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It was as long ago as 1958 that Chew proposed to use the analyticity of the differential cross section for getting information on the strength of the nearest singularity in the $z=\cos\theta$ complex plane by continuing the cross section into the nonphysical region [1]. In the improvement of this technique an important step was the introduction of the optimal conformal mapping $x = x(z)$. By mapping the known singularities on the boundary of the ellipse with its focii on the edge of the physical region (at $x = \pm 1$) it minimizes the number of parameters required for a polynomial approximation to represent the cross section with a given accuracy [2,3]. This made it possible to apply successfully the technique of analytic continuation in nuclear physics [4,5,11].

In this paper we propose to use rational fraction approximations instead of polynomial ones. In a phenomenological way they take into account additional information on the singularities located on the nonphysical sheets of the cross section and, as a result, a considerable improvement in the performance of the continuation procedure is achieved.

It is well-known that the Feynman graphs representing various direct reaction mechanisms have singularities in the $z = \cos\theta$ variable [6]. The graph describing the transfer process of a bound nucleus has a pole, while other triangular and more complicated graphs have branch points. All of these singularities lie on the real axis of the z plane. For large n the deviation of the best polynomial approximation $P_n(z)$ is determined by the sum of the semiaxis a of that ellipse ("ellipse of convergence") which has its focii on the edge of the physical region at $z = \pm 1$, , and which has the nearest singularity on its boundary [2,3,8]:

$$\max |f(z) - P_n(z)| \approx a^{-n} .$$

If we map an additional region of analyticity into the interior of the convergence ellipse, then a increases, i.e., the convergence is accelerated. In particular, the optimal conformal mapping [2,3] maps the upper and lower boundaries of the left- and right-hand cuts in the z plane on the boundary of the convergence ellipse. In a number of cases, however, one can find only a small number of singularities on the boundary of the ellipse, and large regions of analyticity remain unexploited. To improve further the convergence of polynomial approximations, one has to study the structure of the nonphysical sheets in the z plane, and using this information, to map the possible largest region into the interior of the convergence ellipse. In practice this procedure is strongly hindered by the lack of information on the analytic structure of the nonphysical sheets.

We want to call attention to the existence of such functions which are able, at least in principle, to take into account information on the analyticity structure of the function to be approximated in a phenomenological way, and thus to make the convergence faster, as compared to the polynomial approximations. Several functions of this kind are listed in ref. [8]. We have chosen the rational fractions $R_{nm}(x) = P_n(x)/Q_m(x)$, where $P_n(x)$ and $Q_m(x)$ are polynomials. For large n the deviation of the best rational fraction approximation is determined by the sum of the semiaxes β of that ellipse of convergence which would have been received by mapping the entire region of analyticity into its interior [8]:

$$\max |f(x) - R_{nn}(x)| \approx \beta^{-n} .$$

This estimate is given for the physical region, but it is clear that a similar estimate is valid in the neighbourhood of it, too. It is not suitable to describe the known analyticity structure of the physical sheet in a phenomenological way, i.e., by effective poles. Therefore it is always useful to employ the optimal conformal mapping, but if it gives only a small number of physically important singularities on the boundary of the convergence ellipse, then one hopes that rational fraction approximations give better results than polynomial ones. Of course, one can use not only rational fractions, but other functions with similar properties, as well /8/.

To demonstrate the advantage of rational fraction approximations we analysed p-d elastic scattering at $E_p = 6.78$ MeV, where impressively accurate experimental data are available. The analyticity structure of the differential cross section was studied, for instance, in ref. /1/. The nuclear branch points are located at $z_t = 3.8$ and $z_u = -7.1$. The neutron exchange pole is at $z_p = -1.62$, while the elastic Coulomb branch point is at $z = +1$. Using the conformal mapping as proposed in ref. /3/, one can describe the experimental data on the function $(x - x_p)^2 d\sigma/d\Omega$ ($x = x(z)$ is the variable received by the mapping, $x_p = x(z_p)$ is the location of the pole) by a polynomial of degree of eigth $P_8(x)$. The continuation of this approximation to the pole gives the following estimate for the vertex constant of the virtual decay $d \to p + n$: $G = G_d^4 = 0.231 \pm 0.014$ fermi^2 (our determination of the vertex constant and its connection with the differential cross section can be found in ref. /7/).

This estimate significantly differs from the known value of the deuteron vertex constant $G_d^2 = 0.43\,f$ ($G = 0.185\,f^2$) /7/, and it has an unexpectedly large statistical error. This indicates that in the given case the polynomial approximations are not effective enough. The rational fraction approximations $R_{33}(x)$, $R_{52}(x)$ and $R_{51}(x)$ gave the following results: $G = 0.161 \pm 0.002$, 0.161 ± 0.003 and 0.155 ± 0.002 f^2. If one takes into account the absolute normalization error of the experimental data, then the average is $G = 0.159 \pm 0.005$ f^2. A smaller number of fitting parameters and a smaller statistical error of the result points out the advantage of the application of rational fraction approximations.

A detailed study of the location of the poles and zeros of the approximating rational fractions is to be published later . We note only, that the poles can be given a physical interpretation. The effective pole describing the Coulomb singularity is, however, too near to the physical region. Though the result we have got is different from the standard value, nevertheless it does not mean that rational fractions gave a wrong value. A more sophisticated analysis with polynomial approximation, when the Coulomb singularity is suppressed by a factor of $(1-\cos\theta)^2$, carried out at 7 values of the bombarding energy in the interval $E_p = 3$-46 MeV, gives $G = 0.159 \pm 0.004$ f^2, too [10].

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