(0)}$ and contain all the information about the formation ("prehistory") of

the system. Besides, as the collision integral obeys the conservation laws (1), these coefficients have to obey the following relations

$$\begin{aligned} b_0 &= -a \frac{\delta n}{n^{(0)}} \\ b_i &= am \frac{n}{n^{(0)}} u_i \\ \sum_{i=1}^3 b_{ii} &= \frac{am}{2n^{(0)}} \left[\frac{\delta n}{n^{(0)}} \sum_{s=1}^3 P_{ss}^{(0)} - \sum_{s=1}^3 \delta P_{ss} \right] \end{aligned} \quad (6)$$

Only terms of the first-order change (with respect to the ground state) of the density, velocity field and pressure tensor have been kept in (6) as a linearized version of our method is used in present work. We analyze the spreading widths assuming $a = -\frac{1}{\tau} = \text{const}$, b_0 and b_i to obey eqs (6), b_{ij} are supposed diagonal, i.e.

$$b_{ij} = b_{ii} \delta_{ij} = \frac{am}{6n^{(0)}} \left[\frac{\delta n}{n^{(0)}} \sum_{s=1}^3 P_{ss}^{(0)} - \sum_{s=1}^3 \delta P_{ss} \right]$$

and all the other coefficients $b_{[n]} = 0$. This approximation is a "corrected" (in sense that the collision integral conserves density, momentum and kinetic energy) relaxation time approximation widely used in literature [ref.¹⁶]. When the parameter β is zero we have the usual τ approximation.

Due to the relations (6) one can express all the coefficients in terms of the first one and the main problem is to determine the relaxation time. In present calculations we use a relaxation time evaluated microscopically in [refs.^{8,9}] from the equilibration of a distorted momentum distribution in a kinetic approach, as discussed later.

3. Dispersion equation for monopole frequencies

How both damping mechanisms can be described by our method is demonstrated on the example of 0^+ excitations, analyzed in a scheme in which all the deformations up to hexadecapole in configuration and momentum spaces are taken into account. This allows us to obtain not only the spreading widths including collision terms, but to see also the effect of the higher deformations which reflects in the Landau damping. The parameter τ is supposed to be the same for all modes. The velocity field $u(\vec{r}, t)$ and the corresponding displacement field $\xi(\vec{r}, t) = u(\vec{r}, t)\delta t$ are parametrized as follows

$$u_i(\vec{r}, t) = \dot{L}_i(t) + \sum_{j=1}^3 \dot{L}_{i,j}(t)x_j + \sum_{j,k=1}^3 \dot{L}_{i,jk}(t)x_j x_k + \sum_{jkl}^3 \dot{L}_{i,jkl}(t)x_j x_k x_l \quad (7)$$

$$\xi_i(\vec{r}, t) = L_i(t) + \sum_{j=1}^3 L_{i,j}(t)x_j + \sum_{j,k=1}^3 L_{i,jk}(t)x_j x_k + \sum_{jkl}^3 L_{i,jkl}(t)x_j x_k x_l$$

where L 's depend on time only and the dots over them signify time derivatives. Due to the continuity equation the displacement $\xi_i(\vec{r}, t)$ of the elements of the nuclear fluid changes the nuclear density by $\delta n = -\text{div}(n\vec{\xi})$. So, the changes in the nuclear integral characteristics are associated with the displacement field and the complexity of the flow (7) determines the type of excitations we can describe in our scheme.

To see the nature of the collective variables, whose dynamics are determined by the equations for the moments up to four, we use as usual the technique of irreducible tensors of the rotational group. The different cartesian tensors of fourth and second rank which appear in the corresponding virial equations may be represented [ref.¹⁷] as linear combinations of pseudoscalars, tensors of first, second, third and fourth rank. In spherical nuclei the virial equations split into five separate sets of equations for the tensors of different symmetry describing 0^+ , 1^+ , 2^+ , 3^+ and 4^+ excitations. We are interested here in the equations for pseudoscalars only as they describe the 0^+ excitations. These excitations have been excluded in previous paper [ref.¹⁸], where the tensors of fourth rank have been considered in the collisionless variant of the method, assuming the nuclei to be incompressible. The washed up set of equations is quite cumbersome:

$$\sum_{s=1}^3 \dot{V}_{s,s} - \sum_{s=1}^3 \delta \Pi_{s,s} + \delta \int n \sum_{s=1}^3 x_s \frac{\partial U^{(0)}}{\partial x_s} d\vec{r} = 0 \quad (8)$$

$$\sum_{r,s=1}^3 \dot{V}_{r,rss} - \sum_{r,s=1}^3 \delta \Pi_{r,rs}^s - 2 \sum_{r,s=1}^3 \delta \Pi_{r,rs}^s + \delta \int n \sum_{s=1}^3 x_s \frac{\partial U^{(0)}}{\partial x_s} r^2 d\vec{r} = 0 \quad (9)$$

$$\sum_{s=1}^3 \delta \dot{\Pi}_{s,s} + 2 \int n \sum_{s=1}^3 \dot{\xi}_s \frac{\partial U^{(0)}}{\partial x_s} d\vec{r} = a(1-\beta) \sum_{s=1}^3 \delta \Pi_{s,s} \quad (10)$$

$$\begin{aligned} \sum_{r,s=1}^3 \delta \dot{\Pi}_{r,rs}^s - 2 \sum_{r,s=1}^3 \delta \Pi_{r,rs}^s - 2 \int P^{(0)} \sum_{r=1}^3 \dot{\xi}_r x_r d\vec{r} - 4 \int \sum_{r,s=1}^3 P_{rs}^{(0)} \dot{\xi}_r x_s d\vec{r} \\ + 2 \int n \sum_{s=1}^3 \dot{\xi}_s \frac{\partial U^{(0)}}{\partial x_s} r^2 d\vec{r} = a(1-\beta) \sum_{r,s=1}^3 \delta \Pi_{r,rs}^s \end{aligned} \quad (11)$$

$$\sum_{r,s=1}^3 \delta \dot{\Pi}_{rs}^{rs} - 2 \sum_{r,s=1}^3 \delta \Pi_{rs}^{rs} - 2 \int P^{(0)} \sum_{r=1}^3 \dot{\xi}_r x_r d\vec{r} - 4 \int \sum_{r,s=1}^3 P_{rs}^{(0)} \dot{\xi}_r x_s d\vec{r} + 2 \int n \sum_{r=1}^3 \dot{\xi}_r x_r \sum_{s=1}^3 x_s \frac{\partial U^{(0)}}{\partial x_s} d\vec{r} = a \sum_{r,s=1}^3 (\delta \Pi_{rs}^{rs} - \frac{\beta}{3} \delta \Pi_{rs}^{rs}) \quad (12)$$

$$\begin{aligned} & \sum_{r,s=1}^3 \delta \dot{\Pi}_{rs}^{rs} - \sum_{r,s=1}^3 \delta \Pi_{rs}^{rs} + \int P^{(0)} \sum_{r=1}^3 x_r (\dot{\xi}_r + \frac{1}{m} \frac{\partial \delta U}{\partial x_r}) d\vec{r} \\ & + 2 \int \sum_{r,s=1}^3 P_{rs}^{(0)} x_r (\dot{\xi}_s + \frac{1}{m} \frac{\partial \delta U}{\partial x_s}) d\vec{r} \\ & + \frac{1}{m} \int (\sum_{r=1}^3 x_r \frac{\partial U^{(0)}}{\partial x_r} \sum_{s=1}^3 \delta P_{rs} + 2 \sum_{r,s=1}^3 x_r \frac{\partial U^{(0)}}{\partial x_s} \delta P_{rs}) d\vec{r} \\ & - \frac{\hbar^2}{4m^2} \delta \int n \sum_{r,s=1}^3 x_r \frac{\partial^3 U^{(0)}}{\partial x_r \partial x_s^2} d\vec{r} \\ & = a \sum_{r,s=1}^3 \delta \Pi_{rs}^{rs} + a(1-\beta) \int \sum_{r,s=1}^3 (\dot{\xi}_r P_{rs}^{(0)} + 2\dot{\xi}_s P_{rs}^{(0)}) d\vec{r} \quad (13) \end{aligned}$$

$$\begin{aligned} & \sum_{r,s=1}^3 \delta \dot{\Pi}_{rs}^{rs} + \frac{4}{m} \int \sum_{r,s=1}^3 \frac{\partial U^{(0)}}{\partial x_r} (\delta P_{rs} + P_{rs}^{(0)} \dot{\xi}_r + 2P_{rs}^{(0)} \dot{\xi}_s) d\vec{r} \\ & - \frac{\hbar^2}{m^2} \int n \sum_{r,s=1}^3 \dot{\xi}_s \frac{\partial^3 U^{(0)}}{\partial x_s \partial x_r^2} d\vec{r} = a \sum_{r,s=1}^3 \delta \Pi_{rs}^{rs} \\ & - a(1-\beta) \left(\int \frac{\delta n}{n^{(0)}} \sum_{r,s=1}^3 P_{rs}^{(0)} d\vec{r} - \frac{10}{3m} \int \frac{P^{(0)}}{n^{(0)}} \left(\frac{\delta n}{n^{(0)}} P^{(0)} - \delta P \right) d\vec{r} \right) \quad (14) \end{aligned}$$

Here and in the following we assume that there is no collective flow in the ground state, i.e. $u_i^{(0)}$ and consequently $P_{ijk}^{(0)}$ are zero.

The equations (8-14) can be written in terms of the following variables (scalars): $\sum_{s=1}^3 V_{s,s}$,

$\sum_{r,s=1}^3 V_{r,rs}$, $\sum_{s=1}^3 \delta \Pi_{ss}$, $\sum_{r,s=1}^3 \delta \Pi_{rs}^{rs}$, $\sum_{r,s=1}^3 \delta \Pi_{rs}^{rs}$, $\sum_{r,s=1}^3 \delta \Pi_{rs}^{rs}$, $\sum_{r,s=1}^3 \delta \Pi_{rrss}$. The first two variables

seem quite natural in our method as the variation of the integral characteristics we want to describe are directly expressed as a linear combination of them. So, the variation of the mean square radius $\delta \langle r^2 \rangle$ is proportional to $\sum_{s=1}^3 V_{s,s}$ and the variation $\delta \langle r^4 \rangle$ to $\sum_{r,s=1}^3 V_{r,rs}$. There is one-to-one correspondence between the components of the cartesian tensors of second

$$V_{i,j} = m \int n(\vec{r}, t) \xi_i(\vec{r}, t) x_j d\vec{r},$$

and four rank

$$V_{ijkl} = m \int n(\vec{r}, t) \xi_i(\vec{r}, t) x_j x_k x_l d\vec{r}$$

and the parameters $L(t)$ of the velocity field in expansions (7) (see Appendix). The rest four scalars containing variations of pressure tensors

$$\delta \Pi_{ij} = \int \delta P_{ij} d\vec{r},$$

$$\delta \Pi_{ij}^{kl} = \int x_k x_l \delta P_{ij} d\vec{r},$$

$$\delta \Pi_{ijk}^{l} = \int x_l \delta P_{ijk} d\vec{r},$$

$$\delta \Pi_{ijkl} = \int \delta P_{ijkl} d\vec{r}$$

play the role of new integral characteristics needed for the correct description of the evolution of the density moments of multipolarity $l=4$. They reflect the fact that in nuclei having finite size the deformations of the Fermi surface are dynamically coupled with those in configuration space. In the simple scaling approximation [refs.^{19,20}] only quadrupole distortions of the local Fermi surface has been taken into account.

To express all the quantities appearing in equations (9)-(14) in terms of discussed variables and thus to close the system we suppose by analogy with the parametrization (7):

$$\delta P_{ij} = n^{(0)}(\vec{r}) \left[D_{ij}(t) + \sum_{k=1}^3 D_{ij,k}(t) x_k + \sum_{k,l=1}^3 D_{ij,kl}(t) x_k x_l \right]$$

$$\delta P_{ijk} = n^{(0)}(\vec{r}) \left[D_{ijk}(t) + \sum_{l=1}^3 D_{ijk,l}(t) x_l \right] \quad (15)$$

$$\delta P_{ijkl} = n^{(0)}(\vec{r}) D_{ijkl}(t)$$

The expansions (15) are multiplied by the ground state density $n^{(0)}$ to guarantee that the variations of the pressure tensors become zero outside the nucleus.

The coefficients of the equations obtained depend only on the ground state properties. They are integrals over nuclear volume of different powers of the ground state particle densities, kinetic energy densities $P_{ij}^{(0)}$, $P_{ijk}^{(0)}$ and their gradients. Any feasible description of the ground state (Thomas-Fermi, Hartree-Fock, square density) can be used in calculating them. In present work the ground state density is assumed to have a Fermi shape distribution:

$$n^{(0)}(r) = \frac{n_0}{1 + \exp \frac{r-R}{a}} \quad (16)$$

with $R=1.115A^{1/3} - 0.53A^{-1/3}$, $a=0.568fm$ [21]. n_0 is fixed by the condition $\int n^{(0)} d\vec{r}=A$. The ground state values $P_{ij}^{(0)}$ and $\Pi_{ijkl}^{(0)}$ are connected by the virial equations in the state of equilibrium:

$$\Pi_{ijkl}^{(0)} = \frac{1}{m} \int x_l \left(\frac{\partial U^{(0)}}{\partial x_i} P_{jk}^{(0)} \right)_{ijk} d\vec{r} + \frac{\hbar^2}{4m^2} \int n^{(0)} x_l \frac{\partial^3 U^{(0)}}{\partial x_i \partial x_j \partial x_k} d\vec{r} \quad (17)$$

(...) $_{ijk}$ means symmetrization over ijk . In spherical nuclei it is natural to suppose: $P_{ij}^{(0)} = \delta_{ij} P_{ii}^{(0)}$ and if we have $P_{ii}^{(0)}$ the condition (16) may be used to express the ground-state value of the fourth-rank pressure $\Pi_{ijkl}^{(0)}$ in terms of known ground-state values. We suppose $\Pi_{ijkl}^{(0)}$ diagonal in each two indices and proportional to $(n^{(0)})^{\frac{7}{3}}$ by analogy with the Thomas-Fermi approximation $P_{ii}^{(0)} = \gamma (n^{(0)})^{\frac{5}{3}}$ and then eq.(17) gives the proportionality coefficient γ_4 .

Assuming a harmonic time dependence $\exp(i\Omega t)$ of all the amplitudes $V_{i,j}$, $\delta\Pi_{ij}$, etc. we obtain dispersion equation for the eigenfrequencies. The presence of collision terms leads to complex frequencies $\Omega = (E + i\frac{\Gamma}{2})/\hbar$ and the time dependence of the eigenmodes is: $\exp\left[\left(iE - \frac{\Gamma}{2}\right)\frac{t}{\hbar}\right]$. So the centroid of the excitation is $E = \hbar \text{Re}\Omega$ and its width is $\Gamma = 2\hbar \text{Im}\Omega$.

4. Results for the monopole mode in ^{208}Pb

We use the Skyrme type interaction SKM^* [ref.¹⁵] which gives a nuclear matter compressibility coefficient $K = 200MeV$ (soft nuclear equation of state). The density parameter n_0 , eq.(16), comes out to be $0.17fm^{-3}$. Let us discuss first the case without collision term.

If we consider only quadrupole deformations in momentum and coordinate space we came back to the scaling approximation³⁾. The velocity field is linear in \vec{r} and the variations of the pressure tensor do not depend on the position in space. We have only tensors of second rank and we get only one solution of the equations of motion (9-14) with eigenfrequency

$$E(GMR) = 13.31 MeV$$

which exhausts the monopole Energy Weighted Sum Rule ($E0 - EWSR$). When we include also hexadecapole deformations we have three non-zero solutions, the lowest of these states exhausting the most part of $E0 - EWSR$:

$$\begin{aligned} E_1 &= 10.99 MeV & S_1 &= 97.87 \\ E_2 &= 29.59 MeV & S_2 &= 8.13 \\ E_3 &= 33.14 MeV & S_3 &= -6.00 \end{aligned}$$

where E gives the energy and S the corresponding fraction of the $EWSR$. The negative value obtained for the highest energy state comes from some numerical uncertainty for the solutions which correspond to a very small strength.

The centroid frequency, computed using as weights the monopole transition probabilities of each state is very close to the "scaling" result, obtained when only quadrupole deformations are included. In agreement with fully RPA calculations [refs.^{2,23,24}] we get a very small fragmentation of the monopole strength: this is a clear indication that the collisionless damping (Landau damping) should play a little role for the giant monopole mode.

In order to analyse the role of collisions on the monopole spreading width we have performed the full calculations including the relaxation time terms, always with the maximum allowed deformations (up to the hexadecapole).

The relaxation time has been microscopically evaluated solving the Landau Kinetic equation in momentum space for an initially distorted momentum distribution. An average quadrupole deformation has been chosen corresponding to the centroid frequency of the monopole. The relaxation time is computed from a clear exponential decay of the collective energy¹⁸⁾. A parametrization of the free nucleon-nucleon cross-section has been used and the Pauli blocking rearrangement at each time step has been carefully taken into account. We get a value $\tau = 267fm/c$.

With collisions the three states discussed before acquire some width which is actually very much dependent on the imposition of the conservation laws eq.(1). Indeed in the τ -approximation ($\beta = 0$ case, only the first term of the expansion eq.(5)) we get for energies and widths:

$$\begin{aligned} E_1 &= 10.99 MeV & \Gamma_1 &= 1.04 MeV \\ E_2 &= 29.59 MeV & \Gamma_2 &= -0.55 MeV \\ E_3 &= 33.14 MeV & \Gamma_3 &= 0.23 MeV \end{aligned}$$

where again we have some unphysical negative values for states corresponding to a very small strength. With the inclusion of the additional terms in the expansion eq.(5) ($\beta = 1$ case) we have:

$$E_1 = 10.98 \text{ MeV} \quad \Gamma_1 = 0.67 \text{ MeV}$$

$$E_2 = 29.78 \text{ MeV} \quad \Gamma_2 = 0.51 \text{ MeV}$$

$$E_3 = 32.95 \text{ MeV} \quad \Gamma_3 = 0.59 \text{ MeV}$$

It is impressive the strong decrease of the spreading width for the first monopole state, which almost exhausts the $E0 - EWSR$, where our results are certainly computationally correct. This is a nice indication of the importance of conservation laws in the relaxation time method: if we use the simple τ -approximation we are overestimating the damping due to the two-body collisions.

Of course the final monopole widths will be very much depending on the choice of the relaxation time parameter, while the energies will not be much affected. This can be easily seen from the table where we report energies and widths of the three Pb monopole states for various values of the relaxation time τ :

Table 1: Energies and widths of the monopole excitations in ^{208}Pb

τ	E_x	Γ
[fm/c]	[MeV]	[MeV]
	10.72	1.94
50	29.94	3.91
	32.64	2.35
100	10.91	1.17
	29.82	1.78
	32.88	1.32
	10.95	0.90
150	29.79	1.10
	32.93	0.93
	10.97	0.77
	29.78	0.76
200	32.94	0.74
	10.98	0.69
250	29.78	0.56
	32.95	0.62

In principle it could be possible to proceed like in [refs.^{9,22}] and to use the experimental widths to fix the parameter τ . However it is clear that this will not be working for the monopole

case. Indeed in order to reproduce the experimental spreading width in Pb which is of the order of 2 MeV²⁵) we can see from the table that we need a very short and unrealistic relaxation time $\tau \leq 50 \text{ fm/c}$. It seems then that also the effect of collisions is not large enough for the monopole vibrations to account for the experimental damping width.

5. Conclusion

We have shown that the semiclassical method based on the equations of motion of the phase-space moments of the Wigner distribution function is suitable for studying various sources of damping of nuclear collective states. Indeed the inclusion of higher order moments naturally leads to a possible fragmentation of the collective strength and therefore to the study of the Landau Damping. Moreover the collisional dissipation can be easily introduced through a relaxation time approach. In this way the interplay between the two damping mechanisms is also automatically accounted for.

Using realistic Skyrme forces an application is shown to the damping of the giant monopole resonance in ^{208}Pb , including phase space distortions up to the hexadecadupole moment.

The relaxation time is evaluated from a microscopic solution of the Landau equation for a distorted Fermi distribution corresponding to the collective energy. For the mode which exhausts the largest part of the $E0 - EWSR$, corresponding to the first frequency, we get a negligible Landau Damping and a very small effect from the collisional damping particularly when the correct conservation laws are imposed within the τ -approximation.

Since the escape width seems also to be quite small³⁾, from our results we can deduce that the damping of giant monopole vibrations should be related to effect outside our RPA-like picture, like quantum fluctuations of the reference state.

More work is in progress on this line.

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APPENDIX

For spherical nuclei we have the following relations between the component of the tensors of second and four rank and the parameters in the expansions (7) and (15).

$$\frac{4\pi}{3} m \sum_{s=1}^3 L_{s,s} = c_1(r) \sum_{s=1}^3 V_{s,s} + c_2(r) \sum_{r,s=1}^3 V_{r,rs}$$

$$\frac{4\pi}{5} m \sum_{r,s=1}^3 L_{r,rs} = c_3(r) \sum_{s=1}^3 V_{s,s} + c_4(r) \sum_{r,s=1}^3 V_{r,rs}$$

$$\sum_{s=1}^3 D_{ss} = c_5(r) \sum_{r,s=1}^3 \delta \Pi_{ss}^{rr} + c_6(r) \sum_{s=1}^3 \delta \Pi_{ss}$$

$$\sum_{r,s=1}^3 D_{rr,ss} = c_7(r) \sum_{r,s=1}^3 \delta \Pi_{ss}^{rr} + c_8(r) \sum_{s=1}^3 \delta \Pi_{ss}$$

$$\sum_{r,s=1}^3 D_{rs,rs} = \frac{1}{c_{11}} (c_9(r) \sum_{r,s=1}^3 \delta \Pi_{ss}^{rr} + c_{10}(r) \sum_{s=1}^3 \delta \Pi_{ss} + \sum_{r,s=1}^3 \delta \Pi_{rs}^{rs}),$$

where

$$c_1 = \frac{h(3)}{c_{12}}, c_2 = -\frac{h(2)}{c_{12}}, c_3 = c_2,$$

$$c_4 = \frac{h(1)}{c_{12}}, c_5 = \frac{h(1)}{c_{13}}, c_6 = -\frac{h(2)}{c_{13}},$$

$$c_7 = -\frac{3A}{4\pi c_{13}}$$

$$c_8 = 3\frac{h(1)}{c_{13}}, c_9 = -\frac{4\pi}{3}(c_5 h(1) + \frac{c_7}{5} h(2)),$$

$$c_{10} = -\frac{4\pi}{3}(c_6 h(1) + \frac{c_8}{5} h(2)), c_{11} = \frac{8\pi}{15} h(2),$$

$$c_{12} = h(1)h(3) - h(2)^2, c_{13} = 4\pi h(1)^2 - Ah(2)$$

The following radial integrals appear in c_i :

$$h(i) = \int n^{(0)} r^{(2i+2)} dr (i = 1, 2, 3).$$

References

- [1] G. Baym and C. Pethick, The physics of liquid and solid helium (Wiley, NY, 1978) part II, ch.2, pp. 1-222, 123-175
- [2] J. Wambach, Rep.Progr.Phys. **51** (1988) 988
- [3] M. Di Toro, Progr.in Part. and Nucl.Physics, **22** (1991) 385

- [4] V.G. Soloviev, Progr.in Part. and Nucl.Physics, **19** (1987) 107
- [5] B. Schwesinger and J. Wambach, Nucl.Phys.A**426** (1984) 253
- [6] S. Drozd, V. Klemt, J. Speth and J. Wambach, Nucl.Phys. A**451** (1986) 11
- [7] P.F. Bortignon, R.A. Broglia, G.F. Bertsch and J. Pacheco Nucl.Phys. A**460** (1986) 149
- [8] A. Smerzi, A. Bonasera, M. Di Toro Phys.Rev. C**44** (1991) 1713
- [9] G.F. Burgio and M. Di Toro, Nucl.Phys.A**476** (1988) 189
- [10] D.M. Brink, A. Dellafore and M. Di Toro, Nucl.Phys. A**456** (1986) 205
- [11] A. Dellafore and F. Matera, Nucl.Phys.A**460** (1986) 245
- [12] E.B. Balbutsev and I.N. Mikhailov, Journ.Phys. G**14** (1988) 545
- [13] E.B. Balbutsev and I.N. Mikhailov, Description of the Nuclear Collective Dynamics by the Method of Moments, in Nuclear Collective Dynamics (Printing House of Acad. Sci. USSR "Nauka", 1990), p.3-66
- [14] A.A. Abrikosov, I.M. Halatnikov, Yspehi Fiz.Nauk v**66** (1958) 177 (sov.edition)
- [15] J.W. Negele, Rev.Mod.Physics v.**54** (1982) 913
- [16] H. Kohl, P. Schuck and S. Stringari, Nucl.Phys.A**459** (1986) 265
- [17] D.A. Varshalovich, A.N. Moskalev, V.K. Khersovskii, Quantum Theory of Angular Momentum (World Scientific Publishing Co, Inc., 1988)
- [18] E.B. Balbutsev, S.I. Basturkov, I.N. Mikhailov, V.P. Sinichkin and L.Sh. Shekhter, Yadernaja Fizika v.**50** (1990) 1264
- [19] G. Holzwarth, Z.Phys.A**284** (1978) 291
- [20] T. Yukawa and G. Holzwarth, Nucl.Phys.A**364** (1981) 29
- [21] A.M. Bernstein, Adv.Nucl.Phys. ,v.**3** (1969) 325

- [22] E.B. Balbutsev, I.N. Mikhailov and J. Piperova, *Izvestija Acad. Sci. USSR*, v.51 (1987) 890
- [23] G.F. Bertsch and S.F. Tsai, *Phys. Rev C*18 (1975) 125
- [24] T.S. Dumitrescu, C.H. Dasso, F.E. Serr and Toku Suzuki, *Journ.Phys.* G12 (1986) 349
- [25] A. Van Der Woude, *Electric and Magnetic Giant Resonances in nuclei*, *International Review of Nuclear Physics*, ed. J.Speth, 7 (1991)

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