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WEAK INTERACTIONS IN A BILOCAL CHIRAL THEORY
II. NONLEPTONIC DECAYS OF QUARKONIA IN BILOCAL APPROACH

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Слабые взаимодействия в билокальной киральной теории.
II. Нелептонные распады кваркониев в билокальном подходе

Эта часть работы посвящена описанию мезонных констант распада и нелептонным переходам D- и B-мезонов в рамках билокальной теории мезонов.

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Weak Interactions in a Bilocal Chiral Theory.
II. Nonleptonic Decays of Quarkonia in Bilocal Approach

In this part the calculus to describe meson decay constants and the nonleptonic transitions for D and B mesons in a bilocal meson field theory are formulated.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1. Introduction

Heavy quarkonium physics stays e.g. the problem of non-leptonic decays of mesons. This includes the description of $q\bar{q}$-bound states and the evaluation of decay rates. In papers [1]- [5] the bilocal meson theory was constructed including the nonrelativistic nonlocal description of heavy quarkonia and the chiral Lagrangian for light mesons in the bilocal limit [3]- [5]. In paper [5] we discussed the description of mesonic orbital excitation in the bilocal theory.

This paper is devoted to the calculation of meson decay constants and of amplitudes of non-leptonic decays of D and B mesons.

2. Relativistic bound states in QCD

In this section we shortly repeat some statements concerning the construction of relativistic bound states in QCD. Bound states played a fundamental role in the development of quantum theory. The description of the atomic spectrum by E.Schrödinger signified the formation of quantum mechanics as a consistent theory, and the description of the Lamb shift of spectral lines by H.Bethe started the creation of QED and quantum field theory.

To get some insight into the description of relativistic bound states in gauge theory, let us first consider the well- known example of an atom in its rest frame (with the momentum $\mathcal{P}_\mu = (M_A, 0, 0, 0)$. In lowest order with respect to radiative corrections the atom spectrum is described by the action

$$W = \int d^4x \bar{\psi}(x)(i\not{\partial} - m^0)\psi(x) + \frac{1}{2}\int d^4x d^4y \psi(y)\bar{\psi}(x)\mathcal{K}(x,y)\psi(x)\bar{\psi}(y) \ , \tag{1}$$

where $\mathcal{K}$ is the Coulomb kernel

$$\mathcal{K} = (\gamma_0) \cdot V_0(\mathbf{z}) \cdot (\gamma_0)\,\delta(x_0) \ ,$$

$z_\mu = x_\mu - y_\mu$ is the relative space-time. Now the question arises how to describe a moving atom. The wave function of a relativistic atom is constructed by the usual boost operation

$$\begin{aligned} \phi(x,y) \quad &\longrightarrow \quad \phi'(x,y) = e^{i\mathcal{P}'\cdot X}\chi(z^\perp)\cdot\delta(z\cdot\eta') \ , \\ z^\perp_\mu \quad &= \quad z_\mu - \eta'_\mu(z\cdot\eta') \ , X_\mu = \frac{(x+y)_\mu}{2} \ , \end{aligned} \tag{2}$$

where $\mathcal{P}'_\mu$ is the total momentum $\mathcal{P}'_\mu = (\sqrt{\vec{\mathcal{P}}^2 + M_A^2}, \vec{\mathcal{P}} \neq 0) \equiv M_A \cdot \eta'_\mu$.

This relativistic atom bilocal field is described by action (1) with the moving Coulomb kernel

$$\mathcal{K}(x,y) = \mathcal{K}(z|X) = \not{\eta}' \cdot V_0(z^\perp) \cdot \not{\eta}'\,\delta(z\cdot\eta') \ . \tag{3}$$

This means that we choose the new radiative gauge depending on the arbitrary unit time- like vector η' (that one calls the time- axis of quantization) and this vector has been chosen parallel to the total momentum of atoms ($\eta' \sim \mathcal{P}'$).

It is easy to see that the bilocal field (2) satisfies the Yukawa condition [6]

$$z_\mu \frac{\partial}{\partial X_\mu}\phi(z|X) = 0 \ , \tag{4}$$

which means that the bilocal field is an irreducible representation of the Lorentz group (i.e. it has the mass $\mathcal{P}^2 = M_A^2$ and spin).

As a result, we obtain the action (1)with the kernel (3), where the time- axis η' is the unit eigenvector of the bound state total momentum operator

$$\eta'_\mu \phi(z|X) \sim \frac{\partial}{\partial X_\mu}\phi(z|X) \ . \tag{5}$$

These prescriptions lead to a new relativistic potential model [1]- [5] which unifies the potential model for the heavy- quarkonia spectroscopy and the bilocal generalization of the chiral Lagrangian for light quarkonia. It is important to underline that the instantaneous bound state physics (4) not only depends on the gauge but this dependence is necessary for the relativistic covariance of physical observables (in particular, the mass spectrum). In this case, the potential model (1),(3) becomes the relativistic one [1].

Just this recipe should be taken into account for hadron description. As we have seen, the relativistic covariant formulation of instantaneous interaction is the main ingredient to describe bound states. Now the question arises: How to get this picture in the field theory of interacting quarks and gluons. The answer was found by the quantization procedure of the gauge theory which includes the explicit solution of the classical equation for the time- component of gauge field $A_0 = (\eta \cdot A)$ (we called this quantization the "minimal approach" [7]).

In this minimal quantization of QCD the gluon exchange interaction between quarks is naturally divided into two parts: the instantaneous and the retardation contributions. If we define any hadron bound state by formulae (2), and (4) as an irreducible nonlocal representation of the Poincaré group, it is easy to understand that the covariant instantaneous gluon interaction $\sim \delta(\eta \cdot z)$ at the point of existence of bound states $\eta \cdot z = 0$ is greater than the remaining retarded part of this interaction in QCD. This minimal approach allows one to formulate the hadron perturbation theory [1], the lowest order of which is the new relativistic potential model. On the recent level of QCD we can choose the quark- antiquark potential in the form of the sum of the rising and Coulomb ones.

The criterion of the validity of our approach is the description of the light meson physics (spectroscopy and decay constants) in terms of the parameter of the rising potential defined from the quarkonium spectroscopy.

The instantaneous singularities, forming the bound states, cannot be reproduced by a relativistic gauge where all gauge field propagators have singularities only on the light cone. All modern QCD approaches, including the lattice calculations, do not take into account these peculiarities of the problem of bound states. Finally, we note that in the minimal scheme of quantization it can be shown by the explicit solution of constraints [7, 8, 9] that this method contains additional physical information - the topological degeneracy of the colour physical state, as the mechanism of confinement.

3. Meson decay constants

The quadratic part of effective action over bilocal field $\mathcal{M}$ has the form

$$W^{(2)}_{\rm eff} = -i\frac{N_c}{2}{\rm Tr}(G_\Sigma \mathcal{M})^2 \ . \tag{6}$$

Here G_Σ is the Green function $G_\Sigma^{-1} = i\not\partial - \Sigma$, Σ satisfies the Schwinger- Dyson equation and the symbol Tr means both the integration and the trace over discrete indices.

The bilocal field $\mathcal{M}$ can be expanded over creation (a_H^+) and annihilation (a_H) operators

$$\mathcal{M}(x,y) = \mathcal{M}(x-y|\frac{x+y}{2}) = \sum_H \frac{d\vec{\mathcal{P}}_H}{(2\pi)^{3/2}\sqrt{2\omega_H}} * \tag{7}$$

$$* \int \frac{dq}{(2\pi)^4} e^{iq(x-y)}\{e^{i\mathcal{P}(\frac{x+y}{2})}a_H^+\Gamma_H + e^{-i\mathcal{P}(\frac{x+y}{2})}a_H\bar{\Gamma}_H\} \ .$$

$\mathcal{P}_H, \omega_H$ are the total momentum and energy of the bound state with quantum numbers H, respectively. The vertex functions Γ_H satisfy the Bethe- Salpeter equation

$$\Gamma_H = -i\mathcal{K}^\eta(G_\Sigma \Gamma_H G_\Sigma) \ ,$$

where $\mathcal{K}^\eta$ is the instantaneous interaction kernel with the definite time axis η_μ

$$\mathcal{K}^\eta = \not\eta V(x^\perp)\delta(x\cdot\eta)\not\eta$$

($\eta^2 = 1, \eta_\mu = \mathcal{P}_\mu/\sqrt{\mathcal{P}^2}$).

Now we consider the inclusion of weak interactions into the effective action (6). The effective Lagrangian of weak interaction has the form

$$\mathcal{L}_{\rm eff} = \frac{G}{\sqrt{2}}\{V_{ij}(\bar{Q}_i O_\mu q_j)(\bar{l} O_\mu \nu_l) + h.c.\} = \frac{G}{\sqrt{2}}\{V_{ij}(\bar{Q}_i O_\mu q_j) l_\mu + h.c.\}. \tag{8}$$

$G = 10^{-5}/m_p^2$ is the Fermi constant, Q denotes the column of (u, c, t) quarks and q denotes the column of (d, s, b) quarks, V is the Kobayashi- Maskawa mixing matrix, $O_\mu = \gamma_\mu(1+\gamma_5)$, $l_\mu = \bar{l}O_\mu \nu_l$, $l = e, \mu, \tau$, $\nu_l = \nu_e, \nu_\mu, \nu_\tau$.

For the definition of the meson decay constants we will introduce in the bilocal action (6) the local leptonic weak current $\hat{L}$ by the substitution

$$\mathcal{M}(x,y) \to \mathcal{M}(x,y) + \hat{L}(x,y) \ , \tag{9}$$

where

$$\hat{L}(x,y) = \frac{G}{\sqrt{2}}\delta(x-y)\hat{l}e^{i\mathcal{P}_L(\frac{x+y}{2})} \ , \tag{10}$$

$\hat{l} = O_\mu l_\mu$ and $\mathcal{P}_L$ is the total momentum of the leptonic pair.

Inserting (9), (10) in (6), we get the following form for the effective action, which corresponds to weak interaction

$$\mathrm{Tr}(G_\Sigma\mathcal{M})^2 \to 2\mathrm{Tr}(G\mathcal{M}G\hat{L}) = 2\frac{G}{\sqrt{2}}V_{ij}O_\mu(G\mathcal{M}G)_{ji}l_\mu \ , \qquad (11)$$

and the matrix element for the decay of a pseudoscalar meson M into a leptonic pair reads

$$< l\nu|W_{\rm eff}|M > \ = \ (2\pi)^4 i\delta(\mathcal{P}_H-\mathcal{P}_L)\frac{1}{\sqrt{(2\pi)^3 2\omega_H}}\frac{G}{\sqrt{2}}(< l\nu|l_\mu|0>) * \qquad (12)$$

$$* \int\frac{dq}{(2\pi)^4}\mathrm{tr}\left\{V_{ij}O_\mu G_j(q-\frac{\mathcal{P}}{2})\bar{\Gamma}_{ji}G_i(q+\frac{\mathcal{P}}{2})\right\} .$$

The whole dependence on the mesonic properties of this matrix elements is included into the integral term. Using the relations for Green functions [1]

$$G_j(q-\frac{\mathcal{P}}{2}) \ = \ \left(\frac{\Lambda_+^\eta}{q_0-(E_j+\frac{M_H}{2}-i\epsilon)}+\frac{\Lambda_-^\eta}{q_0-(-E_j+\frac{M_H}{2}+i\epsilon)}\right)\not\eta \ , \qquad (13)$$

$$G_j(q+\frac{\mathcal{P}}{2}) \ = \ \not\eta\left(\frac{\bar{\Lambda}_+^\eta}{q_0-(E_i-\frac{M_H}{2}-i\epsilon)}+\frac{\bar{\Lambda}_-^\eta}{q_0-(-E_i-\frac{M_H}{2}+i\epsilon)}\right) ,$$

with

$$\Lambda_\pm^{(a)} \ = \ S_a^{-1}(p)\Lambda_\pm^0 S_a(p) = \frac{1}{2}(1\pm S_a^{-2}(p)\gamma_0) = \frac{1}{2}(1\pm\gamma_0 S_a^2(p)) \ ,$$

$$\bar{\Lambda}_\pm^{(a)} \ = \ S_a(p)\Lambda_\pm^0 S_a^{-1}(p) = \frac{1}{2}(1\pm S_a^2(p)\gamma_0) = \frac{1}{2}(1\pm\gamma_0 S_a^{-2}(p)) \ ,$$

$$S_a^{\pm 2}(p) = \sin\phi_a(p) \pm \hat{p}\cos\phi_a(p) = \exp(\pm 2\hat{p}\nu_a(p)) \ ; \qquad (14)$$

$$\hat{p} \ = \ \hat{p}_i\gamma_i \ , \ \hat{p}_i = \frac{p_i}{|p_i|} \ , \ \hat{p}^2 = -1 \ ;$$

$$\sin\phi_a(p) \ = \ \frac{m_a}{E_a(p)} \ , \ \cos\phi_a(p) = \frac{|\mathbf{p}|}{E_a(p)}$$

$$\nu_a(p) \ = \ \frac{1}{2}(-\phi_a(p)+\frac{\pi}{2}) \ ; \ E_a(p) = \sqrt{\mathbf{p}^2+m_a^2} \ ;$$

$$\Lambda_\pm^0 \ = \ \frac{1}{2}(1\pm\gamma_0) \ ,$$

and the definition of the wave function of bound states

$$i\int\frac{dq_0}{2\pi}(G_j\bar{\Gamma}_{ji}G_i) \equiv \bar{\Psi}_{ji} \ ,$$

we can write

$$\int \frac{dq}{(2\pi)^4}\mathrm{tr}\left\{V_{ij}O_\mu G_j(q-\frac{\mathcal{P}}{2})\bar{\Gamma}_{ji}G_i(q+\frac{\mathcal{P}}{2})\right\} \qquad (15)$$

$$= \ -i\int\frac{d\mathbf{q}}{(2\pi)^3}\mathrm{tr}(V_{ij}O_\mu\bar{\Psi}_{ji}) \ .$$

This compact form is very useful because the whole description of relativistic bound states may represented in the language of the functions Ψ_{ji}. Let us expand the wave function of bound state over the Lorentz matrices. By help of (14) we can write Ψ as the "dressed" wave function $\overset{0}{\Psi}$

$$\bar{\Psi} = \Psi(-\mathcal{P}) \ = \ S_j^{-1}\overset{0}{\Psi}(-\mathcal{P})S_i^{-1} \qquad (16)$$

$$\overset{0}{\Psi}(\mathcal{P}) \ = \ (\overset{0}{\Psi}_1{}^I + \not\eta\overset{0}{\Psi}_2{}^I)\gamma^I$$

$$\overset{0}{\Psi}(-\mathcal{P}) \ = \ (\overset{0}{\Psi}_1{}^I - \not\eta\overset{0}{\Psi}_2{}^I)\gamma^I .$$

In these relations $\gamma^I = \{\gamma^5, \gamma^i, 1\}$, $\overset{0}{\Psi}{}_i{}^I = \{\overset{0}{L}{}_i, \overset{0}{N}{}_i{}^a, \Sigma_i\}$ $(i=1,2)$ correspond to pseudoscalar, vector and scalar mesons, respectively. For pseudoscalar mesons we have $\gamma^I \equiv \gamma^5$ and

$$\overset{0}{\Psi}(-\mathcal{P}) = (\overset{0}{L}_1 - \not\eta\overset{0}{L}_2)\gamma^5 . \qquad (17)$$

The functions $\overset{0}{L}_1$ and $\overset{0}{L}_2$ satisfy the set of two equations [5]

$$M\overset{0}{L}_{(\substack{2\\1})}(p) \ = \ E\overset{0}{L}_{(\substack{1\\2})}(p) - \int\frac{d\mathbf{q}}{(2\pi)^3}V(\mathbf{p}-\mathbf{q})[\mathrm{c}^\mp(p)\mathrm{c}^\mp(q)-\xi\mathrm{s}^\mp(p)\mathrm{s}^\mp(q)]\overset{0}{L}_{(\substack{1\\2})}(q) ; \qquad (18)$$

where

$$\mathrm{c}^\pm(p) \ = \ c_i(p)c_j(p)\mp s_i(p)s_j(p) \equiv \mathrm{c}_{ij}^\pm(p) \ ,$$

$$\mathrm{s}^\pm(p) \ = \ s_i(p)c_j(p)\pm s_j(p)c_i(p) \equiv \mathrm{s}_{ij}^\pm(p) \ ,$$

$$s_{i,j} \ = \ \sin(\nu_{i,j}) \ ,$$

$$c_{i,j} \ = \ \cos(\nu_{i,j}) \ ,$$

$\xi = (\hat{p}\cdot\hat{q})$; $\nu_{i,j}$ are defined in (14), i,j are the flavours of quarks, which form the bound state.

We obtain the expression for the decay constant of the meson with quantum numbers H using the standard definitions for the decay constant of the meson

$$F_H = \frac{4N_c}{M_H}\int\frac{d\mathbf{q}}{(2\pi)^3}(\overset{0}{L}_2)_H\cos(\phi_H) \ . \qquad (19)$$

Here $\cos(\phi_H)=\cos(\phi_{ij})=\cos(\nu_i+\nu_j)$ and i,j denote the quarks in this meson. Specifying these combinations we have

$$F_\pi=\frac{4N_c}{M_\pi}\int\frac{dq}{(2\pi)^3}(\overset{0}{L}_2)_\pi\cos(\nu_d+\nu_u)\ ;$$

$$F_K=\frac{4N_c}{M_K}\int\frac{dq}{(2\pi)^3}(\overset{0}{L}_2)_K\cos(\nu_s+\nu_d)\ ;$$

$$F_D=\frac{4N_c}{M_D}\int\frac{dq}{(2\pi)^3}(\overset{0}{L}_2)_D\cos(\nu_c+\nu_u)\ ;$$

$$F_{D_s}=\frac{4N_c}{M_{D_s}}\int\frac{dq}{(2\pi)^3}(\overset{0}{L}_2)_{D_s}\cos(\nu_c+\nu_d)\ ;$$

$$F_{B_c}=\frac{4N_c}{M_{B_c}}\int\frac{dq}{(2\pi)^3}(\overset{0}{L}_2)_{B_c}\cos(\nu_b+\nu_c)\ ;$$

$$F_{B_u}=\frac{4N_c}{M_{B_u}}\int\frac{dq}{(2\pi)^3}(\overset{0}{L}_2)_{B_u}\cos(\nu_b+\nu_u)\ .$$

4. Nonleptonic transitions D and B mesons

For the description of nonleptonic decays of D and B mesons we shall start from the effective action which has the form

$$(G\mathcal{M})^2\rightarrow$$

$$\rightarrow\ \frac{G}{\sqrt{2}}\frac{1}{2}V_{ik}V^*_{lj}\Big\{[O_\mu(G_k\mathcal{M}_{kl}G_l)O_\mu](\bar{G}_j\mathcal{M}_{ji}\bar{G}_i)+(G_k\mathcal{M}_{kl}G_l)[O_\mu\bar{G}_j\mathcal{M}_{ji}\bar{G}_iO_\mu]\Big\}\ . \tag{20}$$

In this formula V_{ij} are the matrix elements of Kobayashi- Maskawa matrix, $\bar{G}_i$ denotes the Green function of the quarks (u,c,t) and G_i are the Green functions of quarks (d,s,b). The diagrams for the weak transitions $\mathcal{M}\overset{w}{\rightarrow}\mathcal{M}$ are listed in Fig.1 and the corresponding combinations of the matrix elements of Kobayashi- Maskawa matrix are given in Tabl.1-3.

The matrix elements for these transitions can be written in the form

$$<\mathcal{M}(\mathcal{P})|W_{\rm eff}|\mathcal{M}(\mathcal{P}')>=$$

$$=\ i(2\pi)^4\delta(\mathcal{P}-\mathcal{P}')\cdot\frac{N_c}{2}\cdot\frac{1}{\sqrt{(2\pi)^3 2\omega_H}}\cdot\frac{1}{\sqrt{(2\pi)^3 2\omega_{H'}}}\cdot\frac{G}{\sqrt{2}}\cdot\frac{1}{2}\int\frac{dq}{(2\pi)^3}\int\frac{dk}{(2\pi)^3} \tag{21}$$

$$\cdot\ \Big\{{\rm tr}\big[O_\mu V_{ik}\Psi_{kl}(q+k)V^*_{lj}O_\mu\Psi_{ji}(q)\big]+{\rm tr}\big[O_\mu V_{ik}\Psi_{kl}(q)V^*_{lj}O_\mu\Psi_{ji}(q+k)\big]\Big\}\ .$$

After using techniques similar to those applied in section 3.3 the expressions in the brace brackets for different weak transitions have the form

$\underline{\mathrm{P}\overset{w}{\longrightarrow}\mathrm{P}}$

$$\begin{aligned}{\rm Tr}(\Psi O_\mu\bar{\Psi}O_\mu)\ =\ & 8(\mathcal{P}\mathcal{P}')\cdot\int\frac{dp}{(2\pi)^3}\Big\{\Big[\int\frac{dk}{(2\pi)^3}\overset{0}{L}_2(k+p)c^+_{kl}(k+p)\Big]c^+_{ji}(p)\,\overset{0}{\underline{L}}_2(p)+\\ &+\ \overset{0}{L}_2(p)c^+_{kl}(p)\Big[\int\frac{dk}{(2\pi)^3}c^+_{ji}(k+p)\,\overset{0}{\underline{L}}_2(k+p)\Big]\Big\}+\\ &+\ \int\frac{dp}{(2\pi)^3}\Big\{\Big[\int\frac{dk}{(2\pi)^3}\overset{0}{L}_1(k+p)s^-_{kl}(k+p)\Big]s^-_{ji}(p)\,\overset{0}{\underline{L}}_1(p)+\\ &+\ \overset{0}{L}_1(p)s^-_{kl}(p)\Big[\int\frac{dk}{(2\pi)^3}s^-_{ji}(k+p)\,\overset{0}{\underline{L}}_1(k+p)\Big]\Big\}\ ,\end{aligned} \tag{22}$$

$\underline{\mathrm{P}\overset{w}{\longrightarrow}\mathrm{V}}$

$$\begin{aligned}{\rm Tr}(\Psi O_\mu\bar{\Psi}O_\mu)\ =\ & 8(\eta^\perp_\nu)\cdot\int\frac{dp}{(2\pi)^3}\Big\{\Big[\int\frac{dk}{(2\pi)^3}\overset{0}{L}_2(k+p)c^+_{kl}(k+p)\Big]c^-_{ji}(p)\,\overset{0}{V}_1{}^\nu(p)+\\ &+\ \overset{0}{L}_2(p)c^+_{kl}(p)\Big[\int\frac{dk}{(2\pi)^3}c^-_{ji}(k+p)\,\overset{0}{V}_1{}^\nu(k+p)\Big]\Big\}+\\ &+\ \int\frac{dp}{(2\pi)^3}\Big\{\Big[\int\frac{dk}{(2\pi)^3}\overset{0}{L}_1(k+p)s^-_{kl}(k+p)\Big]s^+_{ji}(p)\,\overset{0}{V}_2{}^\nu(p)+\\ &+\ \overset{0}{L}_1(p)s^-_{kl}(p)\Big[\int\frac{dk}{(2\pi)^3}s^+_{ji}(k+p)\,\overset{0}{V}_2{}^\nu(k+p)\Big]\Big\}\ ,\end{aligned} \tag{23}$$

$\underline{\mathrm{P}\overset{w}{\longrightarrow}\mathrm{S}}$

$$\begin{aligned}{\rm Tr}(\Psi O_\mu\bar{\Psi}O_\mu)\ =\ & -8(\mathcal{P}\mathcal{P}')\int\frac{dp}{(2\pi)^3}\Big\{\Big[\int\frac{dk}{(2\pi)^3}\overset{0}{L}_2(k+p)c^+_{kl}(k+p)\Big]c^+_{ji}(p)\,\overset{0}{\Sigma}_2(p)+\\ &+\ \overset{0}{L}_2(p)c^+_{kl}(p)\Big[\int\frac{dk}{(2\pi)^3}c^+_{ji}(k+p)\,\overset{0}{\Sigma}_2(k+p)\Big]\Big\}+\\ &+\ \int\frac{dp}{(2\pi)^3}\Big\{\Big[\int\frac{dk}{(2\pi)^3}\overset{0}{L}_1(k+p)s^-_{kl}(k+p)\Big]s^-_{ji}(p)\,\overset{0}{\Sigma}_1(p)+\end{aligned} \tag{24}$$

$$+\ \overset{0}{L}_1(p)\,s^-_{kl}(p)\left[\int \frac{dk}{(2\pi)^3}\, s^-_{ji}(k+p)\,\overset{0}{\Sigma}_1(k+p)\right]\Bigg\}\ ,$$

$\underline{P \xrightarrow{w} A}$

$$\begin{aligned}
Tr(\Psi O_\mu \bar{\Psi} O_\mu) \;=\; & -8(\eta^\perp_\nu)\int \frac{dp}{(2\pi)^3}\Bigg\{\left[\int \frac{dk}{(2\pi)^3}\,\overset{0}{L}_2(k+p)\,c^+_{kl}(k+p)\right] c^-_{ji}(p)\,\overset{0}{A}_1{}^{\nu}(p)\; + \\
& +\ \overset{0}{L}_2(p)\,c^+_{kl}(p)\left[\int \frac{dk}{(2\pi)^3}\, c^-_{ji}(k+p)\,\overset{0}{A}_1{}^{\nu}(k+p)\right]\Bigg\} + \\
& +\ \int \frac{dp}{(2\pi)^3}\Bigg\{\left[\int \frac{dk}{(2\pi)^3}\,\overset{0}{L}_1(k+p)\,s^-_{kl}(k+p)\right] s^+_{ji}(p)\,\overset{0}{A}_2{}^{\nu}(p)\; + \\
& +\ \overset{0}{L}_1(p)\,s^-_{kl}(p)\left[\int \frac{dk}{(2\pi)^3}\, s^+_{ji}(k+p)\,\overset{0}{A}_2{}^{\nu}(k+p)\right]\Bigg\}\ ,
\end{aligned} \qquad (25)$$

$\underline{P \xrightarrow{we} \gamma}$

$$\begin{aligned}
& Tr(\Psi O_\mu \bar{\Psi} O_\mu) = \\
& = 16e\left\{(\eta_\nu \cdot \epsilon^*_\nu)\int \frac{dp}{(2\pi)^3}[\overset{0}{L}_2(p)\,c^+_{kl}] + \int \frac{dp}{(2\pi)^3}(p^\perp_\nu \cdot \epsilon^*_\nu)[\overset{0}{L}_1(p)\,s^-_{kl}]\right\}\ ,
\end{aligned} \qquad (26)$$

$\underline{S \xrightarrow{we} \gamma}$

$$\begin{aligned}
& Tr(\Psi O_\mu \bar{\Psi} O_\mu) = \\
& = -16e\left\{(\eta_\nu \cdot \epsilon^*_\nu)\int \frac{dp}{(2\pi)^3}[\overset{0}{\Sigma}_2(p)\,c^-_{kl}] + \int \frac{dp}{(2\pi)^3}(p^\perp_\nu \cdot \epsilon^*_\nu)[\overset{0}{\Sigma}_1(p)\,s^+_{kl}]\right\}\ ,
\end{aligned} \qquad (27)$$

$\underline{V \xrightarrow{we} \gamma}$

$$\begin{aligned}
& Tr(\Psi O_\mu \bar{\Psi} O_\mu) = \\
& = -16e\epsilon^*_\nu\left\{\int \frac{dp}{(2\pi)^3}[\overset{0}{V}_1{}^{\mu}(p)\,c^-_{kl}] + \int \frac{dp}{(2\pi)^3}(\eta_\nu p^\perp_\mu + \eta_\mu p^\perp_\nu)[\overset{0}{V}_2{}^{\mu}(p)\,s^+_{kl}]\right\}\ ,
\end{aligned} \qquad (28)$$

$\underline{A \xrightarrow{we} \gamma}$

$$\begin{aligned}
& Tr(\Psi O_\mu \bar{\Psi} O_\mu) = \\
& = 16e\epsilon^*_\nu\left\{\int \frac{dp}{(2\pi)^3}[\overset{0}{A}_1{}^{\mu}(p)\,c^+_{kl}] + \int \frac{dp}{(2\pi)^3}(\eta_\nu p^\perp_\mu + \eta_\mu p^\perp_\nu)[\overset{0}{A}_2{}^{\mu}(p)\,s^-_{kl}]\right\}\ .
\end{aligned} \qquad (29)$$

Transitions	$V_{ik}V^*_{lj}$	ji	kl
$\bar{B}^0_d \to D^0$	$V_{ub}V^*_{cd}$	bd	cu
$\bar{B}^0_d \to \bar{D}^0$	$V_{cb}V^*_{ud}$	bd	uc
$\bar{B}^0_d \to T^0_u$	$V_{ub}V^*_{td}$	bd	tu
$\bar{B}^0_d \to \bar{T}^0_u$	$V_{tb}V^*_{ud}$	bd	ut
$\bar{B}^0_d \to T^0_c$	$V_{cb}V^*_{td}$	bd	tc
$\bar{B}^0_d \to \bar{T}^0_c$	$V_{tb}V^*_{cd}$	bd	ct

Table 1.

For these transitions $(\overset{0}{\underline{L}}_i)$ corresponds to $(\overset{0}{\underline{L}}_i)_{B^0_d}$ and $(\overset{0}{L}_i)$ corresponds to wave functions in the final state.

Transitions	$V_{ik}V^*_{lj}$	ji	kl
$\bar{B}^0_s \to D^0$	$V_{ub}V^*_{cd}$	bs	cu
$\bar{B}^0_s \to \bar{D}^0$	$V_{cb}V^*_{ud}$	bs	uc
$\bar{B}^0_s \to T^0_u$	$V_{ub}V^*_{td}$	bs	tu
$\bar{B}^0_s \to \bar{T}^0_u$	$V_{tb}V^*_{ud}$	bs	ut
$\bar{B}^0_s \to T^0_c$	$V_{cb}V^*_{td}$	bs	tc
$\bar{B}^0_s \to \bar{T}^0_c$	$V_{tb}V^*_{cd}$	bs	ct

Table 2.

For these transitions $(\overset{0}{\underline{L}}_i)$ corresponds to $(\overset{0}{\underline{L}}_i)_{B^0_s}$ and $(\overset{0}{L}_i)$ corresponds to wave functions in the final state.

5. Conclusion

In this paper general expressions for decay constants of quarkonia (D and B mesons) and the weak transition matrix elements for mesonic transitions in bilocal frame are obtained.

The numerical calculations using some kind of "realistic" potentials (may be one should examine the simple Coulomb-plus-linear potential at first) will be considered in the next papers. We think that even a nonrelativistic limit is good enough for a test of the perspectives of this approach. We have the opinion that they are quite good. First numerical investigations based on the proposed framework supported this conclusion [10].

Transitions	$V_{ik}V_{lj}^{*}$	ji	kl
$D^0 \to \bar{K}^0$	$V_{cs}V_{ud}^{*}$	uc	sd
$D^0 \to K^0$	$V_{cd}V_{us}^{*}$	uc	ds
$D^0 \to B_d^0$	$V_{cb}V_{ud}^{*}$	uc	bd
$D^0 \to \bar{B}_d^0$	$V_{cd}V_{ub}^{*}$	uc	db
$D^0 \to B_s^0$	$V_{us}V_{cb}^{*}$	uc	bs
$D^0 \to \bar{B}_s^0$	$V_{cs}V_{ub}^{*}$	uc	sb

Table 3.

For these transitions $(\overset{0}{\underline{L}_i})$ corresponds to $(\overset{0}{\underline{L}_i})_{D^0}$ and $(\overset{0}{L_i})$ corresponds to wave functions in the final state.

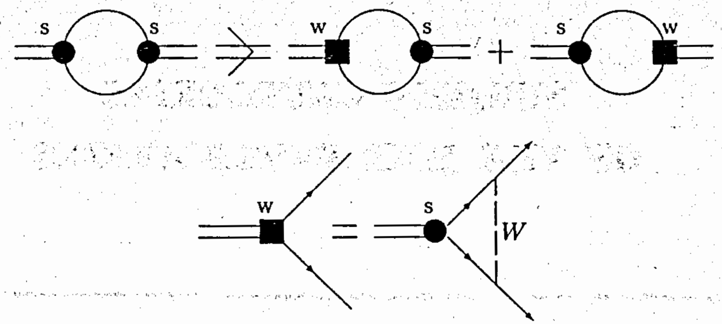

Figure 1.

The diagrams of weak transitions $\mathcal{M} \xrightarrow{w} \mathcal{M}$ in the bilocal approach.

References

[1] V.N.Pervushin, Nucl.Phys.B (Proc.Suppl.) **15**(1990)197.
V.N.Pervushin, Atoms and Hadrons in Gauge Theories, Course of Lectures, JINR E2-91-349, Dubna, 1991.

[2] V.N.Pervushin, Yu.L.Kalinovsky, W.Kallies, N.A.Sarikov, Fortschr. Phys. **38**(1990)323.

[3] Yu.L.Kalinovsky et. al. Sov. J. Nucl. Phys. **49**(1989)1709.

[4] Yu.L.Kalinovsky, L.Kaschluhn, V.N.Pervushin, Phys.Lett. **231B**(1989)288; Fortschr.Phys. **38**(1990)355.
Yu.L.Kalinovsky, W.Kallies, L.Kaschluhn, L.Münchow, V.N.Pervushin, N.A.Sarikov, Few- Body System **10**(1991)87.

[5] Yu.L.Kalinovsky, W.Kallies, L.Münchow, V.N.Pervushin, Weak interactions in a bilocal chiral theory. Part1. Bound state wave functions for given angular momentum, JINR E4-91-501,Dubna,1991.

[6] M.A.Markov, J.Phys. USSR **3**(1940)452;
H.Yukawa, Phys.Rev. **77**(1949)219.

[7] Nguyen Suan Han, V.N.Pervushin, Fortschr.Phys. **3**(1986)611.

[8] V.N.Pervushin, Riv. Nuovo Cim. **8**(1985)10,1.

[9] Nguyen Suan Han, V.N.Pervushin, Preprint CERN TH.5866, 1990.

[10] R.Horvat, D. Kekez, D. Klabučar, D. Palle, Phys.Rev. **D44**(1991)1585.

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