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I.V.Molodtsova*, S.I.Bastrukov*, V.M.Shilov

THE WIDTHS OF MAGNETIC TWIST MODES

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*Saratov State University, 410071 Saratov, Russia

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Молодцова И.В., Баструков С.И., Шилов В.М. E4-92-542
Ширины магнитных твисторных мод

Дается флюид-динамическое описание ширин магнитных резонансов, связанных с длинноволновыми крутильно-подобными колебаниями сферического ядра. Предполагается, что механизм диссипации энергии ядерного коллективного движения обусловлен нуклон-нуклонными столкновениями, приводящими к вязкости. Вычисленная ширина магнитного резонанса, как функция массового числа и мультипольности, имеет вид $\Gamma(M\lambda) = 6{,}2(2\lambda + 3)\ (\lambda - 1)A^{-2/3}$ МэВ.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Molodtsova I.V., Bastrukov S.I., Shilov V.M. E4-92-542
The Widths of Magnetic Twist Modes

Within the dissipative nuclear fluid-dynamics the predictions are presented for the widths of magnetic twist resonances associated with the long wavelength torsion like vibrations of a spherical nucleus. The mechanism of nuclear dissipation is presumed to be caused by the individual nucleon collisions with each other resulting in shear viscosity. The magnetic resonance width as a function of mass number and multipole degree is found to be $\Gamma(M\lambda) = 6.2(2\lambda + 3)\ (\lambda - 1)A^{-2/3}$ MeV.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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1 Preface

In papers [1-3] it has been developed the macroscopic theory of magnetic twist resonances associated with the normal long wavelength torsion-like vibrations of an incompressible nucleus. The method regarded in these articles allowed one to estimate the energy and excitation probability as functions of mass number and multipole degree. Similar problem has been solved in [4-7] in the model of an elastic compressible Fermi-globe without resort to the long wavelength approximation (see also [8,9]). The problem of excitations of these collective modes by inelastically scattered electrons is considered in [10-12]. However the dissipation of transverse collective motions resulting in magnetic resonances has not been discussed in the literature. The purpose of this paper is to analyze the fluid-dynamical approach to this problem based on the two-body mechanism of nuclear dissipation.

The damping mechanism under consideration implies that nuclear dissipation originates from nucleon collisions with each other. As is known from the macroscopic theory of continuum, the two-body dissipation is described in terms of viscous stress tensor. The effect of twisting or shearing viscosity has a volume origin and is characterized by coefficient of dynamical viscosity μ which apriori is unknown. However, as it is pointed out in [13], this coefficient may be adjusted from the data on the kinetic energy of fission fragments. The numerical value for the coefficient of dynamical viscosity given in this paper is

$$\mu = 0.03 \pm 0.01\,\mathrm{TP}, \quad 1\ \mathrm{TP} = 0.948\,\hbar/\mathrm{fm}^3,$$

where abbreviation TP stands for terapoise [14]. In our calculations we will use the coefficient of kinematical (density independent) viscosity ν which is defined as follows $\nu = \mu/\rho_o$, where ρ_o is the equilibrium density of nuclear matter. The used by us method of calculations is relayed on the results of papers [13-15] where presented profound physical arguments for the description of the nuclear damping mechanism in term of two-body viscosity.

In what follows we consider a dissipative fluid dynamical model to obtain an estimate for the magnetic resonance width as a function of mass number, multipole degree and energy.

2 Dissipative nuclear fluid-dynamics

The macroscopic treatment of dissipative nuclear fluid-dynamics is the following. Starting point is the Lagrange equation governing the small-amplitude collective vibrations. The dissipation is described by supplementary term with the Reyleigh's dissipation function F [15,16], so that the basic dynamical equation takes the form

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\alpha}_\lambda} - \frac{\partial L}{\partial \alpha_\lambda} + \frac{\partial F}{\partial \dot{\alpha}_\lambda} = 0 \qquad (2.1)$$

The Lagrangian L of simple-harmonic oscillations is given by

$$L = \frac{B_\lambda(\dot{\alpha}^\lambda)^2}{2} - \frac{C_\lambda(\alpha^\lambda)^2}{2}. \qquad (2.2)$$

The inertia B_λ and stiffness C_λ for the long-wavelength torsion-like motions we write down in the form [17]

$$B_\lambda = M\frac{\lambda(\lambda+1)}{(2\lambda+1)} < r^{2\lambda} >, \quad C_\lambda = \frac{Mv_F^2}{5}\lambda(\lambda^2-1) < r^{2\lambda-2} >, \qquad (2.3)$$

which explicitly take into account the diffuse of nuclear edge. In the sharp edge approximation the value of B_λ and C_λ have been derived in [2]. For $< r^\lambda >$ we use the standard definition of average radius of the order of λ which can be found in [18]. By M is denoted the nucleus mass and v_F stands for the Fermi-velocity.

The Reyleigh's dissipation function F is defined as (see, for instance, eq.(3.8) in [15])

$$F = -\frac{\dot{\alpha}_\lambda^2}{4}\nu \int \rho \left(\frac{\partial a_i^\lambda}{\partial x_j} + \frac{\partial a_j^\lambda}{\partial x_i}\right)^2 d\tau. \qquad (2.4)$$

As in refs.[2,13], by $\mathbf{a}_t^\lambda(\mathbf{r})$ we denote the field of instantaneous displacements and $\alpha_\lambda(t)$ is interpreted as a collective time-dependent amplitude in accord with the Bohr and Mottelson treatment of nuclear vibrations. It is convenient to represent the latter equation in the form

$$F = \frac{\dot{\alpha}^2}{2} D_\lambda, \quad \text{where} \quad D_\lambda = \frac{\nu}{2} \int \rho \left(\frac{\partial a_i^\lambda}{\partial x_j} + \frac{\partial a_j^\lambda}{\partial x_i} \right)^2 d\tau, \qquad (2.5)$$

is the friction coefficient. Substitution of (2.2) and (2.5) into (2.1) yields

$$B_\lambda \ddot{\alpha}_\lambda + D_\lambda \dot{\alpha}_\lambda + C_\lambda \alpha_\lambda = 0, \qquad (2.6)$$

which is the standard equation of damped harmonic oscillator. The energy E_λ and width $\Gamma(M\lambda)$ of a resonance are computed as follows

$$E(M\lambda) = \hbar (C_\lambda / B_\lambda)^{1/2}, \quad \Gamma(M\lambda) = \hbar D_\lambda / B_\lambda. \qquad (2.7)$$

The toroidal field of displacement $\mathbf{a}^\lambda(\mathbf{r})$ which corresponds to the long wavelength torsion-like oscillations resulting in the $M\lambda$ twist resonances is given by [2]

$$\mathbf{a}^\lambda(\mathbf{r}) = \mathrm{rot}\, \mathbf{r}\, r^\lambda P_\lambda(\theta), \qquad (2.8)$$

where $P_\lambda(\theta)$ are the Legendre polynomials. Upon substituting this field into (2.5) we find

$$D_\lambda = M \nu < r^{2\lambda-2} > \lambda(\lambda^2 - 1). \qquad (2.9)$$

The width of $M\lambda$ twist mode is given by

$$\Gamma(M\lambda) = \hbar\nu(2\lambda + 1)(\lambda - 1) \frac{< r^{2\lambda-2} >}{< r^{2\lambda} >}. \qquad (2.10)$$

Taking into account that the energy of twist eigenmode (when the real density is presumed) is given by [17]

$$E(M\lambda) = \hbar \left[\frac{1}{5} v_F^2 (2\lambda + 1)(\lambda - 1) \frac{< r^{2\lambda-2} >}{< r^{2\lambda} >} \right]^{1/2}, \qquad (2.11)$$

we can recalculate $\Gamma(M\lambda)$ in terms of E_λ. As a result we obtain

$$\Gamma(M\lambda) = \frac{5\nu}{\hbar v_F^2} [E(M\lambda)]^2\, \mathrm{MeV}^{-1}. \qquad (2.12)$$

3 Results and discussion

As a first application of the above presented picture we estimate the magnetic resonance width as function of mass number and multipole degree by use of the sharp edge approximation. In this approximation eq.(2.10) take the form

$$\Gamma(M\lambda) = \frac{\hbar\mu}{\rho_o R^2} (2\lambda + 3)(\lambda - 1) = \frac{5\mu}{\hbar v_F^2} [E(M\lambda)]^2\, \mathrm{MeV}^{-1}, \qquad (3.1)$$

where $\rho_o = 3m/(4\pi r_o)$ and $v_F = \hbar/(2mr_o)(9\pi)^{1/3}$. Using the following set of constants

$$r_o = 1.25\,\mathrm{fm}, \; \hbar = 197.32858\,\mathrm{MeV\,fm/c}, \; m = 931.5016\,\mathrm{MeV/c^2}$$

and μ given in Sect.1 the width is given by

$$\Gamma(M\lambda) = 6.23\,(2\lambda + 3)(\lambda - 1)\, A^{-2/3}\,\mathrm{MeV}, \qquad (3.2)$$

so, the two-body damping mechanism relults in the $A^{-2/3}$ dependence of the magnetic resonance width on mass number.

In ref.[2] it is found that the energy of $M\lambda$ twist resonance is given by

$$E(M\lambda) = \frac{\hbar v_F}{R} \left[\frac{1}{5} (2\lambda + 3)(\lambda - 1) \right]^{1/2}. \qquad (3.3)$$

Noticing the similar multipole dependence of both energy and width, we can represent $\Gamma(M\lambda)$ in term of $E(M\lambda)$. As a result we obtain

$$\Gamma(M\lambda) = \frac{5\mu}{\rho\hbar v_F^2} [E(M\lambda)]^2\, \mathrm{MeV}^{-1} = 1.87 \cdot 10^{-2}\, [E(M\lambda)]^2\, \mathrm{MeV}^{-1}. \qquad (3.4)$$

Equations (3.2) and (3.2) are the basic predictions of the paper.

It is interesting to compare the above presented estimates with those for isoscalar electric resonances obtained by Nix and Sierk [13] for the same macroscopic damping mechanism. The result of [13] reads

$$\Gamma(E\lambda) = \frac{2\hbar\mu}{\rho_o R^2}(2\lambda+1)(\lambda-1). \tag{3.5}$$

It worth to notice that the kinetic approach to the damping problem of electric giant resonances developed in refs.[19,20] also leads to the $A^{-2/3}$ mass number dependence of $E(\lambda)$ resonance width. Comparing the spreading widths of magnetic (3.4) and electric (3.5) resonances we find

$$\frac{\Gamma(M\lambda)}{\Gamma(E\lambda)} = \frac{1}{2}\cdot\frac{(2\lambda+3)}{(2\lambda+1)} < 1, \quad \text{for} \quad \lambda \geq 2. \tag{3.6}$$

Thus we found that the width of magnetic resonance turns out always lower than the width of electric resonance.

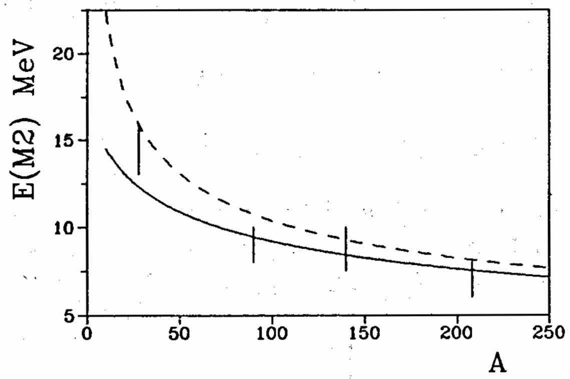

Fig. 1: Calculated and experimental [21,22] energies E(M2) of magnetic quadrupole resonance vs. mass number A. Full curve, calculation with Fermi distribution; broken curve, step densite calculation.

In Figs.1 and 2 the energy and width of quadrupole twist mode as function of mass number are pictured. These figures clearly

display the effect of the nuclear edge diffuse which is to reduce the absolute values of both the energy of resonance and its width. For the electric resonances this effect is stressed in [23,24].

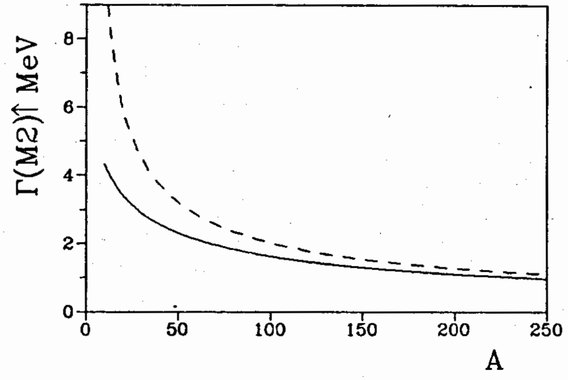

Fig. 2: Computed width Γ(M2)↑ of magnetic quadrupole resonance vs. mass number A. Full curve, calculation with Fermi distribution; broken curve, step densite calculation.

Table 1: Energy E(M2) and width Γ(M2)↑ magnetic quadrupole resonance.

Nucleus	**Energy E(M2), MeV**		**Width Γ(M2)↑, MeV**
	Exp. [21, 22]	**Theory**	**Theory**
^{28}**Si**	13-16	10.5-13.5	2.6-2.7
^{90}**Zr**	8-10	8-10	1.5-1.6
^{140}**Ce**	7.5-10	7.5-9.5	1.2-1.3
^{208}**Pb**	6-8	6.5-8.5	1.0-1.1

In table 1, the list of $E(M2)$ and $\Gamma(M2)$ for several specific spherical nuclei is presented. It is seen that the nuclear fluid-dynamics yields a reasonable description of energies. As to the spreading magnetic resonance widths is concerned, the data by now are completely lacking to be compared with the above presented predictions. In recent paper [25], the first calculations of $M1$ resonance width for deformed nuclei have been performed and found that the width is approximately proportional to square of energy. This conclusion well agrees with the predictions inferred in the present paper for the magnetic resonances with $\lambda \geq 2$ which, as we beleave, will be subjected to experimental tests during the coming years.

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