ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ
ДУБНА

E4 - 9270

V.B.Belyaev, J.Wrzecionko, B.F.Irgaziev

MOTION OF A PARTICLE IN THE AXIAL-SYMMETRIC POTENTIAL WITH ARBITRARY DEFORMATION VALUE

1975

E4 - 9270

V.B.Belyaev, J.Wrzecionko,[1] B.F.Irgaziev[2]

MOTION OF A PARTICLE IN THE AXIAL-SYMMETRIC POTENTIAL WITH ARBITRARY DEFORMATION VALUE

Submitted to ЯФ

[1] Institute for Nuclear Research, Warsaw, Poland.
[2] Moscow State University, Moscow.

It is somewhat difficult to solve three-dimensional equations of motion like the Schrödinger and Lippmann-Schwinger equations in the two-body problem with the noncentral potential. To describe the bound states in the noncentral potential a series of methods has been proposed [1,2,3]. The subject of these methods is to expand a wave function over some complete set of functions which are the solution to the problem with central symmetry. As a result one obtains the system of coupled in orbital moment one-dimensional equations which being cut off may be integrated numerically. However, in such an approach there arise some additional complications. For instance, the expansion over the eigenfunctions of the central Saxon-Woods potential, frequently used in nuclear physics, contains only the eigenfunctions of the discrete spectrum, i.e., the incomplete set. The inclusion of the eigenfunctions of the continuous spectrum complicated essentially the problem.

This difficulty with continuous spectrum disappears if one uses the Hilbert-Schmidt expansion for the kernel of Lippmann-Schwinger equation. Unfortunately the Hilbert-Schmidt functions for almost all known potentials could be calculated numerically.

Apart from the difficulties pointed out, it is necessary to cut off the set of equations. However, such a procedure may appear to be unjustified for a considerable deviation of the potential from the central one.

One may choose another way, namely, replacing the initial noncentral Hamiltonian by a set of auxiliary Hamiltonians being also noncentral but permitting the solution in the explicit form. Such an attempt has been earlier

formulated by authors in ref. [5] to describe the motion in the central potential at high energies and arbitrary angles.

1

In this paper we apply the procedure mentioned to describe the motion of a spinless particle in a deformed potential with the axial symmetry for arbitrary value of deformation. The Lippmann-Schwinger equation in the momentum representation for t-matrix is

$$\langle \vec{k} | t(z) | \vec{k}' \rangle = \langle \vec{k} | V | \vec{k}' \rangle + \int d\vec{q} \langle \vec{k} | V | \vec{q} \rangle G_0(q^2, z) \langle \vec{q} | t(z) | \vec{k}' \rangle, \quad (1)$$

where z is the energy, $G_0(q^2, z)$ is the Green function of noninteracting particles.
Due to the axial symmetry of the potential the projection of the orbital momentum onto the symmetry axis is conserved, and one may reduce the dimension of eq. (1) expanding the matrix elements $\langle \vec{k} | V | \vec{k}' \rangle$ and $\langle \vec{k} | t(z) | \vec{k}' \rangle$ over the eigenfunctions of the orbital moment projection:

$$\begin{aligned} \langle \vec{k} | V | \vec{k}' \rangle &= \sum_{m=-\infty}^{\infty} V_m(\tilde{k}, \tilde{k}') e^{im\phi}, \\ \langle \vec{k} | t(z) | \vec{k}' \rangle &= \sum_{m=-\infty}^{\infty} t_m(\tilde{k}, \tilde{k}', z) e^{im\phi}, \end{aligned} \quad (2)$$

where $\tilde{k}$ is the two-dimensional vector with the components $k\cos\theta$, $k\sin\theta$; θ is the polar angle of the vector $\vec{k}$, ϕ is the difference of the azimuthal angles of the vectors $\vec{k}$, $\vec{k}'$. Substituting (2) into (1) we obtain the two-dimensional equation for the t-matrix

$$t_m(\tilde{k}, \tilde{k}', z) = V_m(\tilde{k}, \tilde{k}') + 2\pi \int d\tilde{q} V_m(\tilde{k}, \tilde{q}) G_0(q^2, z) t_m(\tilde{q}, \tilde{k}', z). \quad (3)$$

To solve eq. (3) we replace the exact potential by the approximate noncentral one by the formula

$$\tilde{V}_m(\tilde{k}, \tilde{k}') = \sum_{i,j}^{N} \| d^{(m)} \|^{-1}_{ij} \eta_i^{(m)}(\tilde{k}) \eta_j^{(m)}(\tilde{k}'). \quad (4)$$

The solution of eq. (3) with the potential (4) has the form:

$$\tilde{t}_m(\tilde{k}, \tilde{k}', z) = \sum_{i,j}^{N} \eta_i^{(m)}(\tilde{k}) [C^{(m)}(z)]^{-1}_{ij} \eta_j^{(m)}(\tilde{k}'), \quad (5)$$

where

$$\begin{aligned} C_{ij}^{(m)}(z) &= d_{ij}^{(m)} - 2\pi \int d\tilde{q} \eta_i^{(m)}(\tilde{q}) G_0(q^2, z) \eta_j^{(m)}(\tilde{q}) = \\ &= d_{ij}^{(m)} - 2\pi I_{ij}^{(m)}(z). \end{aligned} \quad (6)$$

If one substitutes

$$\eta_i^{(m)}(\tilde{k}) = V_m(\tilde{k}, \tilde{s}_i) \quad (7)$$

and

$$d_{ij}^{(m)} = V_m(\tilde{s}_i, \tilde{s}_j),$$

where $\tilde{s}_i$ is the set of fixed two-dimensional vectors, then the expression (4) for the approximate potential is the interpolation of the exact potential V. Indeed if $\tilde{k} = \tilde{s}_j$ then

$$\tilde{V}_m(\tilde{s}_j, \tilde{k}') = V_m(\tilde{s}_j, \tilde{k}').$$

The choice of the functions $\eta_i^{(m)}(\tilde{k})$ is not unique. Nevertheless one can put some constraints which have to be satisfied by them. For example, we can choose these functions from the condition that the approximate t-matrix be close to the exact one in a fixed energy region [6].

2

The concrete calculation of bound state problems and scattering has been performed for a Gaussian potential:

$$V = V_0 e^{-\alpha r^2 \cos^2\theta - \beta r^2 \sin^2\theta} \quad (8)$$

m-th projection of the Fourier transform of this potential has the form:

$$V_m(\tilde{k},\tilde{k}') = \frac{V_0}{8\pi^{3/2}\beta\sqrt{\alpha}} \exp\left[-\frac{(k\cos\theta - k'\cos\theta')^2}{4\alpha} - \frac{(k\sin\theta - k'\sin\theta')^2}{4\beta}\right] \times$$

$$\times \bar{I}_m\left(\frac{kk'\sin\theta\sin\theta'}{2\beta}\right), \qquad (9)$$

where $\bar{I}_m(y) = e^{-y} I_m(y)$, $I_m(y)$ is the Bessel function of the imaginary argument. The parameters α and β obey the relation $\alpha\beta^2 = \alpha_0^3$ which follows from the equality of volumes of central (range $\sim 1/\sqrt{\alpha_0}$) and deformed potentials. The values of the parameters $\sqrt{\alpha_0} = 0.11$. fm^{-1} and $V_0 = -70$ MeV are taken to provide four levels in the central potential. Let us define the deformation parameter as follows:

$$\delta = 1 - \frac{\alpha}{\beta}. \qquad (10)$$

The values of the energies of bound states are determined from the conditions that the determinant of the matrix $C_{ij}^{(m)}(z)$ is zero (6).

Figure 1 gives the dependence of the energies of bound states on deformation. Identification of levels with respect to the orbital moment was achieved by solving the corresponding problem for the central potential.

As was to be expected with appearance of deformation the degeneration in the projection of the orbital moment disappears and the corresponding levels are splitted. The degeneration in the projection sign is concerved according to the axial symmetry.

The convergence of expansion (4) for the eigenvalues was verified by applying this expansion to the calculation of levels in the central potential $V(r) = V_0 e^{-\alpha_0 r^2}$. The results were compared with the energy values obtained by direct solving the Lippmann-Schwinger equation for the fixed value of the orbital moment ℓ. It is found that expansion (4) allows one to find the eigenvalue which differ from the exact ones not more than by 5%.

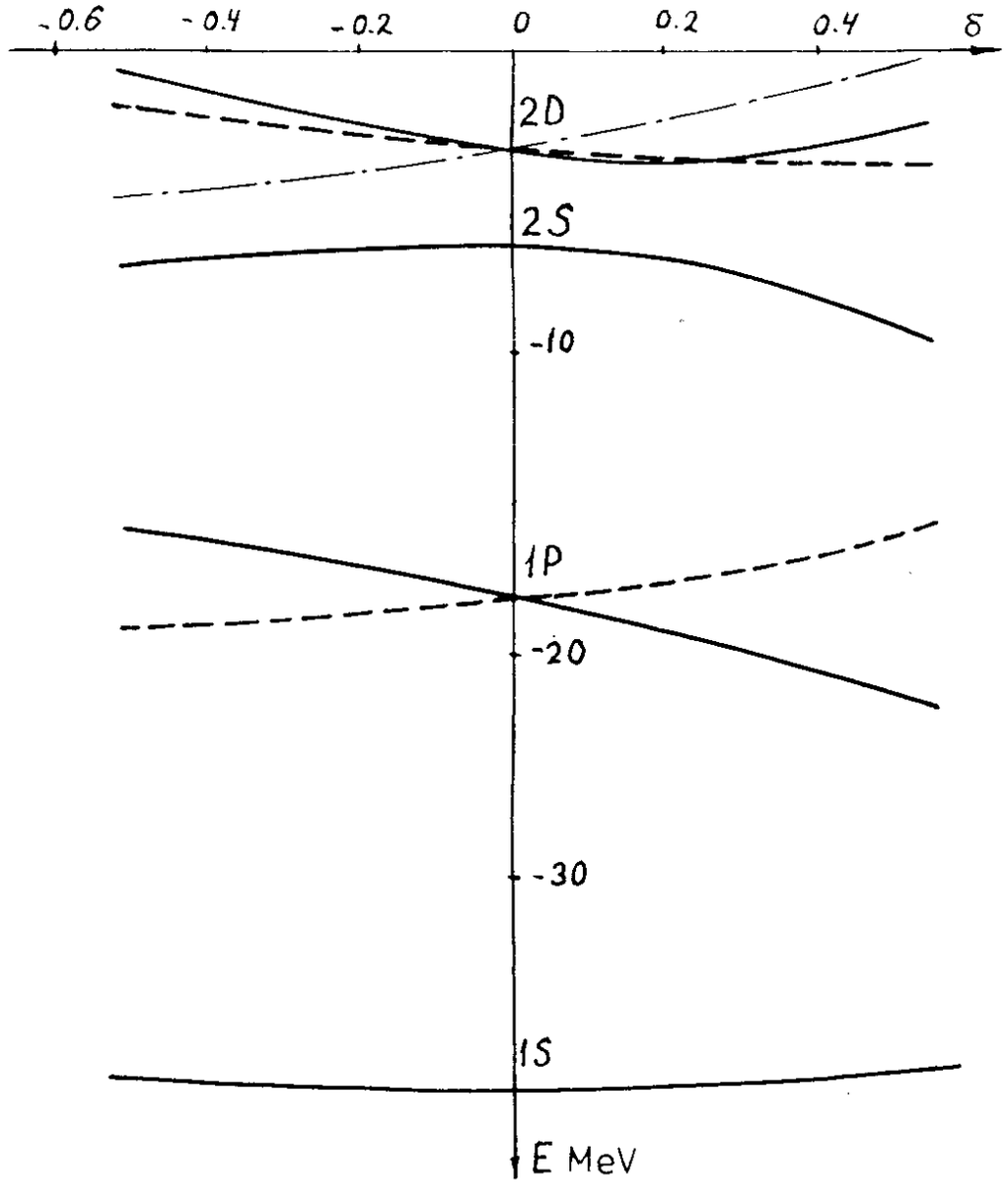

Fig. 1. The energy levels plot against the deformation.
— m = 0; - - - m = 1; —·— m = 2.

When calculating the properties of complex nuclei the condition of orthogonality of approximate functions is used to specify the accuracy of calculations. In the given approach the condition of the orthogonality of approximate solutions holds exactly at any number of separable terms in expansion (4) both in the scattering and bound state problem.

With the same potential the Lippmann-Schwinger equation has been solved in the range of continuous spectrum. The formula (5) allows one to find the dependence of the scattering amplitude on the energy and deformation. In the scattering on the central potential there have been obtained two maxima in the amplitudes with $\ell = 1$ and 3 (fig. 2). When the deformation (potential with $m = 0$) is switched on the maxima are also seen the shape of the maximum depending on deformation. Such a behaviour of the amplitude holds qualitatively in the potentials $V_{m=1}$, $V_{m=2}$. Figure 3 shows the forward scattering amplitude versus energy at different deformation values. A peculiar behaviour of the amplitude is due to the non-monotonic dependence of the potential (9) on the deformation.

3

In conclusion we discuss the convergence of the expansion (4). As has been mentioned above the solution of the two-dimensional equation (3) with the central potential may be used as a criterion to the calculation accuracy of eigenvalues for the deformed potential since from the viewpoint of applicability of expansion (3) to the solution of eq. (3), there is no difference between the central and deformed potentials (in both cases the two-dimensional equation is solved).

Table 1 contains the calculation results of the energy levels in the central potential which has been used for the solution of the two-dimensional equation (3). As is seen from the table for all eigenvalues there is a satisfactory convergence. The calculation of the forward scattering

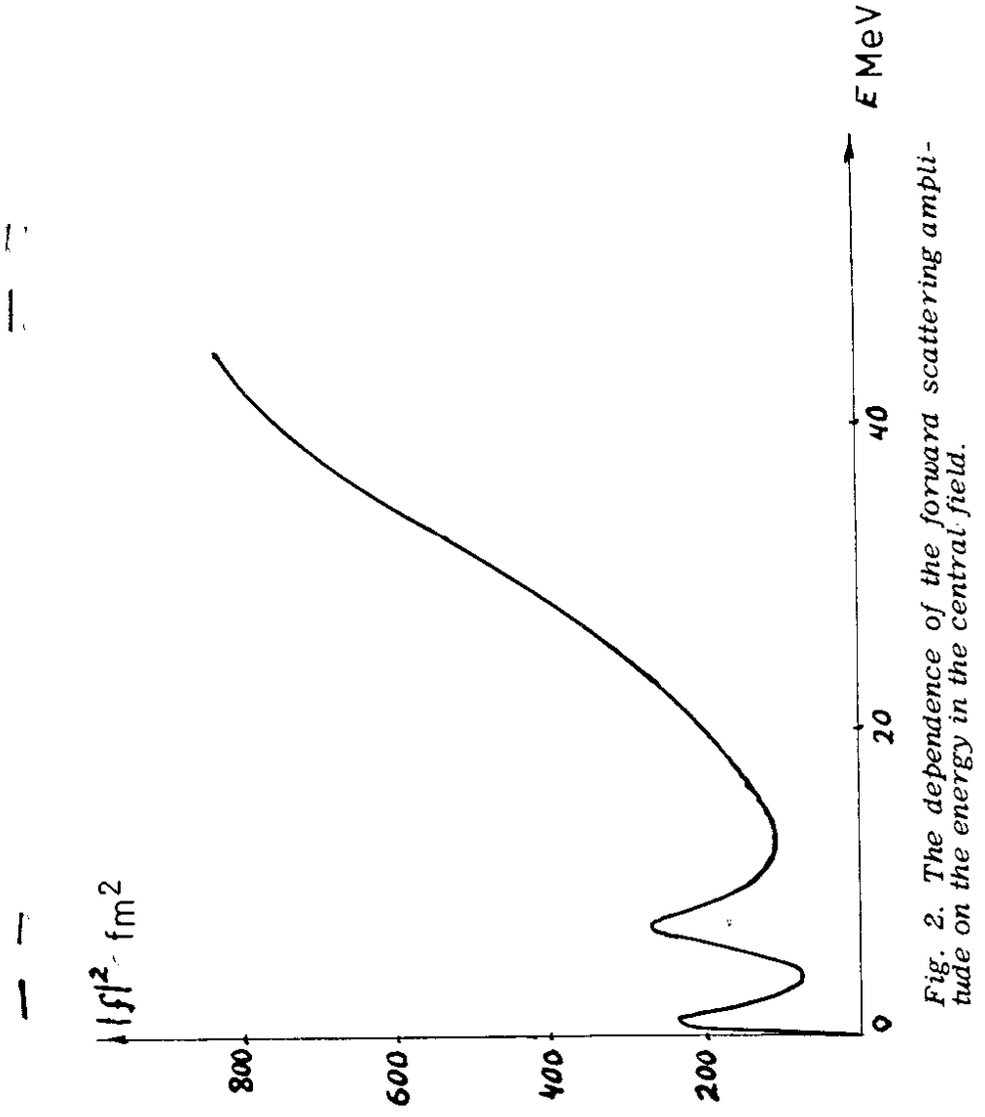

Fig. 2. The dependence of the forward scattering amplitude on the energy in the central field.

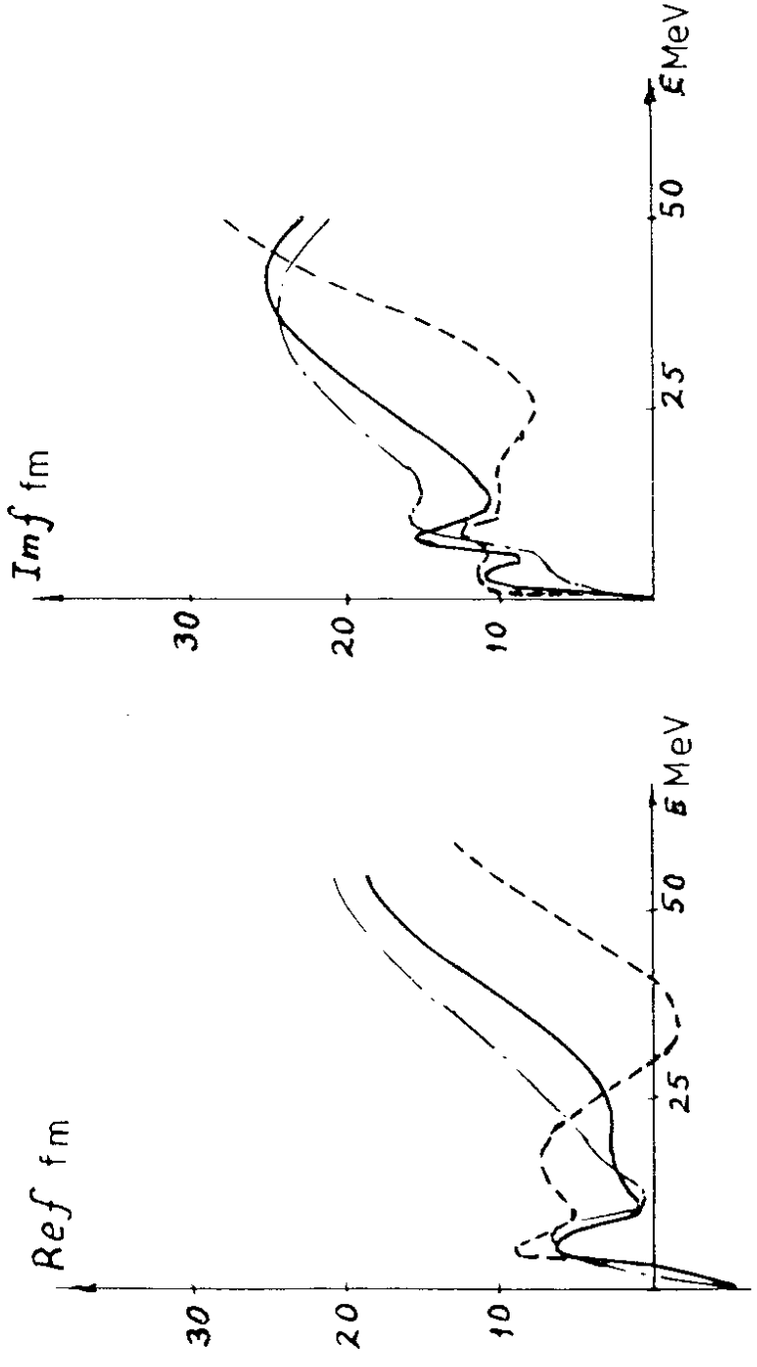

Fig. 3. The imaginary and real parts of the forward scattering amplitude ($\theta = \theta' = 0$, m = 0) at various deformations. —— $\delta = 0$; - - - $\delta = 0.51$; —·—·— $\delta = -0.52$.

Table I

The values of levels in the central potential in dependence of approximation; N is the number equal to the total number of parameters determining the two-dimensional vectors in eq. (7) (m = 0)

N	13	14	16	17	21	23
1s	-	-	<1.5	<1.5	2.8	3.1
1p	2.3	3.3	4.4	4.8	6.4	6.5
2s	14.6	15.7	16.0	15.6	18.0	18.6
2d	35.0	35.3	36.0	36.1	36.9	37.0

amplitude in the central potential in the energy interval $0 \leq E \leq 50$ *MeV* by eq. (5) for N = 23 gives the values which coincide with the exact solution obtained by the summation of partial amplitudes.

Table II

E 50 *MeV*, δ = 0.51, m = 0, $\theta = \theta' = 0$.

N	17	19	21	23
Ref fm	2.45	6.91	9.41	9.41
Imf fm	3.65	7.14	10.32	11.40

The convergence of the calculation of the forward scattering amplitude on the deformed potential (E = 5 *MeV*, δ = 0.51, m = 0) is given in Table II. As well as for the levels, the convergence for the amplitude is obtained when N = 23. The numerical values of the parameters of the approximate potential (4) when N = 23 in the scattering problem are the same as in the bound state problems. The comparison of Tables I and II shows that the change of energy levels in dependence of approximation is more monotone in comparison with the corresponding change of the scattering amplitude. This is due to the fact that the wave functions (amplitudes) are more sensitive to the approximation method of the kernels of the Lippmann-

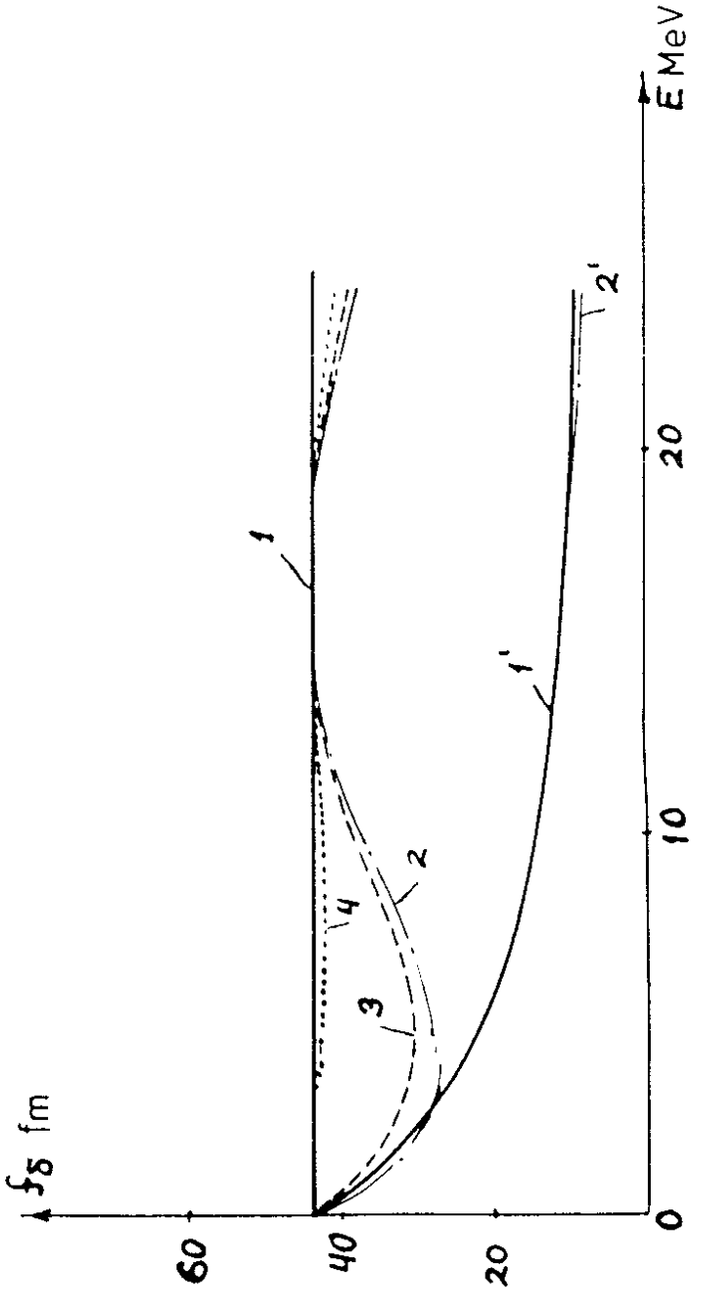

Fig. 4. The dependence of the scattering amplitude in the Born approximation when $\theta = \theta' = 0$ *(1,2,3,4);* $\theta = \theta' = \pi/2$ *(1',2'),* $m = 0$
1 and 1' are the exact Born values.
2 and 2' - N = 13; 3 - N = 18; 4 - N = 23.

Table III

i	S_i	$\cos\theta_i$
1	0	0
2	0.08	-1.0
3	0.08	-0.5
4	0.08	-0.3
5	0.08	0.3
6	0.08	0.5
7	0.08	1.0
8	0.27	-1.0
9	0.27	-0.85
10	0.27	-0.5
11	0.27	-0.25
12	0.27	0.27
13	0.27	0.5
14	0.27	0.85
15	0.27	1.0
16	0.5	-1.0
17	0.5	-0.85
18	0.5	-0.56
19	0.5	0.56
20	0.5	0.85
21	0.5	1.0
22	0.8	-0.5
23	0.8	0.5

Schwinger equation than the energy levels. The nonmonotony of the amplitude can be seen even in the behaviour of the potential at the energy the amplitude has been obtained. It is seen from Fig. 4 that the exact potential in this region ($\tilde{k} = \tilde{k}'$) is well reproduced only when N=23. The parameters of the approximate (table III) potential (4) for N = 23 are given below. It should be emphasized that the aim of the present paper is to investigate the practical applicability of expansion (4) and not to obtain the most accurate numerical results. Thus, it may happen that the used parameters of potential (4) in different approximations in ^{N} are not optimal.

One of the authors (Ir.B.) is grateful to F.Gareev and H.Schultz for valuable discussions and critical remarks.

References

1. *S.G.Nillson. Mat. Fys. Medd. Dan. Vid. Selsk., 29, 16 (1955).*
2. *F.A.Gareev, S.P.Ivanova, B.N.Kalinkin. Preprints JINR P4-2976, P4-3451, Dubna, 1967; Izv. Acad. Nauk. SSSR ser. fiz., 32, 1690 (1968).*
3. *F.A.Gareev, S.P.Ivanova, N.Yu.Shirikova. JINR preprint, P4-5351, Dubna, 1971. THF 8, 97 (1971).*
4. *N.W.Bazeley, D.W.Fox. Phys.Rev., 124, 483 (1961).*
5. *V.B.Belyaev, J.Wrzecionko, B.F.Irgaziev. Yad.Fiz., 20, 1267 (1974).*
6. *A.L.Zubarev, I.M.Ivlieva, A.N.Podkopaev. Preprint JINR, P4-8634, Dubna, 1975.*
   M.M.Musakhanov, A.N.Zubarev. Preprint JINR, E4-8543, Dubna, 1975.

Received by Publishing Department
on October 31, 1975.